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Enhanced Mean Estimation Using Memory Type Estimators With Dual Auxiliary Variables

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Abstract:

- The study investigates how Exponentially Weighted Moving Average (EWMA) is mathematically expressed, what parameter values work best for mean estimation with different data types, and how well it detects slight changes in the process mean. This study suggests a family of memory-type estimators that use two auxiliary variables to estimate the population mean. The suggested class of estimators' mean squared error has been calculated up to the first degree of approximation. Compared to estimators in the literature that use two auxiliary variables, the suggested memory type estimators are found to be more effective. To support the theoretical findings, an empirical and simulation investigation are conducted.

Keywords:

- *exponentially weighted moving average statistic, mean estimation, mean squared error, study variable, two auxiliary variables*

AMS Subject Classification:

- 49A05, 78B26.

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1. INTRODUCTION

A crucial component of survey sampling and statistical inference is estimating the population mean, median and mode , which offers information about a population's central tendency. The efficiency and accuracy of estimation can be greatly increased when auxiliary variables provide information about the population. While utilizing a single auxiliary variable can increase estimation efficiency, doing so with two auxiliary variables can further improve the estimation process, especially in the following situations:

1. Both auxiliary variables have a strong correlation with the study variable.
2. The auxiliary variables offer complementary information, which means they capture distinct dimensions or aspects of the study variable.

The robustness will be improved if the second variable can make up for the lesser correlation between the two auxiliary variables and the study variable. Using two auxiliary variables will increase precision because there is a higher reduction in variance. There are several applications for using two auxiliary variables. For example, in agriculture, it is used to estimate crop yield using rainfall and soil quality as auxiliary variables; in public health, it is used to estimate disease prevalence using socioeconomic and demographic factors; and in economics, it is used to estimate income distribution using employment status and income level as auxiliary variables, among other applications. Lu and Yan (2014) proposed a class of ratio estimators of a finite population mean using two auxiliary variable and Singh and Sharma (2015) suggested a class of exponential ratio estimators of finite population mean using two auxiliary variables. In stratified random sampling, Kumar et al. (2018) suggested a simulation study on estimation of population mean using two auxiliary variables. Triveni and Danish (2024) suggested an efficient population mean estimation via stratified sampling with dual auxiliary information. Shukla et al. (2012), Tailor and Tailor (2008), Kadilar and Cingi (2005) and Abu-Dayyeh et al. (2003) also introduced estimation of population mean using two auxiliary variables. Awan and Shabbir (2014) gave an optimum regression estimator for population mean using two auxiliary variables in simple random sampling. Singh and Nigam (2022) proposed a generalized class of estimators for finite population mean using two auxiliary variables in sample surveys. Grover and Kaur (2021) suggested an improved regression type estimator of population mean with two auxiliary variables and its variant using robust regression method. Sharma and Singh (2014) improved ratio type estimator under second order approximation and Verma et al. (2015) suggested some families of estimators in stratified random sampling using two auxiliary variables.
An Exponentially Weighted Moving Average (EWMA) is a statistical method that gives more weight to recent data points in order to smooth data and identify patterns over time. Older data are given exponentially decreasing weights by the EWMA, which makes it more sensitive to recent changes than a simple moving average (SMA), which gives all observations equal weight. These estimators can greatly increase the efficiency of survey estimates by exploiting auxiliary information and previous data, resulting in reduced mean squared errors when compared to typical estimators. These methods are adaptable to a wide range of survey types and complex survey designs, including those with unequal probability sampling and missing data. This paper focuses on the use of EWMA for mean estimation, emphasizing its

capacity to give reliable and adaptive estimates in dynamic contexts. First, Noor-ul Amin (2020) suggested memory type ratio and product estimators for population mean for time-based surveys then Aslam et al. (2020) proposed memory type ratio and product estimators in stratified sampling. Aslam et al. (2023) gave memory type ratio and product estimators under ranked-based sampling schemes. Noor-ul Amin et al. (2022) suggested variable acceptance sampling plan based on hybrid exponentially weighted moving averages. Aslam et al. (2024) and Noor-ul Amin (2021) also suggested memory type estimators of population mean using exponentially weighted moving averages for time scaled surveys. Sharma et al. (2024) discussed estimation procedures for population mean using EWMA for time scaled survey. Zahid et al. (2023) suggested combination of memory type ratio and product estimators under extended EWMA statistic with application to wheat production. Although using two auxiliary variables to estimate population mean has many benefits, there is a substantial research gap regarding the use of the EWMA statistic in this context. To the best of our knowledge, no studies have looked into this approach, which uses the EWMA statistic to estimate population mean using two auxiliary variables.

In section 1, we first introduced mean estimation, followed by estimation utilizing two auxiliary variables and the EWMA statistic. Then, in section 2 we defined the estimators in the literature for estimation of population mean using two auxiliary variables. We have proposed a generalized family of memory type estimators in section 3. In section 4, we compared the efficiency of the estimators. In section 5, we have done an empirical study and in section 6, a comprehensive simulation study is conducted to compare the efficiency of the estimators. In section 7, we discussed the outcomes of tables and figures. Finally in the last section that is in section 8 we discussed the conclusion.

The EWMA statistic to estimate the population mean based on sample mean for $t > 0$ for the study variable and auxiliary variables are defined as

$V_t = \delta\bar{y}_t + (1-\delta)V_{t-1}$, $W_t = \delta\bar{x}_t + (1-\delta)W_{t-1}$ and $Z_t = \delta\bar{z}_t + (1-\delta)Z_{t-1}$

where δ is weight given to the data known as weight parameter or smoothing constant of current sample observation.

$$E(V_t) = \mu_y \tag{1.1}$$

$$\begin{aligned}
Var(V_t) &= Var\left[\delta\bar{y}_t + (1-\delta)\left(\delta\bar{y}_{t-1} + (1-\delta)V_{t-2}\right)\right] \\
&= Var\left[\delta\bar{y}_t + (1-\delta)\delta\bar{y}_{t-1} + (1-\delta)^2\delta\bar{y}_{t-2} + \cdots\right] \\
&= Var(\bar{y}_t)\delta^2\left[1 + (1-\delta)^2 + (1-\delta)^4 + \cdots\right] \\
&= Var(\bar{y}_t)\delta^2\sum_{n=0}^{\infty}(1-\delta)^{2n}. \qquad (1.2)
\end{aligned}$$

The limiting variance of EWMA statistic V_t is given by

$$Var(V_t) = \frac{\delta^2 Var(\bar{y}_t)}{2\delta - \delta^2} = \Delta Var(\bar{y}_t) \tag{1.3}$$

similarly,

$$E(W_t) = \mu_x \tag{1.4}$$

$$Var(W_t) = \frac{\delta^2 Var(\bar{x}_t)}{2\delta - \delta^2} = \Delta Var(\bar{x}_t) \tag{1.5}$$

$$\mathrm{E}(Z_t) = \mu_z \tag{1.6}$$

$$Var(Z_t) = \frac{\delta^2 \mathrm{Var}(\bar{z}_t)}{2\delta - \delta^2} = \Delta Var(\bar{z}_t) \tag{1.7}$$

where, $\Delta = \frac{\delta}{2-\delta}$.
To derive the MSE of the proposed memory type estimators, we consider the following notations:

$$V_t = \bar{Y}(1 + e_{0t})\ ,\ W_t = \bar{X}(1 + e_{1t}) \text{ and } Z_t = \bar{Z}(1 + e_{2t})$$

such that

$$\begin{aligned}
&E(e_{0t}) = E(e_{1t}) = E(e_{2t}) = 0\\
&E(e_{0t}^2) = \Delta f C_y^2\\
&E(e_{1t}^2) = \Delta f C_x^2\\
&E(e_{2t}^2) = \Delta f C_z^2\\
&E(e_{0t}e_{1t}) = \Delta f \rho_{yx} C_y C_x\\
&E(e_{0t}e_{2t}) = \Delta f \rho_{yz} C_y C_z\\
&E(e_{1t}e_{2t}) = \Delta f \rho_{xz} C_x C_z
\end{aligned} \tag{1.8}$$

where, $f = \frac{1}{n} - \frac{1}{N}$, $C_y^2 = \frac{S_y^2}{\bar{Y}^2}$, $C_x^2 = \frac{S_x^2}{\bar{X}^2}$, $C_z^2 = \frac{S_z^2}{\bar{Z}^2}$ are the coefficient of variation of Y, X and Z.
$S_y^2 = \sum_{i=1}^{N}(Y_i - \bar{Y})^2/(N-1)$, $S_x^2 = \sum_{i=1}^{N}(X_i - \bar{X})^2/(N-1)$ and $S_z^2 = \sum_{i=1}^{N}(Z_i - \bar{Z})^2/(N-1)$ are the population variances of study variable and auxiliary variables respectively.

$\rho_{yx} = \frac{S_{yx}}{\sqrt{S_y^2 S_x^2}}$, $\rho_{yz} = \frac{S_{yz}}{\sqrt{S_y^2 S_z^2}}$ and $\rho_{xz} = \frac{S_{xz}}{\sqrt{S_x^2 S_z^2}}$ are the correlation coefficients between Y and X, Y and Z and X and Z respectively.

2. REVIEW OF ESTIMATORS

Singh (1967) suggested a combination of ratio and product estimator of $\bar{Y}$ using two auxiliary variables for the estimation of population mean as

$$t_1 = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{z}}{\bar{Z}}\right) \tag{2.1}$$

$$\mathrm{MSE}(t_1) = f\bar{Y}^2\left[C_y^2 + C_x^2(1 - 2\rho_{yx}) + C_z^2(1 + 2(\rho_{yz} - \rho_{xz}))\right]. \tag{2.2}$$

Abu-Dayyeh et al. (2003) suggested the following two forms of estimators of a finite population mean using two auxiliary variables.

$$t_2 = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)^{\gamma_1}\left(\frac{\bar{z}}{\bar{Z}}\right)^{\gamma_2} \tag{2.3}$$

$$\text{MSE}_{min}(t_2) = f\bar{Y}^2 C_y^2 \left(1 - \frac{\rho_{yx}^2 + \rho_{yz}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz}}{1 - \rho_{xz}^2}\right). \tag{2.4}$$

The optimum values of γ_1 and γ_2 is given by;

$$\begin{aligned} \gamma_1 &= \frac{-C_y(\rho_{yx} - \rho_{yz}\rho_{xz})}{C_x(1 - \rho_{xz}^2)} \\ \gamma_2 &= \frac{-C_y(\rho_{yz} - \rho_{yx}\rho_{xz})}{C_z(1 - \rho_{xz}^2)} \end{aligned} \tag{2.5}$$

$$t_3 = \lambda_1 \bar{y} \left(\frac{\bar{x}}{\bar{X}}\right)^{\gamma_1} + \lambda_2 \bar{y} \left(\frac{\bar{z}}{\bar{Z}}\right)^{\gamma_2} \tag{2.6}$$

where, $\lambda_1 + \lambda_2 = 1$

$$\begin{aligned} \text{MSE}(t_3) \;=\; & f\bar{Y}^2 \Big(C_y^2 + \lambda_1^2\gamma_1^2 C_x^2 + \lambda_2^2\gamma_2^2 C_z^2 - 2\lambda_1\gamma_2\rho_{yx}C_yC_x \\ & -2\gamma_1\lambda_2\rho_{yz}C_yC_z + 2\lambda_1\lambda_2\gamma_1\gamma_2\rho_{xz}C_xC_z \Big) \end{aligned} \tag{2.7}$$

where,

$$\lambda_1 = \frac{\gamma_2^2 C_z^2 - \gamma_1\rho_{yz}C_yC_z + \gamma_2\rho_{yx}C_yC_x - \gamma_1\gamma_2\rho_{xz}C_xC_z}{\gamma_1^2C_x^2 + \gamma_2^2C_z^2 - 2\gamma_1\gamma_2\rho_{xz}C_xC_z} \text{ and } \lambda_2 = 1 - \lambda_1. \tag{2.8}$$

Lu and Yan (2014) suggested a class of multivariate ratio estimators using information of two auxiliary variables.

$$t_4 = k_1 \bar{y} \left(\frac{a_1\bar{X} + b_1}{a_1\bar{x} + b_1}\right) + k_2 \bar{y} \left(\frac{a_2\bar{Z} + b_2}{a_2\bar{z} + b_2}\right) \tag{2.9}$$

where, $k_1 + k_2 = 1$

$$\begin{aligned} \text{MSE}(t_4) \;=\; & f\bar{Y}^2 \Big(C_y^2 + k_1^2\alpha_1^2 C_x^2 + k_2^2\alpha_2^2 C_z^2 - 2k_1\alpha_1\rho_{yx}C_yC_x \\ & -2k_2\alpha_2\rho_{yz}C_yC_z + 2k_1k_2\alpha_1\alpha_2\rho_{xz}C_xC_z \Big) \end{aligned} \tag{2.10}$$

where,

$$k_1 = \frac{\alpha_2^2C_z^2 + \alpha_1\rho_{yx}C_yC_x - \alpha_2\rho_{yz}C_yC_z - \alpha_1\alpha_2\rho_{xz}C_xC_z}{\alpha_1^2C_x^2 + \alpha_2^2C_z^2 - 2\alpha_1\alpha_2\rho_{xz}C_xC_z} \text{and } k_2 = 1 - k_1 \tag{2.11}$$

$$\alpha_1 = \frac{a_1\bar{X}}{a_1\bar{X} + b_1}, \alpha_2 = \frac{a_2\bar{Z}}{a_2\bar{Z} + b_2}. \tag{2.12}$$

3. PROPOSED ESTIMATOR

To the best of our knowledge no one has proposed the memory type estimator using two auxiliary variables for the estimation of population mean. So, we have looked into this approach and proposed the enhanced generalized family of memory type estimator for the estimation of population mean using two auxiliary variables as

$$t_a = V_t\left[r_1\left(\frac{W_t}{\bar{X}}\right)^{\psi}\exp\left(\frac{p(\bar{X}-W_t)}{p(\bar{X}+W_t)+2q}\right)+r_2\left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right] \tag{3.1}$$

where, r_1 and r_2 are suitable constants to be determined such that the MSE of t_a is minimum, p, ψ and q are either real numbers or functions of the known parameters of auxiliary variables and $r_1 + r_2 \neq 1$, expressing in terms of e_{0t}, e_{1t} and e_{2t} we get,

$$\begin{aligned} t_a &= \bar{Y}(1+e_{0t})\left[r_1\left(\frac{\bar{X}(1+e_{1t})}{\bar{X}}\right)^{\psi}\exp\left(\frac{p(\bar{X}-\bar{X}(1+e_{1t}))}{p(\bar{X}+\bar{X}(1+e_{1t}))+2q}\right)\right. \\ &\quad \left.+r_2\left[1+\log\left(\frac{\bar{Z}(1+e_{2t})}{\bar{Z}}\right)\right]\right] \\ &= \bar{Y}(1+e_{0t})\left[r_1(1+e_{1t})^{\psi}\exp\left(\frac{-ue_{1t}}{2+ue_{1t}}\right)+r_2\left[1+\log(1+e_{2t})\right]\right] \end{aligned} \tag{3.2}$$

where $u = \frac{p\bar{X}}{p\bar{X}+q}$, expanding using Taylor's series expansion upto first order approximation we have

$$\begin{aligned} t_a-\bar{Y} &= (r_1+r_2-1)\bar{Y}+r_1\bar{Y}\left(e_{0t}+e_{1t}(\psi-\tfrac{u}{2})+e_{1t}^2\left(\tfrac{3u^2}{8}-\tfrac{\psi u}{2}+\tfrac{\psi(\psi-1)}{2}\right)+e_{0t}e_{1t}(\psi-\tfrac{u}{2})\right) \\ &\quad +r_2\bar{Y}\left(e_{0t}+e_{2t}+e_{0t}e_{2t}-\tfrac{e_{2t}^2}{2}\right) \end{aligned} \tag{3.3}$$

$$\begin{aligned} (t_a-\bar{Y})^2 &= \bar{Y}^2+r_1^2\bar{Y}^2\left(1+e_{0t}^2+e_{1t}^2\left(\psi-\frac{u}{2}\right)^2+4e_{0t}e_{1t}\left(\psi-\frac{u}{2}\right)\right. \\ &\quad \left.+2e_{1t}^2\left(\frac{3u^2}{8}-\frac{\psi u}{2}+\frac{\psi(\psi-1)}{2}\right)\right) \\ &\quad +r_2^2\bar{Y}^2\left(1+e_{0t}^2+4e_{0t}e_{2t}\right) \\ &\quad +2r_1r_2\bar{Y}^2\left(1+e_{0t}^2+2e_{0t}e_{2t}+e_{0t}e_{1t}\left(\psi-\frac{u}{2}\right)\right. \\ &\quad \left.+e_{1t}e_{2t}\left(\psi-\frac{u}{2}\right)+e_{1t}^2\left(\frac{3u^2}{8}-\frac{\psi u}{2}+\frac{\psi(\psi-1)}{2}\right)-\frac{e_{2t}^2}{2}\right) \\ &\quad -2r_1\bar{Y}^2\left(1+e_{1t}^2\left(\frac{3}{8}u^2-\frac{\psi u}{2}+\frac{\psi(\psi-1)}{2}\right)+e_{0t}e_{1t}\left(\psi-\frac{u}{2}\right)\right) \\ &\quad -2r_2\bar{Y}^2\left(1+e_{0t}e_{2t}+\frac{e_{2t}^2}{2}\right) \end{aligned} \tag{3.4}$$

taking expectations both sides we have;

$$\text{MSE}(t_a) = \bar{Y}^2+r_1^2\alpha+r_2^2\beta+2r_1r_2\gamma-2r_1\theta-2r_2\phi \tag{3.5}$$

where,

$$\alpha = \bar{Y}^2 \left[1 + \Delta f \left(C_y^2 + C_x^2 \left(\psi - \frac{u}{2}\right)^2 + 4\rho_{yx} C_y C_x \left(\psi - \frac{u}{2}\right)\right.\right.$$
$$\left.\left. + 2C_x^2 \left(\frac{3u^2}{8} - \frac{\psi u}{2} + \frac{\psi(\psi - 1)}{2}\right)\right)\right], \tag{3.6}$$

$$\beta = \bar{Y}^2 \left[1 + \Delta f \left(C_y^2 + 4\rho_{yz} C_y C_z\right)\right], \tag{3.7}$$

$$\gamma = \bar{Y}^2 \left[1 + \Delta f \left(C_y^2 + 2\rho_{yz} C_y C_z + \rho_{yx} C_y C_x \left(\psi - \frac{u}{2}\right)\right.\right.$$
$$\left.\left. + \rho_{xz} C_x C_z \left(\psi - \frac{u}{2}\right) + C_x^2 \left(\frac{3u^2}{8} - \frac{\psi u}{2} + \frac{\psi(\psi - 1)}{2}\right) - \frac{C_z^2}{2}\right)\right], \tag{3.8}$$

$$\theta = \bar{Y}^2 \left[1 + \Delta f \left(C_x^2 \left(\frac{3u^2}{8} - \frac{\psi u}{2} + \frac{\psi(\psi - 1)}{2}\right) + \rho_{yx} C_y C_x \left(\psi - \frac{u}{2}\right)\right)\right], \tag{3.9}$$

$$\phi = \bar{Y}^2 \left[1 + \Delta f \left(\rho_{yz} C_y C_z + \frac{C_z^2}{2}\right)\right] \tag{3.10}$$

differentiating with respect to r_1 and r_2 and equating it with zero is given by;

$$r_1 = \frac{\beta\theta - \gamma\phi}{\alpha\beta - \gamma^2} \tag{3.11}$$

$$r_2 = \frac{\alpha\phi - \gamma\theta}{\alpha\beta - \gamma^2} \tag{3.12}$$

substituting the values of r_1 and r_2, the minimum MSE is given by;

$$\mathrm{MSE}_{\min}(t_a) = \bar{Y}^2 - \left(\frac{\alpha\phi^2 + \beta\theta^2 - 2\gamma\theta\phi}{\alpha\beta - \gamma^2}\right). \tag{3.13}$$

A set of estimators generated from t_a using suitable values of r_1, r_2, ψ, p and q are listed above in Table 1 and this table contains conventional parameters like quartile deviation Q_2, coefficient of variation C_x and coefficient of skewness $B_1(x)$.

Subset of proposed estimator	r_1	r_2	ψ	p	q
$t_{a(1)} = V_t$	1	0	0	0	0
$t_{a(2)} = V_t\left[\left(\frac{W_t}{\bar{X}}\right) + \left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	1	1	1	0	1
$t_{a(3)} = V_t\left[\left(\frac{W_t}{\bar{X}}\right)^{\psi} + \left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	1	1	ψ	0	1
$t_{a(4)} = V_t\left[\left(\frac{W_t}{\bar{X}}\right)^{-1} + V_t\left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	1	1	-1	0	1
$t_{a(5)} = V_t\left[1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right]$	0	1	1	1	1
$t_{a(6)} = V_t \exp\left(\frac{\bar{X}-W_t}{\bar{X}+W_t}\right)$	1	0	0	1	0
$t_{a(7)} = V_t\left[r_1\left(\frac{W_t}{\bar{X}}\right) + r_2\left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	r_1	r_2	1	0	1
$t_{a(8)} = V_t\left[r_1 \exp\left(\frac{\bar{X}-W_t}{\bar{X}+W_t}\right) + r_2 V_t\left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	r_1	r_2	0	1	0
$t_{a(9)} = r_1 V_t \exp\left(\frac{(\bar{X}-W_t)}{(\bar{X}+W_t)+2}\right)$	r_1	0	0	1	1
$t_{a(10)} = V_t\left[r_1\left(\frac{W_t}{\bar{X}}\right)\exp\left(\frac{(\bar{X}-W_t)}{(\bar{X}+W_t)+2}\right) + V_t\left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	r_1	1	1	1	1
$t_{a(11)} = V_t\left[\left(\frac{W_t}{\bar{X}}\right)\exp\left(\frac{(\bar{X}-W_t)}{(\bar{X}+W_t)}\right) + r_2 V_t\left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	1	r_2	1	1	0
$t_{a(12)} = r_2 V_t\left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)$	0	r_2	0	0	0
$t_{a(13)} = r_1 V_t \exp\left(\frac{\bar{X}-W_t}{\bar{X}+W_t}\right)$	r_1	0	0	1	0
$t_{a(14)} = V_t\left[r_1 \exp\left(\frac{\bar{X}-W_t}{(\bar{X}+W_t)+2}\right) + \left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	r_1	1	0	1	1
$t_{a(15)} = r_1 V_t\left(\frac{W_t}{\bar{X}}\right)$	r_1	0	1	0	0
$t_{a(16)} = V_t\left(\frac{W_t}{\bar{X}}\right)$	1	0	1	0	0
$t_{a(17)} = V_t\left(\frac{W_t}{\bar{X}}\right)^{-1}$	1	0	-1	0	0
$t_{a(18)} = V_t\left(\frac{W_t}{\bar{X}}\right)^{-\psi}$	1	0	$-\psi$	0	0
$t_{a(19)} = r_1 V_t\left(\frac{W_t}{\bar{X}}\right)^{\psi}$	r_1	0	ψ	0	0
$t_{a(20)} = V_t\left[r_1\left(\frac{W_t}{\bar{X}}\right)^{\psi} + \left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	r_1	1	ψ	0	1
$t_{a(21)} = V_t\left[Q_2 + \left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	Q_2	1	0	0	0
$t_{a(22)} = V_t\left[Q_2\left(\frac{W_t}{\bar{X}}\right)\exp\left(\frac{\bar{X}-W_t}{\bar{X}+W_t}\right) + \left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	Q_2	1	1	1	0
$t_{a(23)} = Q_2 V_t\left(\frac{W_t}{\bar{X}}\right)\exp\left(\frac{\bar{X}-W_t}{\bar{X}+W_t}\right)$	Q_2	0	1	1	0
$t_{a(24)} = Q_2 V_t\left[1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right]$	0	Q_2	0	1	1
$t_{a(25)} = V_t\left[\left(\frac{W_t}{\bar{X}}\right) + Q_2\left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	1	Q_2	1	0	1
$t_{a(26)} = V_t\left[r_1 \exp\left(\frac{\bar{X}-W_t}{(\bar{X}+W_t)+2}\right) + Q_2 V_t\left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	r_1	Q_2	0	1	1
$t_{a(27)} = C_x V_t\left(\frac{W_t}{\bar{X}}\right)$	C_x	0	1	0	0
$t_{a(28)} = V_t\left[C_x\left(\frac{W_t}{\bar{X}}\right)^{\psi}\exp\left(\frac{\bar{X}-W_t}{\bar{X}+W_t}\right) + \left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	C_x	1	ψ	1	0
$t_{a(29)} = C_x V_t\left[1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right]$	0	C_x	0	0	1
$t_{a(30)} = V_t\left[B_1(x)\left(\frac{W_t}{\bar{X}}\right)\exp\left(\frac{\bar{X}-W_t}{(\bar{X}+W_t)+2}\right) + \left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	$B_1(x)$	1	1	1	1
$t_{a(31)} = V_t\left[r_1\left(\frac{W_t}{\bar{X}}\right)^{\psi}\exp\left(\frac{\bar{X}-W_t}{\bar{X}+W_t}\right) + B_1(x)\left(1+\log\left(\frac{Z_t}{\bar{Z}}\right)\right)\right]$	r_1	$B_1(x)$	ψ	1	0

Table 1: Set of estimators generated from the class of estimator (t_a).

4. EFFICIENCY COMPARISON

We compared the efficiency of the proposed memory type estimator t_a with the estimators in the literature without EWMA for mean estimation utilizing two auxiliary variables in order to demonstrate the superiority of the suggested estimators over the others.

$$\begin{aligned} \mathrm{MSE}(t_1) - \mathrm{MSE}(t_a) &= f\bar{Y}^2\left[C_y^2 + C_x^2\left(1-2\rho_{yx}\right) + C_z^2\left(1+2(\rho_{yz}-\rho_{xz})\right)\right] \\ &\quad - \left(\bar{Y}^2 + r_1^2\alpha + r_2^2\beta + 2r_1r_2\gamma - 2r_1\theta - 2r_2\phi\right) \geq 0. \end{aligned} \tag{4.1}$$

$$\begin{aligned} \mathrm{MSE}(t_2)_{\min} - \mathrm{MSE}(t_a) &= f\bar{Y}^2C_y^2\left(1 - \frac{\rho_{yx}^2 + \rho_{yz}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz}}{1-\rho_{xz}^2}\right) \\ &\quad - \left(\bar{Y}^2 + r_1^2\alpha + r_2^2\beta + 2r_1r_2\gamma - 2r_1\theta - 2r_2\phi\right) \geq 0 \end{aligned} \tag{4.2}$$

$$\begin{aligned} \mathrm{MSE}(t_3) - \mathrm{MSE}(t_a) &= f\bar{Y}^2\left[C_y^2 + \lambda_1^2\gamma_1^2C_x^2 + \lambda_2^2\gamma_2^2C_z^2 - 2\lambda_1\gamma_1\rho_{yx}C_yC_x - 2\lambda_2\gamma_2\rho_{yz}C_yC_z\right] \\ &\quad - \left(\bar{Y}^2 + r_1^2\alpha + r_2^2\beta + 2r_1r_2\gamma - 2r_1\theta - 2r_2\phi\right) \geq 0 \end{aligned} \tag{4.3}$$

$$\begin{aligned} \mathrm{MSE}(t_4) - \mathrm{MSE}(t_a) &= f\bar{Y}^2\left[C_y^2 + k_1^2\alpha_1^2C_x^2 + k_2^2\alpha_2^2C_z^2 - 2k_1\alpha_1\rho_{yx}C_yC_x - 2k_2\alpha_2\rho_{yz}C_yC_z\right. \\ &\quad \left. +2k_1k_2\alpha_1\alpha_2\rho_{xz}C_xC_z\right] \\ &\quad - \left(\bar{Y}^2 + r_1^2\alpha + r_2^2\beta + 2r_1r_2\gamma - 2r_1\theta - 2r_2\phi\right) \geq 0. \end{aligned} \tag{4.4}$$

Equation 4.1, 4.2, 4.3 and 4.4 will always be greater than zero.

5. EMPIRICAL STUDY

To compare the performance of the proposed class of memory type estimators and the estimators in literature, three population data sets has been taken.

Population 1 : The data has been taken from the book Feng and Shi (1996) and statistics are calculated from the raw data.

Population 2 : Data has been taken from Steel and Torrie (1960). The log of leaf burn in sec is taken as the study variable Y. Potassium and chlorine percentage is taken as the auxiliary variables X and Z respectively.

Y = log of leaf burn in sec
X = potassium percentage
Z = clorine percentage

Population 3 : Data has been taken from Choudhury and Singh (2012). The number of placebo children is taken as the study variable Y. The number of paralytic polio cases in the placebo group and the number of paralytic polio cases in the 'not inoculated' group is taken as the auxiliary variables X and Z, respectively.

Y = number of 'placebo' children
X = number of paralytic polio cases in the placebo group
Z = number of paralytic polio cases in the 'not inoculated' group

Parameters	Population 1	Population 2	Population 3
N	180	30	34
n	70	6	10
$\bar{Y}$	13.9951	0.6860	4.92
$\bar{X}$	27.3981	4.6537	2.59
$\bar{Z}$	38.7167	0.8077	2.91
C_y	0.4180	0.4803	1.01232
C_x	0.4254	0.2295	1.23189
C_z	0.3339	0.7493	1.07203
ρ_{yx}	0.5630	0.1794	0.73259
ρ_{yz}	0.5273	0.4996	0.43796
ρ_{xz}	0.2589	0.4074	1.00366

Table 2: Population characteristics.

Population 1:

λ_1	λ_2	γ_1	γ_2	MSE(t_3)
1.897944147	−0.897944147	0	1	**0.452474178**
0.406682515	0.593317485	−1	0	**0.638501618**
0.102055853	0.897944147	0	−1	**0.452474178**
0.220182044	0.779817956	−1	−1	**0.722936698**
0.879691935	0.120308065	−1	1	**0.256776778**
−0.06951526	1.06951526	1	−1	**0.234692155**

Table 3: MSE of estimator (t_3) at different values of γ_1 and γ_2.

a_1	a_2	b_1	b_2	MSE(t_4)
0	1	1	1	**0.215693307**
1	0	1	1	**0.204064345**
0	1	1	0	**0.215693307**
−1	0	−1	−1	**0.204064345**
−1	0	1	−1	**0.204064345**
0	−1	1	−1	**0.215693307**
0	−1	1	0	**0.215693307**

Table 4: MSE of estimator (t_4) at different values of a_1, a_2, b_1 and b_2.

t_a				
$\boldsymbol{\delta}$	**0.2**	**0.5**	**0.7**	**0.9**
$\boldsymbol{\alpha}$	195.8923612	195.9514357	196.005966	196.0803255
$\boldsymbol{\beta}$	195.9519495	196.1302006	196.2947401	196.519112
$\boldsymbol{\gamma}$	195.9132834	196.0142023	196.1073581	196.2343888
$\boldsymbol{\theta}$	195.8662071	195.8729731	195.8792188	195.8877355
$\boldsymbol{\phi}$	195.8873974	195.9365441	195.9819103	196.0437732
$\boldsymbol{r_1}$	0.984594605	0.984005659	0.983462069	0.98272089
$\boldsymbol{r_2}$	0.015270251	0.015588929	0.015883065	0.016284116
MSE($\boldsymbol{t_a}$)	**0.022763519**	**0.068269094**	**0.110248845**	**0.167454675**

Table 5: MSE of proposed estimator (t_a) at different values of δ.

Population 2:

$\boldsymbol{\lambda_1}$	$\boldsymbol{\lambda_2}$	$\boldsymbol{\gamma_1}$	$\boldsymbol{\gamma_2}$	MSE($\boldsymbol{t_3}$)
0.6991	0.3009	1	1	**0.012604807**
1.0352	−0.0352	0	1	**0.014518489**
3.4137	−2.4137	−1	0	**0.006997223**
0.9648	0.0352	0	−1	**0.014518489**
1.3743	−0.3743	−1	−1	**0.016094696**
1.1019	−0.1019	−1	1	**0.020275662**
0.5727	0.4273	1	−1	**0.008777011**

Table 6: MSE of estimator (t_3) at different values of γ_1 and γ_2.

$\boldsymbol{a_1}$	$\boldsymbol{a_2}$	$\boldsymbol{b_1}$	$\boldsymbol{b_2}$	MSE($\boldsymbol{t_4}$)
1	1	1	1	**0.011103102**
0	1	1	1	**0.010861877**
1	0	1	1	**0.014008924**
0	1	1	0	**0.010861877**
−1	1	1	1	**0.011271042**
−1	1	−1	1	**0.011103102**
−1	−1	−1	−1	**0.011103102**
−1	0	−1	−1	**0.014008924**
−1	0	1	−1	**0.014008924**
1	−1	1	−1	**0.011103102**
0	−1	1	−1	**0.010861877**
0	−1	1	0	**0.010861877**

Table 7: MSE of estimator (t_4) at different values of a_1, a_2, b_1 and b_2.

t_a				
$\boldsymbol{\delta}$	**0.2**	**0.5**	**0.7**	**0.9**
$\boldsymbol{\alpha}$	0.47229578	0.475695341	0.478833397	0.483112565
$\boldsymbol{\beta}$	0.477218435	0.490463305	0.502689339	0.519361203
$\boldsymbol{\gamma}$	0.473832298	0.480304894	0.486279598	0.494426922
$\boldsymbol{\theta}$	0.470664768	0.470802305	0.470929263	0.471102386
$\boldsymbol{\phi}$	0.47380669	0.480228069	0.486155496	0.49423835
$\boldsymbol{r_1}$	0.120512971	0.097529431	0.076386945	0.047668717
$\boldsymbol{r_2}$	0.873192905	0.88362208	0.893215845	0.906247233
MSE($\boldsymbol{t_a}$)	**0.000150151**	**0.000338794**	**0.000381361**	**0.000237016**

Table 8: MSE of proposed estimator (t_a) at different values of δ.

Population 3:

$\boldsymbol{\lambda_1}$	$\boldsymbol{\lambda_2}$	γ_1	γ_2	MSE($\boldsymbol{t_3}$)
0.3132	0.6868	-1	0	**3.121005343**
0.2050	0.7950	0	-1	**3.632242825**
0.7266	0.2734	-1	1	**0.54254996**
0.2042	0.7958	1	-1	**1.711681397**

Table 9: MSE of estimator (t_3) at different values of γ_1 and γ_2.

$\boldsymbol{a_1}$	$\boldsymbol{a_2}$	$\boldsymbol{b_1}$	$\boldsymbol{b_2}$	MSE($\boldsymbol{t_4}$)
0	1	1	1	**1.415200544**
1	0	1	1	**0.811273057**
0	1	1	0	**1.415200544**
-1	1	1	1	**1.598161655**
-1	0	-1	-1	**0.811273057**
-1	0	1	-1	**0.811273057**
0	-1	1	-1	**1.415200544**
0	-1	1	0	**1.415200544**

Table 10: MSE of estimator (t_4) at different values of a_1, a_2, b_1 and b_2.

t_a				
$\boldsymbol{\delta}$	**0.2**	**0.5**	**0.7**	**0.9**
$\boldsymbol{\alpha}$	26.59415131	31.36965394	35.77781021	41.7889324
$\boldsymbol{\beta}$	24.7619172	25.87295161	26.89852183	28.29702667
$\boldsymbol{\gamma}$	25.36209907	27.67349722	29.80709551	32.71654772
$\boldsymbol{\theta}$	24.71868208	25.74324623	26.68899776	27.97865894
$\boldsymbol{\phi}$	24.40573345	24.80440035	25.17240057	25.67421905
$\boldsymbol{r_1}$	-0.451303419	-0.44474598	-0.438678911	-0.430383815
$\boldsymbol{r_2}$	1.44785782	1.434396723	1.421942255	1.404914097
MSE$(\boldsymbol{t_a})$	**0.02599369**	**0.076254698**	**0.120600458**	**0.177889707**

Table 11: MSE of proposed estimator (t_a) at different values of δ.

		MSE		
Estimators	$\boldsymbol{\delta}$	**Population1**	**Population2**	**Population3**
$\boldsymbol{t_1}$		0.55274	0.05832	0.28675
$\boldsymbol{t_2}$		0.20406	0.01085	0.86728
$\boldsymbol{t_3}$		0.23469	0.00699	0.54255
$\boldsymbol{t_4}$		0.20406	0.01086	0.81127
$\boldsymbol{t_a}$	**0.2**	0.02276	0.00015	0.02599
	0.5	0.06827	0.00034	0.07625
	0.7	0.11025	0.00038	0.12060
	0.9	0.16745	0.00024	0.17789

Table 12: MSEs of estimators for different populations.

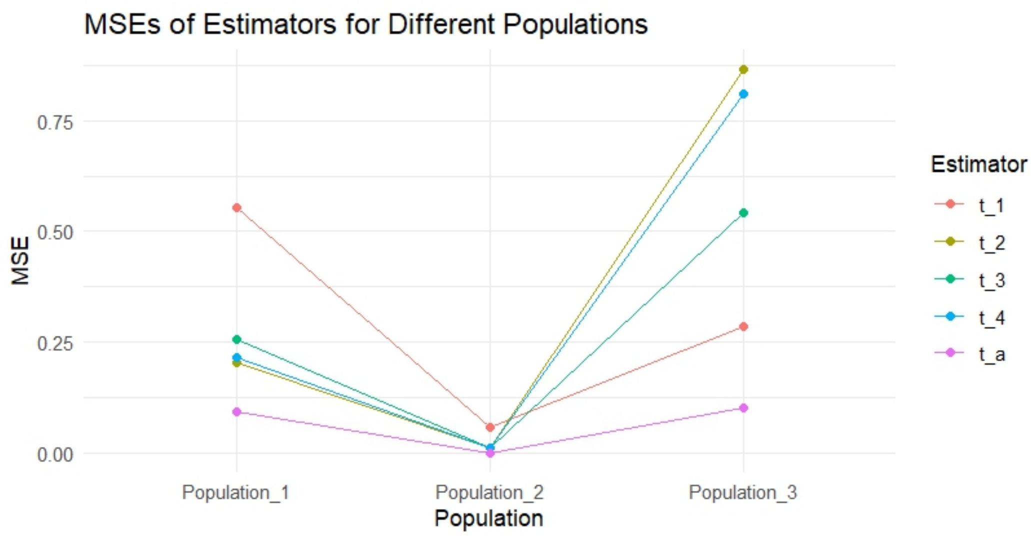

Figure 1: MSEs of estimators for different populations

6. SIMULATION STUDY

A comprehensive simulation study is carried out to assess the effectiveness of the suggested estimator over the estimators in literature. The following steps are used to compute the mean squared error (MSE) and bias of the proposed estimator and the estimators in literature.

a) Generate a population of size $N = 1000$. Let the mean vector $\mu = [50\ 30\ 20]$ for the variables Y, X and Z and standard deviation vector $\sigma = [10\ 5\ 5]$.

b) Select samples by using SRSWOR of size $n = 50, 100, 200, 500$.

c) To observe the effect of the smoothing constant, we have employed a range of δ values i.e., $\delta = 0.01, 0.05, 0.10, 0.50$.

d) MSE for each estimator is obtained by Equation 6.1 and Bias for each estimator is obtained by Equation 6.2.
Compute the Bias and Mean Squared Error as

$$\mathbf{Bias(k)} = \frac{1}{1000}\sum_{i=1}^{1000}(k_i - \bar{y}) \tag{6.1}$$

$$\mathbf{MSE(k)} = \frac{1}{1000}\sum_{i=1}^{1000}(k_i - \bar{y})^2 \tag{6.2}$$

where, $k = t_1, t_2, t_3, t_4$ and t_a

Estimators	**Sample size (n)**	$\boldsymbol{\delta}$	**Bias**	**MSE**
$\boldsymbol{t_1}$	50		0.0075	5.2262
	100		0.0040	2.0959
	200		0.0159	1.0405
	500		−0.0016	0.2910
$\boldsymbol{t_2}$	50		0.0318	6.8404
	100		−0.0507	2.8423
	200		0.0246	1.3570
	500		−0.0005	0.3504
$\boldsymbol{t_3}$	50		0.0076	3.7761
	100		−0.0429	1.5930
	200		0.0199	0.7590
	500		−0.0002	0.1954
$\boldsymbol{t_4}$	50		−0.0107	1.6115
	100		−0.0247	0.7019
	200		0.0157	0.3316
	500		0.0002	0.0874
$\boldsymbol{t_a}$	50	0.01	−0.0127	0.0099
		0.05	−0.1085	0.0641
		0.10	0.0558	0.1476
		0.50	−0.0415	1.0598
	100	0.01	0.0051	0.0039
		0.05	0.0072	0.0445
		0.10	0.0475	0.0851
		0.50	0.0268	0.4520
	200	0.01	0.0083	0.0019
		0.05	0.0559	0.0163
		0.10	0.0149	0.0286
		0.50	0.0177	0.2183
	500	0.01	−0.0028	0.0005
		0.05	−0.0008	0.0031
		0.10	0.0052	0.0108
		0.50	−0.0050	0.0564

Table 13: MSE of the estimators at different sample sizes.

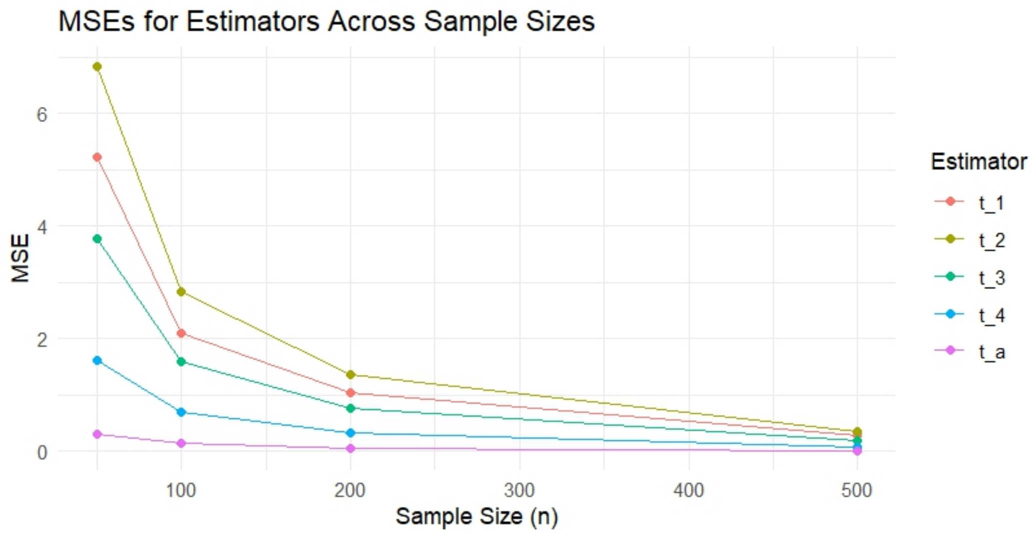

Figure 2: MSEs of estimators for different populations

7. DISCUSSION

Table 2 summarizes the population characteristics of the three populations. Table 3-Table 11 shows the calculated findings for mean squared errors. Table 12 shows a comparison of the suggested memory type estimator to estimators from the literature. The MSE of the estimator (t_3) at different values of γ_1 and γ_2 is calculated in Table 3, Table 6 and Table 9 for the population 1, 2 and 3 respectively. In Table 4, Table 7 and Table 10, the MSE of estimator (t_4) is calculated at different values of a_1, a_2, b_1 and b_2 for the population 1, 2 and 3 respectively. The MSE of the proposed memory type estimator (t_a) at different values of δ is calculated in Table 5, Table 8 and Table 11 for the population 1, 2 and 3 respectively. Table 3 shows that when $\gamma_1 = 1$ and $\gamma_2 = -1$, the estimator (t_3) has the optimal MSE value, Table 6 yields the optimum MSE value of the estimator (t_3) when $\gamma_1 = -1$ and $\gamma_2 = 0$ and Table 9 shows that the optimum MSE value of the estimator (t_3) occurs when $\gamma_1 = -1$ and $\gamma_2 = 1$ for the population 1, 2 and 3 respectively. Table 4 shows the optimum MSE value of the estimator t_4 when $a_1 = 1$, $a_2 = 0$, $b_1 = 1$ and $b_2 = 1$, Table 7 yields the optimum MSE value of the estimator (t_4) when $a_1 = 0$, $a_2 = 1$, $b_1 = 1$ and $b_2 = 0$ and from Table 10 it can be seen that the optimum MSE value of the estimator (t_4) is when $a_1 = -1$, $a_2 = 0$, $b_1 = 1$ and $b_2 = -1$ for the population 1, 2 and 3 respectively.

The MSEs of the proposed class of memory type estimators for each δ value are smaller than those of the standard estimator for predicting population mean utilizing two auxiliary variables of the three populations, as shown in Table 12. Every population taken into consideration has higher MSE values as the δ values increases from 0.01 to 0.50. The smoothing constant δ is employed to make use of both past and current observations. Table 12 illustrates the impact of δ on efficiency. When the value of δ decreases, the efficiency of the suggested EWMA estimator increases, as shown in Table 12.

Figure 1 displays the MSEs of the proposed class of memory type estimator and the estimators in the literature for the three populations, namely Population 1, 2, and 3. In this figure, t_a represents the proposed class of memory type estimator, and the graph clearly shows that the proposed class of estimator has less MSE than other estimators in the literature, implying that the proposed class of memory type estimator is more efficient than estimators in literature using two auxiliary variables.

Table 13 has been obtained by simulation study, it shows the MSEs of the estimators in literature at different sample sizes ($n = 50, 100, 200, 500$) and the MSE of the proposed class of memory type estimator at different values of δ ($\delta = 0.01, 0.05, 0.10, 0.50$) and different sample sizes ($n = 50, 100, 200, 500$). It can be seen from Table 13 that as the sample size increases for an estimator, the value of MSE decreases and for the proposed class of memory type estimator as the value of δ increases, the value of MSE also increases. Here, δ lies between 0 and 2. When δ is high, EWMA places more weight on the current observation and less on past observations. As a result, the smoothed value is more sensitive to random fluctuations and responsive to recent changes, which increases variability and leading to higher MSE. This supports the concept of the EWMA and sampling. Figure 2 represents the MSEs of estimators at different sample sizes and as we can see that the proposed class of memory type estimator has less MSEs as compared to the estimators in literature. So, the proposed class of memory type estimator is more efficient.

8. CONCLUSIONS

This study expands on the classic EWMA framework by including two auxiliary variables that increase mean estimation precision and accuracy. Auxiliary variables are correlated with the variable of interest and give additional information that improves the effectiveness of estimate processes. The suggested method combines EWMA's adaptability with the predictive capacity of auxiliary variables to provide a robust estimator capable of detecting tiny changes in the process mean. In order to estimate the population mean using two auxiliary variables, we proposed a class of memory type estimators. It has been theoretically demonstrated that the proposed class of memory type estimators are more efficient than Abu-Dayyeh et al. (2003), Singh (1967) and Lu and Yan (2014) standard estimators. Additionally, the outcomes of three numerical cases meet these theoretical requirements. The results emphasize how crucial EWMA is for uses including environmental monitoring, financial forecasting, and industrial quality control. This study advances our knowledge of adaptive statistical techniques and how they function in real-time decision-making.

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