REVSTAT – Statistical Journal
Volume 0, Number 0, Month 0000, 000–000
https://doi.org/00.00000/revstat.v00i0.000

# On study of correlated measurement errors using ranked set sampling based logarithmic imputation methods

Authors: Shashi Bhushan
– Department of Statistics, University of Lucknow, Lucknow, India, 226007
shashi.py@gmail.com

Anoop Kumar ✉
– Department of Statistics, Central University of Haryana, Mahendergarh, India, 123031
anoop.asy@gmail.com

Received: Month 0000 Revised: Month 0000 Accepted: Month 0000

Abstract:

- A small amount of research has been done to assess the measurement errors in the presence of missing data using a few sampling approaches, but no research has been done to assess the correlated measurement errors in the presence of missing data. The principal objective of this research is to suggest certain logarithmic imputation methods and the resulting estimators when there are missing data in ranked set sampling (RSS), given that the data are tainted by correlated measurement errors. The developed resulting estimators' mean square error is determined to the first order approximation. A thorough simulation experiment using an artificial population is done to evaluate the effectiveness of the suggested imputation methods and the accompanying derived estimators. The findings suggest that the proposed imputation methods and the resulting estimators repress the traditional imputation methods and the resulting estimators. The proposed imputation methods are also supplied with a real-life data application.

Keywords:

- *Logarithmic imputation methods; Correlated measurement errors; Simulation; Application.*

AMS Subject Classification:

- 49A05, 78B26.

✉ Corresponding author

## 1. INTRODUCTION

In the conventional theory of survey sampling, it is presumed during the data collection that the observed value is the actual value of the data; but in practise, these assumptions are not met, and the observed value of the data fluctuates with the measurement errors. The reader is referred to Murthy, 1967 and Cochran, 1968 for the thorough study of measurement errors. The discrepancy between the variable's true and observed values is known as the measurement error which is arises due to the faulty instrument, faulty experimental techniques, interviewers, participants, and data interpreters used. The effects of measurement error mixing with data on the statistical characteristics of the estimates of parameter are typically not covered in books. The effects of measurement error in linear and nonlinear regression modelling have been covered by Cheng and Van Ness, 1999 and Carroll et al., 2006 in their books, but not in the context of sample surveys. Shalabh, 1997 examined the impact of measurement errors on the ratio estimators. Later on, the impact of the measurement errors was examined by several authors by proposing various improved and modified estimators for the parameters of interest. Manisha and Singh, 2000 investigated a class of estimators of population mean under measurement errors. Sahoo et al., 2006 proposed the regression and ratio estimators under the case of measurement errors. Singh and Karpe, 2009 introduced the estimation procedure of the ratio and product of two population means utilizing auxiliary information in the presence of measurement errors. Tariq et al., 2021 proposed the variance estimators in the presence of measurement errors utilizing auxiliary information, while, Tariq et al., 2022 developed a generalized variance estimator utilizing auxiliary information in the presence and absence of measurement errors. Bhushan et al., 2023b suggested novel logarithmic type estimators in the case of measurement errors.

All these authors assumed that the measurement errors in the study and auxiliary variables are independent, but assuming measurement errors in both study and auxiliary variables may be inadequate because the same surveyor collects the data on both the variables. As a result, they will be correlated and dependent, and the underlying comportment of the data may be accountable for this dependability in the measurement errors. Shalabh and Tsai, 2017 was the pioneer who developed the concept of correlated measurement errors. They examined the effect of correlated measurement errors over the efficiency of the traditional ratio and product estimators of population mean. Bhushan et al., 2023d evaluated the performance of the novel logarithmic estimators under correlated measurement errors, while, Bhushan et al., 2023c developed some classes of robust estimators to assess the effect of correlated measurement errors. Kumar et al., 2023 studied the impact of correlated measurement errors utilizing some efficient classes of estimators.

McIntyre, 1952 developed the concept of ranked set sampling to efficiently estimate the production of the mean pasture. It has been proved an efficient alternative of the simple random sampling. It can be used where the ranking of the observations is much easier than obtaining their exact values. These situations frequently occur in the fields of medicine, forestry, environmental monitoring, reliability estimation, etc. Halls and Dell, 1966 investigated the RSS for forage yields. Chen et al., 2007 developed improved procedures for disease prevalence estimation under RSS. Das et al., 2018 introduced the Bayesian estimation of measles vaccination coverage under RSS. Zamanzade, 2019 developed EDF-based tests of exponentiality under pair RSS. Zamanzade et al., 2020 suggested an efficient estimation of cumulative distribution function utilizing moving extreme RSS with application to reliability. Apart from this, the ranked set sampling has also been applied to almost all statistical problems including estimation of the population mean, population variance, population proportion, cumulative distribution function, statistical quality control, etc. Al-Omari and Bouza, 2014 proposed ratio estimators of population mean with missing values under RSS. Alam et al., 2022 proposed estimation of population variance under RSS utilizing ratio of auxiliary information with study variable. Al-Omari and Haq, 2012 suggested improved quality control charts for monitoring the process mean under double RSS. Ahmed et al., 2019 suggested predictive estimation of population mean under RSS.

In various decision making problems, a complete structure of the data must be required to draw the valid and effective conclusions. But situations may arise when the structure of the data are incomplete and if the conclusion is drawn it will be invalid and inherently affect the statistical properties such as unbiasedness and efficiency. Imputation is a well-known technique to dealt with the missing data. There are several imputation methods that have been proposed by different authors for the estimation of the parameter of interests. Lee et al., 1994 suggested variance estimation in survey sampling with imputed values. Heitjan and Basu, 1996 discriminated between 'missing at random' and 'missing completely at random' approaches. Bhushan and Pandey, 2018 examined optimality of ratio type estimation methods for population mean in presence of missing data. Yadav and Prasad, 2023 suggested exponential methods of estimation under robust regression quantile regression methods. Bhushan et al., 2022 developed the estimation of population mean in presence of missing data under simple random sampling. But no work is available to dealt with issue of missing data provided the data are tainted with the correlated measurement errors.

In this study, we have three objectives. The first objective is to construct the fundamental theory under missing completely at random for the ranked set sampling in the case of missing data provided the data are tainted with the correlated measurement errors. The second objective is to adapt imputations such as the usual mean, ratio, logarithmic imputation methods and to propose Searls type logarithmic imputation methods for estimating the population mean without altering the initial responses while imputing the missing values. The third objective is to study the effect of the correlated measurement errors on the performance of the adapted and suggested imputation methods.

In Section 2, we develop the theory and terminology considered throughout this article. In Section 3, we adapt some fundamental imputations such as mean, ratio, and logarithmic imputation methods. In Section 4, we suggest Searls type logarithmic imputation methods and their properties to estimate the population mean as well as to evaluate the impact of the correlated measurement errors under ranked set sampling. In Section 5, we establish the efficiency comparison to study the performance of theimputation methods. Section 6 presents a thorough simulation and the key notes of the simulation results, while the application of the adapted and proposed methods is presented in Section 7. The conclusion is presented in Section 8.

## 2. Methodology and notation

The methodology and notations are described under following subsections.

### 2.1. Methodology

Correlated measurement errors represent systematic biases or inaccuracies in measurements that consistently occur throughout multiple data points or observations. These errors may stem from different sources, namely, instrumental bias, environmental conditions, observer bias, sampling bias, or data processing errors. These errors may occur in different real-life situations across different fields. Some related examples are discussed below:

- In medical diagnosis, the correlated measurement errors may occur when several diagnostic tests for the same condition produce consistent but inaccurate results. For example, if multiple blood tests for a certain disease consistently produce higher or lower values due to a systematic error in the testing equipment or procedure, it may lead to correlated measurement errors across patients.
- The correlated measurement errors may arise in climate data collection when multiple weather stations in a region are affected by the same environmental factors, like urban heat islands or nearby industrial activities. If these factors systematically bias temperature or precipitation measurements across stations, it can lead to correlated measurement errors in the recorded climate data.
- In financial reporting, the correlated measurement errors may occur when companies within the same industry adopt similar accounting practices or face similar economic challenges. For instance, if multiple companies overstate their revenues due to aggressive revenue recognition policies during economic downturns, it can lead to correlated measurement errors in financial statements across the industry.
- In educational testing, the correlated measurement errors may arise when students from similar socio-economic backgrounds or educational backgrounds perform consistently better or worse on standardized tests due to factors unrelated to their actual knowledge or abilities. For example, if students from affluent neighborhoods receive better preparation or resources for standardized tests compared to students from disadvantaged communities, it can lead to correlated errors in test scores across schools or districts.
- The correlated measurement errors may occur in market research surveys when respondents from the same demographic group or geographic region provide systematically biased responses due to shared cultural norms or experiences. For example, if respondents from a particular age group tend to overestimate their willingness to purchase a certain product due to social pressure or peer influence, it can lead to correlated errors in market research data across surveys.

## 2.2. Notation

In the standard ranked set sampling, one initially defines a set size $H$. Then, $H^2$ sample units are randomly selected from the population and separated into $H$ sets, each having size $H$. The units in each set are given to judgment ranking without getting real measurements. The $i^{th}$ judgement order statistic is selected from the $i^{th}$ $(i = 1, 2, ..., H)$ set, and the remaining unquantified units are replaced to the population. This develops a cycle of ranked set sampling. The complete cycle may be replicated $K$ times to obtain a ranked set sample of size $n = HK$. The $i^{th}$ judgement order statistic for the $l^{th}$ cycle is $X_{[i]l}$. The ranking procedure may contain some errors, which is shown by the usage of square brackets, whereas the square brackets are changed to round brackets if the rankings are perfect. As compared to SRS of equivalent size, the RSS frequently results in more effective inference. This is due to the fact that a ranked set sample includes information from both the preliminary rankings and the quantified observations in addition to the information provided by the observations themselves.

Suppose that the true measurements on $j^{th}$ unit of the study and auxiliary variables are $X_j$ and $Y_j$, respectively. Although true measurements on these units cannot be obtained due to some reasons, they may still be measured as $x_j$ and $y_j$ using the ME $v_j$ and $u_j$ for the $j^{th}$ unit of the corresponding variables. Let $x_j = X_j + v_j$ and $y_j = Y_j + u_j$, where $j = 1, 2, ..., n$. Let $(\bar{y}, \bar{x})$ be the sample means, $(\mu_y, \mu_x)$ be the population means, $(\sigma_y^2, \sigma_x^2)$ be the population variances, $(C_y, C_x)$ be the population coefficients of variation of variables $(y, x)$, respectively, and $\rho_{xy}$ be the coefficient of correlation between variables $x$ and $y$. The MEs $(u_j, v_j)$ are also unobservable having means of $(0, 0)$, variances $(\sigma_u^2, \sigma_v^2)$, the population coefficients of variation $(C_u, C_v)$, and cor relation coefficient of $\rho_{uv}$.

Let the number of responding units be $H_1 = HP$ out of the drawn $H$ units where $P$ is the probability that $i^{th}$ respondent follow the group of the responding units $g_r$ and $(1-P)$ be the probability that $i^{th}$ respondent follow the non-responding group $\bar{g}_r$ provided that $s = g_r \cup \bar{g}_r$. Let $r = HKP$ be the responding units out of sampled $n$ units such that $n > r$. The value $Y_i,\ i \in g_r$ is observable for each unit, except for the units $i \in \bar{r}_u$ the values are missing and require imputation to construct the full structure of the data to make a correct conclusion. The known auxiliary variable population data assist to execute the imputation of missing $Y$ values. Suppose $\bar{x}_{r,rss} = \sum_{i=1}^{H_1} \sum_{l=1}^{K} x_{(i:i)l}/HKP$ and $\bar{y}_{r,rss} = \sum_{i=1}^{H_1} \sum_{l=1}^{K} y_{[i:i]l}/HKP$ are the traditional estimators of $\mu_x$ and $\mu_y$, respectively, such that $x_{(i:i)j}$ and $y_{[i:i]l}$ are the $i^{th}$ order statistics and $i^{th}$ judgement order in the $i^{th}$ sample of size $H$ in the cycle $l$ for variables $X$ and $Y$, respectively. For easiness, we have denoted $(x_{(i:i)l}, y_{[i:i]l})$ by $(x_{(i)}, y_{[i]})$, respectively. To determine the mean square error (MSE) of the proposed estimators under MEs, we use the following notations: $\bar{y}_{r,rss} = \mu_y(1+\delta_0)$, $\bar{x}_{r,rss} = \mu_x(1+\delta_1)$, and $\bar{x}_{n,rss} = \mu_x(1+\delta_2)$ provided that $E(\delta_0) = E(\delta_1) = E(\delta_2) = 0$ and

$$
\begin{aligned}
E(\delta_0^2) &= (\propto^* C_y^2 - W_y^{2*} + \propto^* C_u^2 - W_u^{2*}) = V_{02}^*,\\
E(\delta_1^2) &= (\propto^* C_x^2 - W_x^{2*} + \propto^* C_v^2 - W_v^{2*}) = V_{20}^*,\\
E(\delta_2^2) &= (\propto C_x^2 - W_x^2 + \propto C_v^2 - W_v^2) = V_{20},\\
E(\delta_0\delta_1) &= (\propto^* \rho_{xy} C_x C_y - W_{xy}^* + \propto^* \rho_{uv} C_u C_v - W_{uv}^*) = V_{11}^*,\\
E(\delta_0\delta_2) &= (\propto \rho_{xy} C_x C_y - W_{xy} + \propto \rho_{uv} C_u C_v - W_{uv}) = V_{11},\\
E(\delta_1\delta_2) &= E(\delta_2^2) = (\propto C_x^2 - W_x^2 + \propto C_v^2 - W_v^2) = V_{20},
\end{aligned}
$$

where $\propto = 1/HK$, $\propto^{*} = 1/HKP$, $C_x = S_x/\mu_x$, $C_v = S_v/\mu_x$, $C_y = S_y/\mu_y$, $C_u = S_u/\mu_y$, $W_x^2 = \sum_{i=1}^{H}(\mu_{x_{(i)}} - \mu_x)^2/H^2K\mu_x^2$, $W_x^{2*} = \sum_{i=1}^{H}(\mu_{x_{(i)}} - \mu_x)^2/H^2KP\mu_x^2$, $W_y^{2*} = \sum_{i=1}^{H}(\mu_{y_{[i]}} - \mu_y)^2/H^2KP\mu_y^2$, $W_{xy} = \sum_{i=1}^{H}(\mu_{x_{(i)}} - \mu_x)(\mu_{y_{[i]}} - \mu_y)/H^2K\mu_x\mu_y$, $W_{xy}^{*} = \sum_{i=1}^{H}(\mu_{x_{(i)}} - \mu_x)(\mu_{y_{[i]}} - \mu_y)/H^2KP\mu_x\mu_y$, $W_u^2 = \sum_{i=1}^{H}(\mu_{u_{[i]}} - \mu_u)^2/H^2K\mu_y^2$, $W_u^{2*} = \sum_{i=1}^{H}(\mu_{u_{[i]}} - \mu_u)^2/H^2KP\mu_y^2$, $W_v^2 = \sum_{i=1}^{H}(\mu_{v_{(i)}} - \mu_v)^2/H^2K\mu_x^2$, $W_v^{2*} = \sum_{i=1}^{H}(\mu_{v_{(i)}} - \mu_v)^2/H^2KP\mu_x^2$, $W_{uv_{[i]}} = \sum_{i=1}^{H}(\mu_{u_{[i]}} - \mu_y)(\mu_{x_{(i)}} - \mu_v)/H^2K\mu_u\mu_v$, $W_{uv}^{*} = \sum_{i=1}^{H}(\mu_{u_{[i]}} - \mu_y)(\mu_{x_{(i)}} - \mu_v)/H^2KP\mu_u\mu_v$, $\mu_{x_{(i)}} = E(x_{(i)})$, and $\mu_{y_{[i]}} = E(y_{[i]})$.

The above results can be easily extended from Al-Omari and Bouza (2014).

## 3. Adapted imputation methods

This part adapts some basic imputations to dealt with the missing data problems when the data are tainted with the correlated measurement errors.

### 3.1. Mean imputation method

We propose mean imputation of the population mean by extending the results of Lee et al., 1994, for single value imputation, when $y$ values of the $i^{th}$ sample unit under ranked set sampling is missing and requires imputation. The techniques of imputation for population mean are given as

$$y_{.i_m} = \begin{cases} y_{[i]} & \text{for } i \in g_r \\ \bar{y}_{r,rss} & \text{for } i \in \bar{g}_r \end{cases}$$

The resultant estimator is given by

$$t_m = \bar{y}_{r,rss}$$

The variance of the estimator $t_m$ is given by

$$Var(t_m) = \mu_y^2 V_{02}^{*} \tag{3.1}$$

Considering the additional auxiliary information into account, the imputation methods are categorized as

Strategy I: If $\mu_x$ is available and $\bar{x}_{n,rss}$ is utilized.

Strategy II: If $\mu_x$ is available and $\bar{x}_{r,rss}$ is utilized.

Strategy III: If $\mu_x$ is unavailable and $\bar{x}_{n,rss}$, $\bar{x}_{r,rss}$ are utilized.

### 3.2. Traditional ratio imputation methods

Motivated by Ahmed et al., 2006 and Shalabh and Tsai, 2017, the classical ratio imputation methods in presence of correlated measurement errors under RSS are given as

Strategy I

$$y_{.i_{r_1}} = \begin{cases} y_{[i]} & \text{for } i \in g_r \\ \frac{1}{n-r}\left[n\bar{y}_{r,rss}\frac{\mu_x}{\bar{x}_{n,rss}} - r\bar{y}_{r,rss}\right] & \text{for } i \in \bar{g}_r \end{cases}$$

Strategy II

$$y_{.i_{r_2}} = \begin{cases} y_{[i]} & \text{for } i \in g_r \\ \frac{1}{n-r}\left[n\bar{y}_{r,rss}\frac{\mu_x}{\bar{x}_{r,rss}} - r\bar{y}_{r,rss}\right] & \text{for } i \in \bar{g}_r \end{cases}$$

Strategy III

$$y_{.i_{r_3}} = \begin{cases} y_{[i]} & \text{for } i \in g_r \\ \frac{1}{n-r}\left[n\bar{y}_{r,rss}\frac{\bar{x}_{n,rss}}{\bar{x}_{r,rss}} - r\bar{y}_{r,rss}\right] & \text{for } i \in \bar{g}_r \end{cases}$$

The resulting estimators under the above strategies will be given by

$$t_{r_1} = \bar{y}_{r,rss}\frac{\mu_x}{\bar{x}_{n,rss}}$$
$$t_{r_2} = \bar{y}_{r,rss}\frac{\mu_x}{\bar{x}_{r,rss}}$$
$$t_{r_3} = \bar{y}_{r,rss}\frac{\bar{x}_{n,rss}}{\bar{x}_{r,rss}}$$

The $MSE$ equations of the above resultant estimators are given by

$$MSE(t_{r_1}) = \mu_y^2(V_{02}^* + V_{20} - 2V_{11}) \tag{3.2}$$
$$MSE(t_{r_2}) = \mu_y^2(V_{02}^* + V_{20}^* - 2V_{11}^*) \tag{3.3}$$
$$MSE(t_{r_3}) = \mu_y^2\{V_{02}^* + (V_{20}^* - V_{20}) - 2(V_{11}^* - V_{11})\} \tag{3.4}$$

## 3.3. Traditional logarithmic imputation methods

Following Bhushan et al., 2022 and Bhushan et al., 2023a, the traditional logarithmic imputation methods in the case of correlated measurement errors under RSS are as follows:
Strategy I

$$y_{.i_{l_1}} = \begin{cases} y_{[i]} & \text{for } i \in g_r \\ \frac{1}{n-r}\left\{n\bar{y}_{r,rss}\left\{1+\log\left(\frac{\bar{x}_{n,rss}}{\mu_x}\right)\right\}^{\Lambda_1} - r\bar{y}_{r,rss}\right\} & \text{for } i \in \bar{g}_r \end{cases}$$

Strategy II

$$y_{.i_{l_2}} = \begin{cases} y_{[i]} & \text{for } i \in g_r \\ \frac{1}{n-r}\left\{n\bar{y}_{r,rss}\left\{1+\log\left(\frac{\bar{x}_{r,rss}}{\mu_x}\right)\right\}^{\Lambda_2} - r\bar{y}_{r,rss}\right\} & \text{for } i \in \bar{g}_r \end{cases}$$

Strategy III

$$y_{.i_{l_3}} = \begin{cases} y_{[i]} & \text{for } i \in g_r \\ \frac{1}{n-r}\left\{n\bar{y}_{r,rss}\left\{1+\log\left(\frac{\bar{x}_{r,rss}}{\bar{x}_{n,rss}}\right)\right\}^{\Lambda_3} - r\bar{y}_{r,rss}\right\} & \text{for } i \in \bar{g}_r \end{cases}$$

The resulting estimators under the above strategies will be given by

$$t_{l_1} = \bar{y}_{r,rss}\left\{1+\log\left(\frac{\bar{x}_{n,rss}}{\mu_x}\right)\right\}^{\Lambda_1}$$
$$t_{l_2} = \bar{y}_{r,rss}\left\{1+\log\left(\frac{\bar{x}_{r,rss}}{\mu_x}\right)\right\}^{\Lambda_2}$$
$$t_{l_3} = \bar{y}_{r,rss}\left\{1+\log\left(\frac{\bar{x}_{r,rss}}{\bar{x}_{n,rss}}\right)\right\}^{\Lambda_3}$$

where $\Lambda_1$, $\Lambda_2$, and $\Lambda_3$ are duly selected scalars.
The minimum mean square error of the resultant logarithmic estimators $t_{g_1}$, $t_{g_2}$, and $t_{g_3}$ at the optimum values of $\Lambda_{1(opt)} = V_{11}/V_{20}$, $\Lambda_{2(opt)} = V_{11}^*/V_{20}^*$, and $\Lambda_{3(opt)} = (V_{11}^* - V_{11})/(V_{20}^* - V_{20})$, respectively, is expressed by

$$min.MSE(t_{l_1}) = \mu_y^2\left(V_{02}^* - \frac{V_{11}^2}{V_{20}}\right) \tag{3.5}$$

$$min.MSE(t_{l_2}) = \mu_y^2\left(V_{02}^* - \frac{V_{11}^{*2}}{V_{20}^*}\right) \tag{3.6}$$

$$min.MSE(t_{l_3}) = \mu_y^2\left\{V_{02}^* - \frac{(V_{11}^* - V_{11})^2}{(V_{20}^* - V_{20})}\right\} \tag{3.7}$$

## 4. Suggested imputation methods

The survey researchers are always intended to improve the efficiency of their proposed estimators. Searls, 1964 proposed in his noteworthy research that multiplying a tuning parameter in the estimator, improves its efficiency. Taking Searls proposal into consideration, we multiplied a tuning parameter in the traditional logarithmic imputation methods and developed Searls type logarithmic imputation methods for the estimation of population mean in the presence of missing data when the data are tainted with the correlated measurement errors. The developed imputation methods are as follows:
Strategy I

$$y_{.i_{sl_1}} = \begin{cases} y_{[i]} & \text{for } i \in g_r \\ \frac{1}{n-r}\left\{n\Psi_1\bar{y}_{r,rss}\left\{1+\log\left(\frac{\bar{x}_{n,rss}}{\mu_x}\right)\right\}^{\Lambda_1} - r\bar{y}_{r,rss}\right\} & \text{for } i \in \bar{g}_r \end{cases}$$

Strategy II

$$y_{.i_{sl_2}} = \begin{cases} y_{[i]} & \text{for } i \in g_r \\ \frac{1}{n-r}\left\{n\Psi_2\bar{y}_{r,rss}\left\{1+\log\left(\frac{\bar{x}_{r,rss}}{\mu_x}\right)\right\}^{\Lambda_2} - r\bar{y}_{r,rss}\right\} & \text{for } i \in \bar{g}_r \end{cases}$$

Strategy III

$$y_{.i_{sl_3}} = \begin{cases} y_{[i]} & \text{for } i \in g_r \\ \frac{1}{n-r}\left\{n\Psi_3\bar{y}_{r,rss}\left\{1+\log\left(\frac{\bar{x}_{r,rss}}{\bar{x}_{n,rss}}\right)\right\}^{\Lambda_3} - r\bar{y}_{r,rss}\right\} & \text{for } i \in \bar{g}_r \end{cases}$$

The resulting estimators under the above strategies will be given by

$$T_{sl_1} = \mathit{\Psi}_1 \bar{y}_{r,rss} \left\{1 + \log\left(\frac{\bar{x}_{n,rss}}{\mu_x}\right)\right\}^{\Lambda_1}$$
$$T_{sl_2} = \mathit{\Psi}_2 \bar{y}_{r,rss} \left\{1 + \log\left(\frac{\bar{x}_{r,rss}}{\mu_x}\right)\right\}^{\Lambda_2}$$
$$T_{sl_3} = \mathit{\Psi}_3 \bar{y}_{r,rss} \left\{1 + \log\left(\frac{\bar{x}_{r,rss}}{\bar{x}_{n,rss}}\right)\right\}^{\Lambda_3}$$

where $\mathit{\Psi}_j,\ j = 1, 2, 3$ and $\Lambda_j$ are appropriately selected scalars.

**Remark 4.1.** The logarithmic imputation methods $y_{.i_{l_j}}$ and the resultant logarithmic estimators $t_{l_j}$ are special cases of the Searls type logarithmic imputation methods $y_{.i_{sl_j}}$ and the resultant Searls type logarithmic estimators $T_{sl_j}$ for $\mathit{\Psi}_j = 1$.

**Theorem 4.1.** *The minimum MSE of the resultant Searls type logarithmic estimators* $T_{sl_j},\ j = 1, 2, 3$ *is stated as*

$$min.MSE(T_{sl_j}) = \mu_y^2 \left(1 - \frac{G_j^2}{F_j}\right) \tag{4.1}$$

**Proof:** Expressing the estimator $T_{sl_1}$ in terms of errors as

$$\begin{aligned}
T_{sl_1} &= \mathit{\Psi}_1 \mu_y (1 + \delta_0) \left\{1 + \log\left(\frac{\mu_x(1+\delta_2)}{\mu_x}\right)\right\}^{\Lambda_1} \\
&= \mathit{\Psi}_1 \mu_y (1 + \delta_0) \left\{1 + \delta_2 - \frac{\delta_1^2}{2} + ...\right\}^{\Lambda_1} \\
&= \mathit{\Psi}_1 \mu_y (1 + \delta_0) \left\{1 + \Lambda_1\left(\delta_2 - \frac{\delta_2^2}{2}\right) + \frac{\Lambda_1(\Lambda_1 - 1)}{2!}\delta^2\right\} \\
&= \mathit{\Psi}_1 \mu_y \left\{1 + \delta_0 + \Lambda_1\delta_2 + \Lambda_1\delta_0\delta_2 - \frac{\Lambda_1}{2}\delta_2^2 + \frac{\Lambda_1(\Lambda_1-1)}{2}\delta_2^2\right\} \qquad (4.2)
\end{aligned}$$

Subtracting $\mu_y$ both sides to (4.2) furnishes (4.3):

$$T_{sl_1} - \mu_y = \mu_y \left[\mathit{\Psi}_1 \left\{1 + \delta_0 + \Lambda_1\delta_2 + \Lambda_1\delta_0\delta_2 - \frac{\Lambda_1}{2}\delta_2^2 + \frac{\Lambda_1(\Lambda_1-1)}{2}\delta_2^2\right\} - 1\right] \tag{4.3}$$

Taking the expectation on each side of (4.3), we obtain

$$Bias(T_{sl_1}) = \mu_y \left[\mathit{\Psi}_1 \left\{1 + \left(\frac{\Lambda_1^2}{2} - \Lambda_1\right) V_{20} + \Lambda_1 V_{11}\right\} - 1\right]$$

Do square and take the expectation on each side of (4.3), we obtain

$$\begin{aligned}
MSE(T_{sl_1}) &= \mu_y^2 \begin{bmatrix} 1 + \mathit{\Psi}_1^2\{1 + V_{02}^* + 2\Lambda_1(\Lambda_1-1)V_{20} + 4\Lambda_1 V_{11}\} \\ -2\mathit{\Psi}_1\left\{1 + \left(\frac{\Lambda_1^2}{2} - \Lambda_1\right)V_{20} + \Lambda_1 V_{11}\right\} \end{bmatrix} \\
&= \mu_y^2(1 + \mathit{\Psi}_1^2 F_1 - 2\mathit{\Psi}_1 G_1) \qquad (4.4)
\end{aligned}$$

where $F_1 = 1 + V_{02}^* + 2\Lambda_1(\Lambda_1 - 1)V_{20} + 4\Lambda_1 V_{11}$ and $G_1 = 1 + \left(\frac{\Lambda_1^2}{2} - \Lambda_1\right) V_{20} + \Lambda_1 V_{11}$. Minimization of the (4.4) w.r.t. $\Lambda_1$ provides:

$$\Psi_{1(opt)} = \frac{G_1}{F_1}$$

Use of $\Psi_{1(opt)}$ in (4.4) provides:

$$minMSE(T_{sl_1}) = \mu_y^2 \left(1 - \frac{G_1^2}{F_1}\right)$$

In the same way, we can obtain the minimum MSE of the rest of the resultant proposed estimators $T_{sl_j}$, $j = 2, 3$. We may usually write

$$MSE(T_{sl_j}) = \mu_y^2(1 + \Psi_j^2 F_j - 2\Psi_j G_j) \tag{4.5}$$

where $F_2 = 1+V_{02}^*+2\Lambda_2(\Lambda_2-1)V_{20}^*+4\Lambda_2 V_{11}^*$, $G_2 = 1+\left(\frac{\Lambda_2^2}{2} - \Lambda_2\right) V_{20}^*+\Lambda_2 V_{11}^*$, $F_3 = 1+V_{02}^*+2\Lambda_3(\Lambda_3-1)(V_{20}^*-V_{20})+4\Lambda_3(V_{11}^*-V_{11})$, and $G_3 = 1+\left(\frac{\Lambda_3^2}{2} - \Lambda_3\right)(V_{20}^*-V_{20})+\Lambda_3(V_{11}^*-V_{11})$. Minimization of (4.5) w.r.t. $\Psi_j$ provides:

$$\Psi_{j(opt)} = \frac{G_j}{F_j}$$

Use of $\Psi_{j(opt)}$ in (4.5) provides:

$$minMSE(T_{sl_j}) = \mu_y^2 \left(1 - \frac{G_j^2}{F_j}\right)$$

Note that minimizing $\Psi_j$ and $\Lambda_j$ simultaneously is a tough task. Thus, putting $\Psi_j = 1$ in the respective estimators and minimizing the MSE w.r.t. $\Lambda_j$ provides the optimum values of $\Lambda_j$ as

$$\begin{aligned}
\Lambda_{1(opt)} &= -\frac{V_{11}}{V_{20}}, \\
\Lambda_{2(opt)} &= -\frac{V_{11}^*}{V_{20}^*}, \\
\Lambda_{3(opt)} &= -\frac{(V_{11}^* - V_{11})}{(V_{20}^* - V_{20})}.
\end{aligned}$$

□

**Remark 4.2.** By setting $\rho_{uv} = 0$ in the above results provides the case of uncorrelated measurement errors. These results are more extensive and all-encompassing, and they specifically contain the results of the uncorrelated measurement errors.

## 5. Efficiency conditions

In this section, the proposed Searls type logarithmic imputation methods are compared with the adapted imputation methods and the efficiency conditions are derived under the following lemmas:

**Lemma 5.1.** *The performance of the suggested estimators $T_{sl_j}$ is better than the usual mean estimator $t_m$, if*

$$
\begin{aligned}
MSE(T_{sl_j}) &< Var(t_m) \\
\implies \mu_y^2\left(1-\frac{G_j^2}{F_j}\right) &< \mu_y^2 V_{02}^* \\
\implies \frac{G_j^2}{F_j} &> 1 - V_{02}^*
\end{aligned}
$$

**Lemma 5.2.** *(i). The performance of the suggested estimators $T_{sl_1}$ is better than the ratio estimator $t_{r_1}$ under strategy I, if*

$$\frac{G_1^2}{F_1} > 1 - V_{02}^* - V_{20} + 2V_{11}$$

*(ii). The performance of the suggested estimators $T_{sl_2}$ is better than the the ratio estimator $t_{r_2}$ under strategy II, if*

$$\frac{G_2^2}{F_2} > 1 - V_{02}^* - V_{20}^* + 2V_{11}^*$$

*(iii). The performance of the suggested estimators $T_{sl_3}$ is better than the ratio estimator $t_{r_3}$ under strategy III, if*

$$\frac{G_3^2}{F_3} > 1 - V_{02}^* - (V_{20}^* - V_{20}) + 2(V_{11}^* - V_{11})$$

**Lemma 5.3.** *(i). The performance of the suggested estimators $T_{sl_1}$ is better than the traditional logarithmic estimator $t_{l_1}$ under strategy I, if*

$$\frac{G_1^2}{F_1} > 1 - V_{02}^* + \frac{V_{11}^2}{V_{20}}$$

*(ii). The performance of the suggested estimators $T_{sl_2}$ is better than the traditional logarithmic estimator $t_{l_2}$ under strategy II, if*

$$\frac{G_2^2}{F_2} > 1 - V_{02}^* + \frac{V_{11}^{2*}}{V_{20}^*}$$

*(iii). The performance of the suggested estimators $T_{sl_3}$ is better than the traditional logarithmic estimator $t_{l_3}$ under strategy III, if*

$$\frac{G_3^2}{F_3} > 1 - V_{02}^* + \frac{(V_{11}^* - V_{11})^2}{(V_{20}^* - V_{20})}$$

The suggested imputation methods perform better than the adapted imputation methods if the efficiency conditions derived under the above lemmas are satisfied. These conditions are further evaluated through a comprehensive simulation.

## 6. Simulation

A thorough simulation is undertaken on an artificial population to enhance the theoretical results as well as to see the impact of the correlated measurement errors on the adapted and proposed imputation methods and the resulting estimators. The following points delineate the simulation algorithm:

(i). A population of size $N = 1200$ is generated utilizing a four-variable multivariate normal distribution in $R$ software as $W = (X, Y, u, v)'$ with the mean vector $\mu_w = (\mu_x, \mu_y, 0, 0)'$ and covariance matrix:

$$\begin{pmatrix} \sigma_x^2 & \rho_{xy}\sigma_x\sigma_y & 0 & 0 \\ \rho_{xy}\sigma_x\sigma_y & \sigma_y^2 & 0 & 0 \\ 0 & 0 & \sigma_u^2 & \rho_{uv}\sigma_u\sigma_v \\ 0 & 0 & \rho_{uv}\sigma_u\sigma_v & \sigma_v^2 \end{pmatrix}.$$

The parameters considered to generate the population are given as: $\mu_y = 7$, $\mu_x = 16$, $\sigma_y^2 = 125$, $\sigma_x^2 = 181$, $\rho_{xy} = (-0.9,\ -0.5,\ -0.1,\ 0,\ 0.5,\ 0.9)$, $\sigma_u^2 = (5, 10)$, $\sigma_v^2 = (5, 10)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$.

(ii). Select 15000 ranked set samples of size $n = 15$ with set size $H = 3$ and number of cycle $K = 5$ from the above population by employing ranked set sampling.

(iii). Consider the samples selected in step $(ii)$, the 15,000 values of each resulting estimator are obtained.

(iv). Utilizing the parameters considered in $(i)$, the MSE of the several estimators is determined for the responding probability $P = 0.67$ (when a cycle contains one missing value) as well as for different combinations of $MEs$ such as 5%, 10%, 15%, and 20%. The MSE is calculated employing the following expression:

$$MSE = \frac{1}{15,000}\sum_{i=1}^{15,000}(T_i^* - \mu_y)^2$$

where $T_i^* = t_m,\ t_{r_1},\ t_{r_2},\ t_{r_3},\ t_{l_1},\ t_{l_2},\ t_{l_3},\ T_{sl_1},\ T_{sl_2},\ T_{sl_3}$.

Tables 1-2 contains the findings of the simulation.

### 6.1. Key notes

Table 1 and Table 2 provide the simulation findings (MSE) of the resultant adapted and suggested propositions. The key notes are construed in the following points:

(1). The findings of Table 1 are given for different combinations of $(\sigma_u^2,\ \sigma_v^2)$. For $(\sigma_u^2,\ \sigma_v^2)$=(5,5), it is observed that:

- The sequential increase in the values of $\rho_{xy}$ from 0 to 0.9 reduces the MSE of the ratio estimator $t_{r_1}$ under strategy I, (For example: it can be seen from Table 1 that for $\rho_{xy} = 0$ and $\rho_{uv} = -0.9$, the MSE of the estimator $t_{r_1}$ is 14.64, whereas for $\rho_{xy} = 0.9$ and $\rho_{uv} = -0.9$, the MSE of the estimator $t_{r_1}$ is 12.69). The sequential increase in the size and direction of $\rho_{uv}$ from $-0.9$ to 0.9 also reduces the MSE, (For example: it can be seen from Table 1 that for $\rho_{xy} = 0$, when $\rho_{uv} = -0.9$, the MSE of the estimator $t_{r_1}$ is 14.64 and when $\rho_{uv} = 0.9$, the MSE of the estimator $t_{r_1}$ is 14.23). All these findings are evident from Figure 1. Furthermore, the MSE of the ratio estimators $t_{r_2}$ and $t_{r_3}$ for the strategies II and III, respectively, follow a similar style for which the figures may be supplied, if necessary.
- The sequential increase in the values of $\rho_{xy}$ from 0 to 0.9 reduces the MSE of the traditional logarithmic estimator $t_{l_1}$ under strategy I, (For example: it is noticed from Table 1 that for $\rho_{xy} = 0$ and $\rho_{uv} = -0.9$, the MSE of the estimator $t_{l_1}$ is 11.57, whereas for $\rho_{xy} = 0.9$ and $\rho_{uv} = -0.9$, the MSE of the estimator $t_{l_1}$ is 11.07). The sequential increase in the size and direction of $\rho_{uv}$ from $-0.9$ to 0 increases the MSE, while decreases for the values of $\rho_{uv}$ from 0 to 0.9, (For example: it is noticed from Table 1 that for $\rho_{xy} = 0.5$, when $\rho_{uv} = -0.9$, the MSE of the estimator $t_{l_1}$ is 11.37 and when $\rho_{uv} = 0.9$, the MSE of the estimator $t_{l_1}$ is 11.25). All these findings are evident from Figure 2. The MSE of the traditional logarithmic estimators $t_{l_2}$ and $t_{l_3}$ for strategies II and III, respectively, follow a similar style for which the figures may be supplied, if necessary.
- The sequential increase in the values of $\rho_{xy}$ from 0 to 0.9 grows the MSE of the suggested Searls type logarithmic estimator $T_{sl_1}$ under strategy I, (For example: it is observed from Table 1 that for $\rho_{xy} = 0$ and $\rho_{uv} = -0.9$, the MSE of the estimator $T_{sl_1}$ is 8.10, whereas for $\rho_{xy} = 0.9$ and $\rho_{uv} = -0.9$, the MSE of the estimator $T_{sl_1}$ is 8.30). But the sequential increase in the size and direction of $\rho_{uv}$ from $-0.9$ to 0.9 reduces the MSE of the estimator $T_{sl_1}$, (For example: it is noticed from Table 1 that for $\rho_{xy} = 0.5$, when $\rho_{uv} = -0.9$, the MSE of the estimator $T_{sl_1}$ is 8.31 and when $\rho_{uv} = 0.9$, the MSE of the estimator $T_{sl_1}$ is 8.20). All these findings are evident from Figure 3. The MSE of the ratio estimators $T_{sl_2}$ and $T_{sl_3}$ for strategies II and III, respectively, follow a similar style for which the figures may be supplied, if necessary.
- Additionally, the ratio and logarithmic estimators under the corresponding strategies for various values of correlation coefficients are repressed by the suggested Searls type logarithmic estimators.
- The above observations may easily be noticed for the rest of the combinations of $\sigma_u^2$ and $\sigma_v^2$, i.e., ($\sigma_u^2$, $\sigma_v^2$)=(5, 10), ($\sigma_u^2$, $\sigma_v^2$)=(10, 5), and ($\sigma_u^2$, $\sigma_v^2$)=(10, 10).

(2). The findings of Table 2 are given for different percentages of measurement errors such as 5%, 10%, 15%, and 20%. For ME=5%, it is observed that:

- The sequential increase in the values of $\rho_{xy}$ from 0 to 0.9 reduces the MSE of the ratio estimator $t_{r_1}$ under strategy I, (For example: we can see from Table 2 that for $\rho_{xy} = 0$ and $\rho_{uv} = -0.9$, the MSE of the estimator $t_{r_1}$ is 14.90, whereas for $\rho_{xy} = 0.9$ and $\rho_{uv} = -0.9$, the MSE of the estimator $t_{r_1}$ is 12.98). The sequential increase in the size and direction of $\rho_{uv}$ from $-0.9$ to 0.9 also reduces the MSE, (For example: we can see from Table 2 that for $\rho_{xy} = 0.5$, when $\rho_{uv} = -0.9$, the MSE of the estimator $t_{r_1}$ is 13.70 and when $\rho_{uv} = 0.9$, the MSE of the estimator

$t_{r_1}$ is 13.05). All these findings are evident from Figure 4. Furthermore, the MSE of the ratio estimators $t_{r_2}$ and $t_{r_3}$ for the strategies II and III, respectively, follow a similar style for which the figures may be supplied, if necessary.

- The sequential increase in the values of $\rho_{xy}$ from 0 to 0.9 reduces the MSE of the traditional logarithmic estimator $t_{l_1}$ under strategy I, (For example: it is noticed from Table 2 that for $\rho_{xy} = 0$ and $\rho_{uv} = -0.9$, the MSE of the estimator $t_{l_1}$ is 11.69, whereas for $\rho_{xy} = 0.9$ and $\rho_{uv} = -0.9$, the MSE of the estimator $t_{r_1}$ is 11.25). The sequential increase in the size and direction of $\rho_{uv}$ from $-0.9$ to 0 increases the MSE, while decreases for the values of $\rho_{uv}$ from 0 to 0.9, (For example: it is noticed from Table 2 that for $\rho_{xy} = 0.5$, when $\rho_{uv} = -0.9$, the MSE of the estimator $t_{l_1}$ is 11.52 and when $\rho_{uv} = 0.9$, the MSE of the estimator $t_{l_1}$ is 11.34). All these findings are evident from Figure 5. The MSE of the traditional logarithmic estimators $t_{l_2}$ and $t_{l_3}$ for strategies II and III, respectively, follow a similar style for which the figures may be supplied, if necessary.
- The sequential increase in the values of $\rho_{xy}$ from 0 to 0.9 grows the MSE of the suggested Searls type logarithmic estimator $T_{sl_1}$ under strategy I, (For example: it is observed from Table 2 that for $\rho_{xy} = 0$ and $\rho_{uv} = -0.9$, the MSE of the estimator $T_{sl_1}$ is 8.18, whereas for $\rho_{xy} = 0.9$ and $\rho_{uv} = -0.9$, the MSE of the estimator $T_{sl_1}$ is 8.42). But the sequential increase in the size and direction of $\rho_{uv}$ from $-0.9$ to 0.9 reduces the MSE of the estimator $T_{sl_1}$, (For example: it is noticed from Table 2 that for $\rho_{xy} = 0.5$, when $\rho_{uv} = -0.9$, the MSE of the estimator $T_{sl_1}$ is 8.41 and when $\rho_{uv} = 0.9$, the MSE of the estimator $T_{sl_1}$ is 8.24). All these findings are evident from Figure 6. The MSE of the ratio estimators $T_{sl_2}$ and $T_{sl_3}$ for strategies II and III, respectively, follow a similar style for which the figures may be supplied, if necessary.
- Additionally, the ratio and logarithmic estimators under the corresponding strategies for various values of correlation coefficients are repressed by the suggested Searls type logarithmic estimators.
- The above observations may easily be noticed for the rest of the percentages of ME, i.e., 10%, 15%, and 20%.

## 7. Real data application

The real data example is illustrative in nature. The measurement error can be estimated with help of cross checks performed during the collection of data, such checks are routine in large scale surveys. In this part, the adapted and suggested imputation methods are executed over a real data set based on the humidity of Lahore, Pakistan. The climate of Lahore is a local steppe climate. A little rainfall take place during the year in Lahore. The mean yearly temperature observed in Lahore is recorded to be 75.1 $F$. Approximately 25.0 inch of rainfall occurs on a yearly basis. Lahore is located in the northern hemisphere. The balmy days of Summer commence at the end of June and conclude in September. This period encompasses the months: June, July, August, September. In this article, we considered daily based maximum percentage of humidity. The humidity (%) in the year 2022 is considered as study variable $y$, while in the year 2021 is considered as auxiliary variable $x$. The imputation

**Table 1**: MSE of the estimators for different values of $\sigma_u^2$ and $\sigma_v^2$

| | | | | | Strategy I | | | Strategy II | | | Strategy III | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\sigma_u^2$ | $\sigma_v^2$ | $\rho_{xy}$ | $\rho_{uv}$ | $t_m$ | $t_{r_1}$ | $t_{l_1}$ | $T_{sl_1}$ | $t_{r_2}$ | $t_{l_2}$ | $T_{sl_2}$ | $t_{r_3}$ | $t_{l_3}$ | $T_{sl_3}$ |
| 5 | 5 | 0 | -0.9 | 12.10 | 14.64 | 11.57 | 8.10 | 15.89 | 11.31 | 8.57 | 13.35 | 11.84 | 8.33 |
| | | | -0.5 | 12.12 | 14.56 | 11.60 | 8.10 | 15.76 | 11.34 | 8.56 | 13.32 | 11.86 | 8.31 |
| | | | 0 | 12.17 | 14.49 | 11.64 | 8.12 | 15.64 | 11.38 | 8.57 | 13.31 | 11.91 | 8.31 |
| | | | 0.5 | 12.12 | 14.35 | 11.59 | 8.08 | 15.44 | 11.33 | 8.52 | 13.22 | 11.86 | 8.26 |
| | | | 0.9 | 12.11 | 14.23 | 11.57 | 8.05 | 15.28 | 11.31 | 8.49 | 13.15 | 11.84 | 8.22 |
| | | 0.5 | -0.9 | 12.07 | 13.43 | 11.37 | 8.31 | 14.10 | 11.03 | 8.86 | 12.74 | 11.72 | 8.46 |
| | | | -0.5 | 12.08 | 13.35 | 11.37 | 8.30 | 13.97 | 11.01 | 8.87 | 12.71 | 11.73 | 8.44 |
| | | | 0 | 12.13 | 13.28 | 11.38 | 8.29 | 13.84 | 11.01 | 8.89 | 12.70 | 11.76 | 8.44 |
| | | | 0.5 | 12.08 | 13.13 | 11.30 | 8.24 | 13.64 | 10.92 | 8.85 | 12.60 | 11.70 | 8.38 |
| | | | 0.9 | 12.07 | 13.01 | 11.25 | 8.20 | 13.47 | 10.85 | 8.84 | 12.53 | 11.67 | 8.34 |
| | | 0.9 | -0.9 | 12.21 | 12.69 | 11.07 | 8.30 | 12.93 | 10.50 | 9.22 | 12.45 | 11.65 | 8.51 |
| | | | -0.5 | 12.23 | 12.60 | 11.04 | 8.27 | 12.79 | 10.45 | 9.23 | 12.41 | 11.64 | 8.49 |
| | | | 0 | 12.27 | 12.53 | 11.02 | 8.26 | 12.65 | 10.40 | 9.27 | 12.40 | 11.66 | 8.48 |
| | | | 0.5 | 12.23 | 12.36 | 10.92 | 8.18 | 12.43 | 10.27 | 9.25 | 12.30 | 11.58 | 8.42 |
| | | | 0.9 | 12.21 | 12.24 | 10.84 | 8.13 | 12.25 | 10.16 | 9.25 | 12.22 | 11.54 | 8.39 |
| 5 | 10 | 0 | -0.9 | 12.11 | 14.78 | 11.58 | 8.12 | 16.10 | 11.32 | 8.59 | 13.42 | 11.85 | 8.35 |
| | | | -0.5 | 12.13 | 14.67 | 11.61 | 8.12 | 15.92 | 11.36 | 8.57 | 13.38 | 11.88 | 8.33 |
| | | | 0 | 12.17 | 14.55 | 11.65 | 8.12 | 15.72 | 11.40 | 8.56 | 13.34 | 11.92 | 8.31 |
| | | | 0.5 | 12.14 | 14.37 | 11.62 | 8.08 | 15.47 | 11.37 | 8.51 | 13.24 | 11.89 | 8.25 |
| | | | 0.9 | 12.11 | 14.21 | 11.58 | 8.04 | 15.24 | 11.32 | 8.47 | 13.14 | 11.85 | 8.20 |
| | | 0.5 | -0.9 | 12.07 | 13.58 | 11.41 | 8.34 | 14.33 | 11.09 | 8.86 | 12.81 | 11.74 | 8.48 |
| | | | -0.5 | 12.09 | 13.46 | 11.40 | 8.32 | 14.14 | 11.07 | 8.87 | 12.77 | 11.75 | 8.46 |
| | | | 0 | 12.13 | 13.34 | 11.40 | 8.30 | 13.93 | 11.04 | 8.88 | 12.72 | 11.77 | 8.44 |
| | | | 0.5 | 12.10 | 13.15 | 11.32 | 8.24 | 13.67 | 10.94 | 8.85 | 12.62 | 11.72 | 8.38 |
| | | | 0.9 | 12.07 | 12.98 | 11.24 | 8.18 | 13.43 | 10.83 | 8.83 | 12.52 | 11.66 | 8.32 |
| | | 0.9 | -0.9 | 12.21 | 12.86 | 11.14 | 8.35 | 13.18 | 10.61 | 9.21 | 12.53 | 11.68 | 8.54 |
| | | | -0.5 | 12.24 | 12.73 | 11.09 | 8.31 | 12.97 | 10.53 | 9.23 | 12.48 | 11.67 | 8.51 |
| | | | 0 | 12.27 | 12.59 | 11.05 | 8.27 | 12.75 | 10.45 | 9.25 | 12.43 | 11.67 | 8.48 |
| | | | 0.5 | 12.25 | 12.39 | 10.94 | 8.19 | 12.47 | 10.29 | 9.24 | 12.32 | 11.60 | 8.42 |
| | | | 0.9 | 12.22 | 12.21 | 10.82 | 8.11 | 12.21 | 10.14 | 9.24 | 12.21 | 11.53 | 8.36 |
| 10 | 5 | 0 | -0.9 | 12.53 | 15.14 | 11.99 | 8.35 | 16.43 | 11.72 | 8.83 | 13.82 | 12.26 | 8.59 |
| | | | -0.5 | 12.53 | 15.02 | 12.01 | 8.34 | 16.24 | 11.74 | 8.80 | 13.76 | 12.27 | 8.55 |
| | | | 0 | 12.52 | 14.85 | 12.00 | 8.32 | 16.00 | 11.74 | 8.76 | 13.67 | 12.26 | 8.50 |
| | | | 0.5 | 12.53 | 14.72 | 12.00 | 8.30 | 15.79 | 11.74 | 8.74 | 13.61 | 12.27 | 8.47 |
| | | | 0.9 | 12.52 | 14.56 | 11.98 | 8.27 | 15.57 | 11.71 | 8.70 | 13.53 | 12.25 | 8.43 |
| | | 0.5 | -0.9 | 12.49 | 13.94 | 11.82 | 8.58 | 14.66 | 11.48 | 9.12 | 13.2 | 12.16 | 8.73 |
| | | | -0.5 | 12.50 | 13.81 | 11.79 | 8.55 | 14.45 | 11.44 | 9.11 | 13.14 | 12.15 | 8.70 |
| | | | 0 | 12.48 | 13.64 | 11.73 | 8.51 | 14.21 | 11.37 | 9.09 | 13.05 | 12.11 | 8.65 |
| | | | 0.5 | 12.50 | 13.49 | 11.70 | 8.47 | 13.98 | 11.30 | 9.09 | 12.99 | 12.10 | 8.61 |
| | | | 0.9 | 12.48 | 13.33 | 11.63 | 8.42 | 13.75 | 11.21 | 9.08 | 12.90 | 12.06 | 8.57 |
| | | 0.9 | -0.9 | 12.63 | 13.21 | 11.53 | 8.60 | 13.50 | 10.99 | 9.48 | 12.92 | 12.09 | 8.80 |
| | | | -0.5 | 12.64 | 13.07 | 11.47 | 8.55 | 13.28 | 10.90 | 9.48 | 12.85 | 12.06 | 8.76 |
| | | | 0 | 12.63 | 12.89 | 11.38 | 8.48 | 13.02 | 10.76 | 9.48 | 12.75 | 12.01 | 8.71 |
| | | | 0.5 | 12.64 | 12.73 | 11.30 | 8.42 | 12.77 | 10.64 | 9.50 | 12.68 | 11.98 | 8.67 |
| | | | 0.9 | 12.63 | 12.55 | 11.20 | 8.35 | 12.52 | 10.49 | 9.50 | 12.59 | 11.92 | 8.62 |
| 10 | 10 | 0 | -0.9 | 12.52 | 15.32 | 11.99 | 8.36 | 16.70 | 11.72 | 8.85 | 13.90 | 12.26 | 8.61 |
| | | | -0.5 | 12.56 | 15.16 | 12.04 | 8.37 | 16.44 | 11.79 | 8.83 | 13.84 | 12.31 | 8.59 |
| | | | 0 | 12.65 | 15.03 | 12.14 | 8.40 | 16.20 | 11.88 | 8.83 | 13.82 | 12.40 | 8.58 |
| | | | 0.5 | 12.56 | 14.74 | 12.04 | 8.31 | 15.81 | 11.78 | 8.74 | 13.64 | 12.31 | 8.47 |
| | | | 0.9 | 12.53 | 14.51 | 11.99 | 8.26 | 15.48 | 11.72 | 8.69 | 13.51 | 12.27 | 8.40 |
| | | 0.5 | -0.9 | 12.49 | 14.12 | 11.85 | 8.62 | 14.93 | 11.54 | 9.12 | 13.29 | 12.17 | 8.77 |
| | | | -0.5 | 12.52 | 13.96 | 11.85 | 8.59 | 14.67 | 11.52 | 9.12 | 13.23 | 12.19 | 8.73 |
| | | | 0 | 12.62 | 13.82 | 11.88 | 8.59 | 14.41 | 11.52 | 9.16 | 13.21 | 12.25 | 8.72 |
| | | | 0.5 | 12.52 | 13.52 | 11.72 | 8.48 | 14.01 | 11.33 | 9.09 | 13.01 | 12.13 | 8.61 |
| | | | 0.9 | 12.49 | 13.28 | 11.61 | 8.40 | 13.66 | 11.18 | 9.07 | 12.88 | 12.06 | 8.54 |
| | | 0.9 | -0.9 | 12.70 | 13.44 | 11.69 | 8.69 | 13.81 | 11.20 | 9.48 | 13.06 | 12.20 | 8.88 |
| | | | -0.5 | 12.70 | 13.22 | 11.61 | 8.62 | 13.48 | 11.07 | 9.48 | 12.96 | 12.16 | 8.83 |
| | | | 0 | 12.76 | 13.08 | 11.53 | 8.57 | 13.23 | 10.93 | 9.54 | 12.92 | 12.16 | 8.78 |
| | | | 0.5 | 12.70 | 12.67 | 11.33 | 8.41 | 12.66 | 10.65 | 9.50 | 12.69 | 12.02 | 8.69 |
| | | | 0.9 | 12.69 | 12.44 | 11.18 | 8.31 | 12.31 | 10.44 | 9.52 | 12.57 | 11.95 | 8.65 |

methods are applied over the humidity data of Lahore. Initially, we have generated the values of measurement errors U and V from normal distribution with 0 means and 5% variance of the $\sigma_y^2$ and $\sigma_x^2$. Then, the values of $\sigma_u^2$, $\sigma_v^2$, and $\rho_{uv}$ are obtained. Following are the necessary descriptive statistics: $N$=365, $n = 15$, $H = 3$, $K = 5$, $P = 0.67$, $\bar{Y}$= 87.04, $\bar{X}$=86.09, $\sigma_y^2$=230.03, $\sigma_x^2$=179.84, $\sigma_u^2 = 116.38$, $\sigma_v^2 = 80.13$, $\rho_{xy}$=0.83, and $\rho_{uv} = 0.05$.

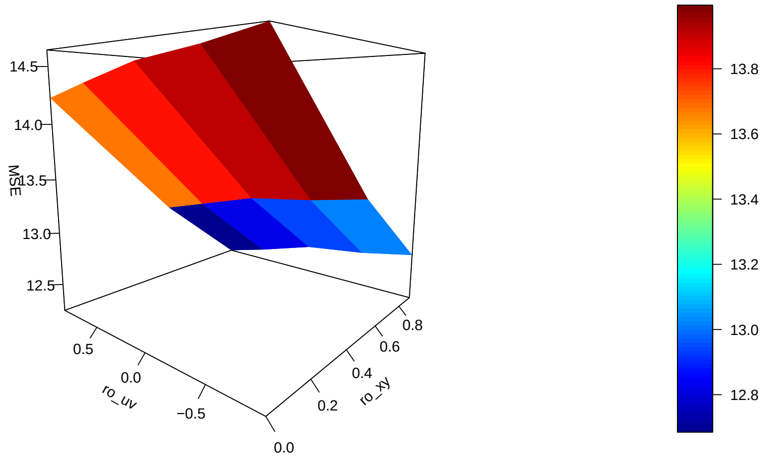


**Figure 1**: MSE of the estimator $t_{r_1}$ revealed in Table 1 for $(\sigma_u^2,\ \sigma_v^2)$=(5, 5), $\rho_{xy} = (0,\ 0.5,\ 0.9)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$

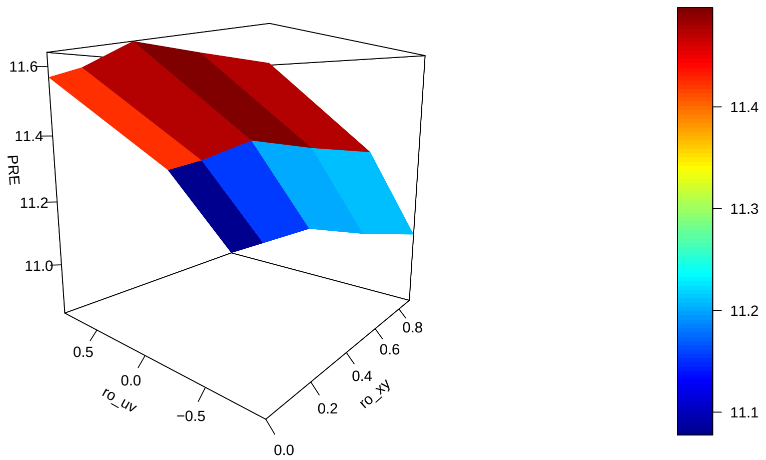


**Figure 2**: MSE of the estimator $t_{l_1}$ revealed in Table 1 for $(\sigma_u^2,\ \sigma_v^2)$=(5, 5), $\rho_{xy} = (0,\ 0.5,\ 0.9)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$

For the above data, the MSE of the proposed estimators is calculated and the outcomes are displayed in Table 3 which show the outperformance of the suggested estimators over the adapted estimators.

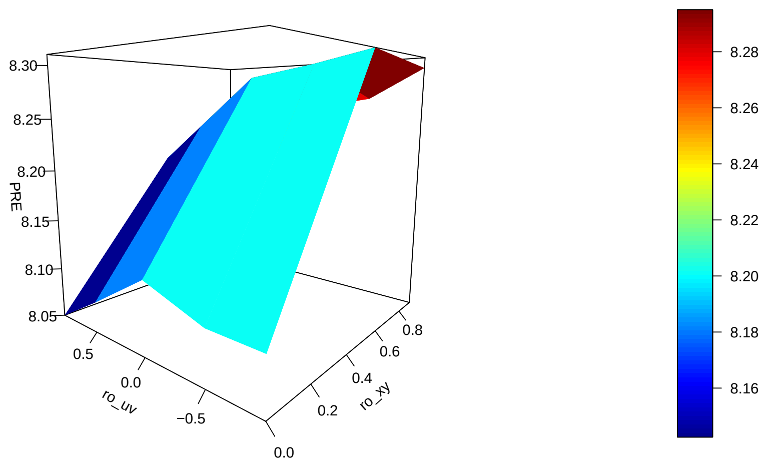


**Figure 3**: MSE of the estimator $T_{sl_1}$ revealed in Table 1 for $(\sigma_u^2,\ \sigma_v^2)$=(5, 5), $\rho_{xy} = (0,\ 0.5,\ 0.9)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$

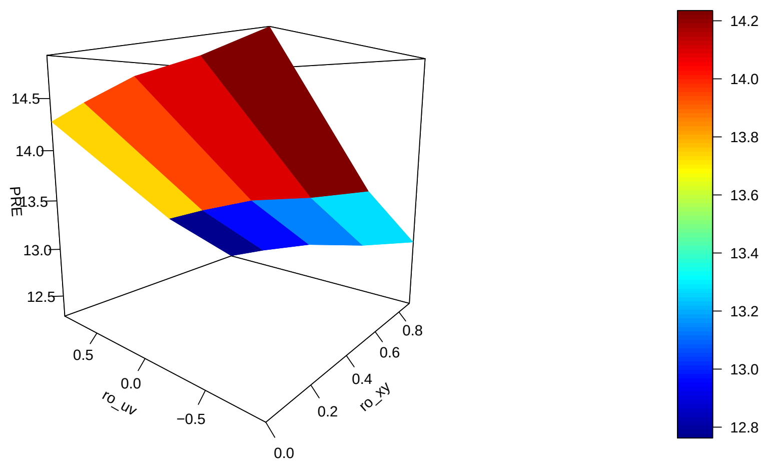


**Figure 4**: MSE of the estimator $t_{r_1}$ revealed in Table 2 for ME=5%, $\rho_{xy} = (0,\ 0.5,\ 0.9)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$

## 8. Conclusion

In survey sampling, the estimation of population mean in the presence of missing data when the data are tainted with the correlated measurement errors, is a challenging task. As mentioned earlier such problems are easily identified while performing cross checks done by a supervisor over the data collected by an investigator. The researcher can easily identify such

**Table 2**: MSE of the estimators for different percentages of ME

| | | | | Strategy I | | | Strategy II | | | Strategy III | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| % of ME | $\rho_{xy}$ | $\rho_{uv}$ | $t_m$ | $t_{r_1}$ | $t_{l_1}$ | $T_{sl_1}$ | $t_{r_2}$ | $t_{l_2}$ | $T_{sl_2}$ | $t_{r_3}$ | $t_{l_3}$ | $T_{sl_3}$ |
| 5% | 0 | -0.9 | 12.22 | 14.90 | 11.69 | 8.18 | 16.22 | 11.43 | 8.66 | 13.54 | 11.96 | 8.42 |
| | | -0.5 | 12.23 | 14.77 | 11.71 | 8.18 | 16.02 | 11.46 | 8.63 | 13.48 | 11.98 | 8.39 |
| | | 0 | 12.29 | 14.66 | 11.77 | 8.19 | 15.83 | 11.52 | 8.63 | 13.46 | 12.04 | 8.38 |
| | | 0.5 | 12.25 | 14.46 | 11.72 | 8.14 | 15.55 | 11.47 | 8.57 | 13.34 | 11.99 | 8.31 |
| | | 0.9 | 12.22 | 14.28 | 11.68 | 8.10 | 15.30 | 11.42 | 8.53 | 13.23 | 11.95 | 8.25 |
| | 0.5 | -0.9 | 12.18 | 13.70 | 11.52 | 8.41 | 14.45 | 11.20 | 8.93 | 12.93 | 11.86 | 8.55 |
| | | -0.5 | 12.20 | 13.56 | 11.51 | 8.38 | 14.24 | 11.17 | 8.93 | 12.87 | 11.86 | 8.52 |
| | | 0 | 12.25 | 13.45 | 11.52 | 8.37 | 14.03 | 11.15 | 8.95 | 12.84 | 11.89 | 8.51 |
| | | 0.5 | 12.21 | 13.24 | 11.42 | 8.30 | 13.75 | 11.03 | 8.91 | 12.72 | 11.82 | 8.44 |
| | | 0.9 | 12.18 | 13.05 | 11.34 | 8.24 | 13.49 | 10.92 | 8.89 | 12.61 | 11.76 | 8.38 |
| | 0.9 | -0.9 | 12.32 | 12.98 | 11.25 | 8.42 | 13.30 | 10.72 | 9.28 | 12.64 | 11.79 | 8.61 |
| | | -0.5 | 12.34 | 12.83 | 11.20 | 8.38 | 13.07 | 10.63 | 9.29 | 12.58 | 11.78 | 8.58 |
| | | 0 | 12.40 | 12.70 | 11.17 | 8.34 | 12.85 | 10.56 | 9.33 | 12.55 | 11.79 | 8.56 |
| | | 0.5 | 12.35 | 12.48 | 11.03 | 8.25 | 12.54 | 10.38 | 9.31 | 12.41 | 11.70 | 8.48 |
| | | 0.9 | 12.32 | 12.28 | 10.91 | 8.16 | 12.26 | 10.21 | 9.31 | 12.30 | 11.63 | 8.42 |
| 10% | 0 | -0.9 | 12.74 | 15.83 | 12.19 | 8.51 | 17.36 | 11.92 | 9.01 | 14.26 | 12.47 | 8.79 |
| | | -0.5 | 12.78 | 15.59 | 12.28 | 8.51 | 16.97 | 12.02 | 8.96 | 14.16 | 12.53 | 8.73 |
| | | 0 | 12.90 | 15.36 | 12.40 | 8.54 | 16.58 | 12.15 | 8.95 | 14.11 | 12.65 | 8.71 |
| | | 0.5 | 12.81 | 14.97 | 12.29 | 8.44 | 16.03 | 12.04 | 8.84 | 13.87 | 12.55 | 8.58 |
| | | 0.9 | 12.74 | 14.61 | 12.18 | 8.34 | 15.53 | 11.91 | 8.76 | 13.66 | 12.46 | 8.46 |
| | 0.5 | -0.9 | 12.70 | 14.65 | 12.12 | 8.79 | 15.61 | 11.83 | 9.26 | 13.66 | 12.41 | 8.95 |
| | | -0.5 | 12.75 | 14.39 | 12.12 | 8.76 | 15.20 | 11.81 | 9.25 | 13.56 | 12.44 | 8.89 |
| | | 0 | 12.86 | 14.16 | 12.15 | 8.74 | 14.80 | 11.80 | 9.28 | 13.50 | 12.51 | 8.86 |
| | | 0.5 | 12.77 | 13.74 | 11.96 | 8.60 | 14.22 | 11.56 | 9.21 | 13.25 | 12.37 | 8.72 |
| | | 0.9 | 12.70 | 13.37 | 11.76 | 8.46 | 13.69 | 11.30 | 9.17 | 13.03 | 12.24 | 8.60 |
| | 0.9 | -0.9 | 12.93 | 14.05 | 12.05 | 8.92 | 14.60 | 11.61 | 9.61 | 13.48 | 12.50 | 9.09 |
| | | -0.5 | 12.95 | 13.72 | 11.95 | 8.83 | 14.11 | 11.46 | 9.60 | 13.33 | 12.45 | 9.02 |
| | | 0 | 12.97 | 13.34 | 11.80 | 8.71 | 13.52 | 11.22 | 9.62 | 13.15 | 12.39 | 8.95 |
| | | 0.5 | 12.94 | 12.89 | 11.55 | 8.53 | 12.87 | 10.86 | 9.62 | 12.92 | 12.26 | 8.81 |
| | | 0.9 | 12.93 | 12.53 | 11.32 | 8.38 | 12.34 | 10.53 | 9.65 | 12.73 | 12.14 | 8.73 |
| 15% | 0 | -0.9 | 13.26 | 16.77 | 12.67 | 8.82 | 18.50 | 12.38 | 9.39 | 14.99 | 12.97 | 9.16 |
| | | -0.5 | 13.33 | 16.40 | 12.83 | 8.83 | 17.91 | 12.57 | 9.29 | 14.85 | 13.08 | 9.07 |
| | | 0 | 13.50 | 16.07 | 13.02 | 8.87 | 17.33 | 12.78 | 9.27 | 14.77 | 13.26 | 9.04 |
| | | 0.5 | 13.33 | 15.46 | 12.82 | 8.70 | 16.50 | 12.57 | 9.08 | 14.38 | 13.08 | 8.82 |
| | | 0.9 | 13.28 | 14.94 | 12.68 | 8.58 | 15.76 | 12.39 | 9.00 | 14.10 | 12.99 | 8.67 |
| | 0.5 | -0.9 | 13.22 | 15.61 | 12.68 | 9.16 | 16.78 | 12.42 | 9.61 | 14.40 | 12.96 | 9.34 |
| | | -0.5 | 13.30 | 15.22 | 12.72 | 9.12 | 16.17 | 12.43 | 9.57 | 14.24 | 13.01 | 9.26 |
| | | 0 | 13.46 | 14.87 | 12.78 | 9.10 | 15.56 | 12.44 | 9.61 | 14.16 | 13.13 | 9.21 |
| | | 0.5 | 13.30 | 14.22 | 12.45 | 8.87 | 14.68 | 12.04 | 9.49 | 13.75 | 12.88 | 8.99 |
| | | 0.9 | 13.24 | 13.69 | 12.19 | 8.68 | 13.91 | 11.67 | 9.46 | 13.46 | 12.72 | 8.83 |
| | 0.9 | -0.9 | 13.52 | 15.14 | 12.76 | 9.37 | 15.94 | 12.38 | 9.97 | 14.32 | 13.14 | 9.54 |
| | | -0.5 | 13.54 | 14.65 | 12.65 | 9.26 | 15.20 | 12.21 | 9.94 | 14.09 | 13.10 | 9.43 |
| | | 0 | 13.34 | 13.79 | 12.20 | 8.89 | 14.02 | 11.64 | 9.74 | 13.56 | 12.78 | 9.06 |
| | | 0.5 | 13.54 | 13.41 | 12.08 | 8.84 | 13.34 | 11.36 | 9.95 | 13.47 | 12.82 | 9.09 |
| | | 0.9 | 13.51 | 12.87 | 11.72 | 8.59 | 12.55 | 10.84 | 9.99 | 13.19 | 12.63 | 9.02 |
| 20% | 0 | -0.9 | 13.81 | 17.73 | 13.15 | 9.14 | 19.66 | 12.83 | 9.79 | 15.74 | 13.49 | 9.54 |
| | | -0.5 | 13.88 | 17.22 | 13.37 | 9.15 | 18.86 | 13.11 | 9.62 | 15.53 | 13.63 | 9.41 |
| | | 0 | 14.11 | 16.77 | 13.64 | 9.20 | 18.08 | 13.40 | 9.58 | 15.42 | 13.88 | 9.36 |
| | | 0.5 | 13.93 | 15.98 | 13.41 | 9.00 | 16.98 | 13.15 | 9.37 | 14.94 | 13.67 | 9.10 |
| | | 0.9 | 13.81 | 15.27 | 13.15 | 8.79 | 15.99 | 12.82 | 9.24 | 14.53 | 13.49 | 8.87 |
| | 0.5 | -0.9 | 13.78 | 16.59 | 13.24 | 9.52 | 17.97 | 12.98 | 9.99 | 15.16 | 13.51 | 9.74 |
| | | -0.5 | 13.85 | 16.05 | 13.30 | 9.47 | 17.13 | 13.03 | 9.90 | 14.93 | 13.58 | 9.61 |
| | | 0 | 14.07 | 15.58 | 13.40 | 9.45 | 16.32 | 13.07 | 9.93 | 14.81 | 13.74 | 9.55 |
| | | 0.5 | 13.89 | 14.75 | 13.01 | 9.17 | 15.17 | 12.57 | 9.79 | 14.31 | 13.46 | 9.28 |
| | | 0.9 | 13.78 | 14.01 | 12.60 | 8.89 | 14.13 | 12.01 | 9.74 | 13.89 | 13.19 | 9.05 |
| | 0.9 | -0.9 | 14.10 | 16.23 | 13.41 | 9.80 | 17.28 | 13.08 | 10.34 | 15.15 | 13.76 | 9.98 |
| | | -0.5 | 14.13 | 15.58 | 13.32 | 9.68 | 16.30 | 12.92 | 10.28 | 14.85 | 13.73 | 9.83 |
| | | 0 | 13.86 | 14.44 | 12.76 | 9.21 | 14.72 | 12.22 | 10.02 | 14.14 | 13.32 | 9.37 |
| | | 0.5 | 14.12 | 13.92 | 12.60 | 9.13 | 13.82 | 11.85 | 10.26 | 14.02 | 13.37 | 9.41 |
| | | 0.9 | 14.10 | 13.20 | 12.11 | 8.80 | 12.76 | 11.13 | 10.32 | 13.66 | 13.12 | 9.30 |

measurement or response errors if he is cautious enough. Moreover, some characteristics warrant such anticipation on the part of researcher. Apart from this, a researcher can determine whether data contains MEs by employing several techniques. The researcher can analyse the data for outliers or inconsistencies that deviate from expected patterns. The researcher can perform consistency checks by comparing responses to related questions can help identify dis-

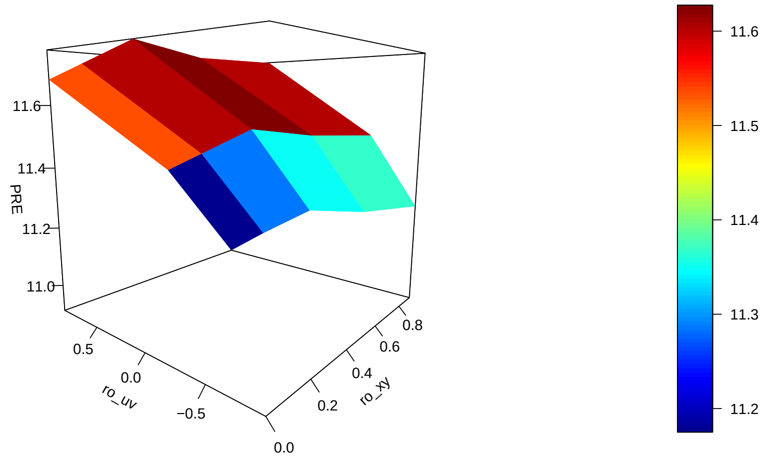


**Figure 5**: MSE of the estimator $t_{l_1}$ revealed in Table 2 for ME=5%, $\rho_{xy} = (0,\ 0.5,\ 0.9)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$

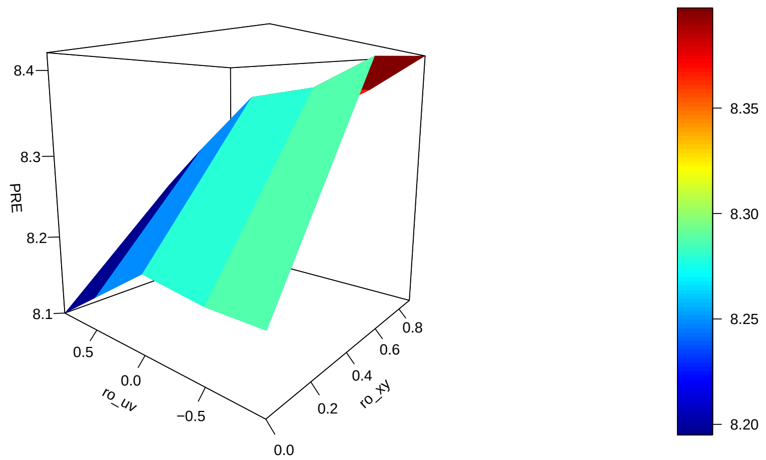


**Figure 6**: MSE of the estimator $T_{sl_1}$ revealed in Table 2 for ME=5%, $\rho_{xy} = (0,\ 0.5,\ 0.9)$, and $\rho_{uv} = (-0.9,\ -0.5,\ 0,\ 0.5,\ 0.9)$

**Table 3**: $MSE$ of the adapted and suggested estimators for real data

| | Strategy I | | | Strategy II | | | Strategy III | | |
|---|---|---|---|---|---|---|---|---|---|
| $t_m$ | $t_{r_1}$ | $t_{l_1}$ | $T_{sl_1}$ | $t_{r_2}$ | $t_{l_2}$ | $T_{sl_2}$ | $t_{r_3}$ | $t_{l_3}$ | $T_{sl_3}$ |
| 31.04 | 20.98 | 20.34 | 20.23 | 16.02 | 15.06 | 14.97 | 26.08 | 25.77 | 21.74 |

crepancies. Cross-validation with other datasets or using alternative measurement methods can provide additional insights. Reviewing the survey design and administration process is crucial to uncover potential sources of error, such as ambiguous questions or interviewer bias.

Very few studies are available for the estimation of population mean in the presence of missing data provided the data are tainted with measurement errors under simple random sampling. However, no work is available for the estimation of population mean in the presence of missing data given that the data are contaminated with the correlated measurement errors under ranked set sampling. This article is a fundamental effort to adapt the classical mean, ratio, and traditional logarithmic imputation methods and to propose Searls type logarithmic imputation methods along with their resultant estimators. The MSE of the adapted and proposed resultant estimators is determined employing the first order approximation. The comparison of the proposed and adapted estimators is carried out to establish the efficiency conditions under which the proposed estimators would surpass the adapted estimators. A thorough simulation is performed to enhance the theoretical results that additionally examines the impact of the correlated measurement errors on the performance of the resultant estimators. Table 1 and Table 2 present the simulation results for diffferent values of ($\sigma_u^2$, $\sigma_v^2$) and different percentages of MEs. Table 1 indicates that the MSE of the proposed Searls type logarithmic estimators $T_{sl_j},\ j = 1, 2, 3$ grows as $\rho_{xy}$ varies from 0 to 0.9 which is also rely on the direction and magnitude of $\rho_{uv}$. Further, the MSE of the proposed estimators reported in Table 2 for various percentages of MEs indicates the same pattern. In addition, a real data application of the adapted and suggested imputations is also provided. The results of the real data application show that the proposed estimators dominate the adapted estimators under each strategy. Therefore, the adapted and suggested imputation methods and the resulting estimators are strongly recommended to the survey persons to deal with the real-life challenges of the correlated measurement errors.

## ACKNOWLEDGMENTS

The authors gratefully acknowledge the support rendered by DST SERB, India vide project number SUR/2022/003187 for this work.
The authors are extremely grateful to the learned referee for his valuable comments and Editor-in-Chief.

## Disclosure statement

No potential competing interest was reported by the authors.

## REFERENCES

Ahmed, M. S., Al-Titi, O., Al-Rawi, Z., and Abu-Dayyeh, W. (2006). Estimation of a population mean using different imputation methods. *Statistics in Transition*, 7(6):1247–1264.

Ahmed, S., Shabbir, J., and Gupta, S. (2019). Predictive estimation of population mean in ranked set sampling. *REVSTAT-Statistical Journal*, 17(4):551–562.

Al-Omari, A. I. and Bouza, C. (2014). Ratio estimators of the population mean with missing values using ranked set sampling. *Environmetrics*, 26:67–76.

Al-Omari, A. I. and Haq, A. (2012). Improved quality control charts for monitoring the process mean using double-ranked set sampling methods. *Journal of Applied Statistics*, 39(4):745–763.

Alam, R., Hanif, M., Shahbaz, S. H., and Shahbaz, M. Q. (2022). Estimation of population variance under ranked set sampling method by using the ratio of supplementary information with study variable. *Scientific Reports*, 12:21203.

Bhushan, S., Kumar, A., Pandey, A. P., and Singh, S. (2022). Estimation of population mean in presence of missing data under simple random sampling. *Communications in Statistics - Simulation and computation*, 52(12):6048–6069.

Bhushan, S., Kumar, A., and Shukla, S. (2023a). Impact assessment of correlated measurement errors using logarithmic-type estimators. *Statistics*, 57(5):1010–1036.

Bhushan, S., Kumar, A., and Shukla, S. (2023b). Novel logarithmic type estimators in presence of measurement errors. *Journal of Statistical Theory and Practice*, 17(3):35.

Bhushan, S., Kumar, A., and Shukla, S. (2023c). On classes of robust estimators in presence of correlated measurement errors. *Measurement*, 220:113383.

Bhushan, S., Kumar, A., and Shukla, S. (2023d). Performance evaluation of novel logarithmic estimators under correlated measurement errors. *Communications in Statistics - Theory and Methods*, 53(15):5353–5363.

Bhushan, S. and Pandey, A. P. (2018). Optimality of ratio type estimation methods for population mean in presence of missing data. *Communication in Statistics-Theory and Methods*, 47(11):2576–2589.

Carroll, R. J., Ruppert, D., Stefanski, L. A., and Crainiceanu, C. M. (2006). *Measurement error in nonlinear models: A modern perspective (2nd edn.)*. Boca Raton: Chapman & Hall/CRC.

Chen, H., Stasny, E. A., and Wolfe, D. A. (2007). Improved procedures for estimation of disease prevalence using ranked set sampling. *Biometric Journal*, 49:530–538.

Cheng, C. L. and Van Ness, J. W. (1999). *Statistical regression with measurement error*. London: Arnold and New York: Oxford University Press.

Cochran, W. G. (1968). Errors of measurement in statistics. *Technometrics*, 10(4):637–666.

Das, R., Verma, V., and Nath, D. C. (2018). Bayesian estimation of measles vaccination coverage under ranked set sampling. *Statistics in Transition*, 18(4):589–608.

Halls, L. K. and Dell, T. R. (1966). Trial of ranked-set sampling for forage yields. *Forest Science*, 12(1):22–26.

Heitjan, D. F. and Basu, S. (1996). Distinguishing 'missing at random' and 'missing completely at random'. *The American Statistician*, 50:207–213.

Kumar, A., Bhushan, S., Shukla, S., Emam, W., Tashkandy, Y. A., and Gupta, R. (2023). Impact of correlated measurement errors on some efficient classes of estimators. *Journal of Mathematics*, 2023(1):8140831.

Lee, H., Rancourt, E., and Sarndal, C. E. (1994). Experiments with variance estimation from survey data with imputed values. *Journal of Official Statistics*, 10:231–243.

Manisha and Singh, R. K. (2000). An estimation of population mean in the presence of measurement errors. *Journal of the Indian Society of Agricultural Statistics*, 54(1):13–18.

McIntyre, G. A. (1952). A method for unbiased selective sampling using ranked set sampling. *Australian Journal of Agricultural Research*, 3:385–390.

Murthy, M. N. (1967). *Sampling: Theory and Methods*. Statistical Publishing Society, Calcutta.

Sahoo, L. N., Sahoo, R. K., and Senapati, S. C. (2006). An empirical study on the accuracy of ratio and regression estimators in the presence of measurement errors. *Monte Carlo Methods and Applications*, 12(5):495–501.

Searls, D. T. (1964). The utilization of a known coefficient of variation in the estimation procedure. *Journal of Americal Statistical Association*, 59:1225–1226.

Shalabh (1997). Ratio method of estimation in the presence of measurement errors. *Journal of the Indian Society of Agricultural Statistics*, 50(2):150–155.

Shalabh and Tsai, J. R. (2017). Ratio and product methods of estimation of population mean in the presence of correlated measurement errors. *Communications in Statistics-Simulation and Computation*, 46(7):5566–5593.

Singh, H. P. and Karpe, N. (2009). On the estimation of ratio and product of two population means using supplementary information in presence of measurement errors. *Statistica*, 69(1):27–47.

Tariq, M. U., Qureshi, M. N., and Hanif, M. (2021). Variance estimators in the presence of measurement errors using auxiliary information. *Thailand Statistician*, 19(3):606–616.

Tariq, M. U., Qureshi, M. N., and Hanif, M. (2022). Generalized variance estimator using auxiliary information in the presence and absence of measurement error. *Scientia Iranica*, 29(4):1868–1879.

Yadav, V. K. and Prasad, S. (2023). Exponential method of estimation in sampling theory under robust quantile regression methods. *Communications in Statistics-Theory and Methods*, 53(17):6285–6298.

Zamanzade, E. (2019). Edf-based tests of exponentiality in pair ranked set sampling. *Statistical Papers*, 60(6):2141–2159.

Zamanzade, E., Mahdizadeh, M., and Samawi, H. (2020). Efficient estimation of cumulative distribution function using moving extreme ranked set sampling with application to reliability. *AStA Advances in Statistical Analysis*, 104(3):485–502.