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The Neutrosophic Negative Binomial Distribution: Algorithms and Practical Application

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Abstract:

- This paper introduces the negative binomial distribution using the novel concept of the neutrosophic random variable. It explores properties of the neutrosophic random variable based on measures such as expectation and variance and outlines fundamental characteristics of the proposed negative binomial distribution. The paper proceeds to introduce algorithms for generating neutrosophic data based on this distribution. The application of the proposed negative binomial distribution is demonstrated in the context of oil exploration data, with comparative analysis against existing negative binomial distribution results from classical statistics. Extensive simulations and comparative studies reveal discrepancies between the outcomes obtained using the proposed distribution and those from the traditional negative binomial distribution in classical statistics. The study concludes that the proposed negative binomial distribution proves applicable in uncertain environments.

Keywords:

- *discrete distribution; uncertainty; simulation; imprecise data; probability.*

AMS Subject Classification:

- 62A86.

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1. INTRODUCTION

The negative binomial distribution, a discrete probability distribution, is employed to determine the likelihood of achieving more than one success before reaching a fixed number of trials. It serves as an extension of the geometric distribution, which is utilized to calculate the probability of the first success. Widely applied across various fields such as ecology, geology, industry, and biological sciences, the negative binomial distribution has found diverse applications. Yun and Youlin (1996) introduced the Q-control chart based on the negative binomial distribution. Lloyd-Smith (2007) utilized the negative binomial distribution in analyzing infection data. Urbieta et al. (2017) proposed memory-based control charts using the negative binomial distribution. Stoklosa et al. (2022) provided a comprehensive review of the negative binomial distribution's applications in biodiversity and ecology. Conceição et al. (2022) discussed the generalization of the negative binomial distribution and presented its properties. Liu et al. (2023) applied the negative binomial distribution to analyze count data from psychology. Park et al. (2023) presented probability limit-based control charts using both geometric and negative binomial distributions. Doi et al. (2024) focused on improving interval estimation through negative binomial parameters. Further details can be found in Chew et al. (2024). Neutrosophic statistics, a branch of mathematical science, is utilized for the analysis, presentation, and inference of imprecise data. Smarandache (2014) is credited with introducing neutrosophic statistics, and Smarandache (2022) demonstrated its efficiency compared to interval statistics. Neutrosophic statistics serve as a generalization of classical statistics, incorporating an additional parameter known as the measure of indeterminacy. It finds application in situations where data is fuzzy or exhibits imprecision. Chen et al., 2017a and Chen et al. (2017b) provided the methodology to analyze the neutrosophic data and compare the efficiency of the two methods. Adepoju et al. (2019) proposed the negative binomial distribution using the fuzzy approach. Granados (2022) and Granados et al. (2022) proposed neutrosophic discrete and continuous distributions. Alvaracín Jarrín et al. (2021) applied the neutrosophic statistics in the social sciences. Khan et al. (2021) presented the neutrosophic Rayleigh distribution. AlAita and Aslam (2023) discussed the application of the neutrosophic statistics in the experiment design. Vishwakarma and Singh (2022) proposed the neutrosophic ranked set method. Ahsan-ul Haq (2022) presented neutrosophic Kumaraswamy distribution and discussed the application in engineering. Delcea et al. (2023) provided the bolometric on the applications of neutrosophic statistics. The development of algorithms and distributions using the neutrosophic statistical methods can be seen in Guo and Sengur (2015), Garg et al. (2020), Granados (2022), Aslam (2023b), Granados et al. (2023), Aslam (2023a), Aslam (2023c) and Aslam and Alamri (2023). Upon reviewing the existing literature, it is evident that a substantial body of work exists on the negative binomial distribution within the realm of classical statistics. However, to the best of the author's knowledge, there is a notable absence of research on algorithms utilizing the neutrosophic negative binomial distribution. This paper seeks to fill this gap by introducing the novel concept of the neutrosophic random variable. The paper unfolds by presenting the new notion of the neutrosophic random variable, followed by an exploration of its properties. Subsequently, the proposed neutrosophic negative binomial distribution is introduced, incorporating the innovative concept of the neutrosophic random variable. The basic properties of this novel negative binomial distribution are then outlined. Two algorithms are proposed for generating imprecise negative binomial data. The paper proceeds to present simulation and comparative studies, applying the proposed distribution to oil exploration data. Anticipated are results that differ from those obtained using the existing negative binomial distribution within the framework of classical statistics.

2. NEUTROSOPHIC RANDOM VARIABLE

In this section, we will present the concept of the neutrosophic random variable. Let X_N be defined as $X_L + X_L I_N$, where X_L represents the determinate part and $X_L I_N$, represents the indeterminate part, with $I_N \epsilon [I_L, I_U]$ being the indeterminate. We assume that the random variable X_L has a mean μ_X and variance σ_X^2. It is crucial to highlight that when $I_L = 0$, the proposed neutrosophic random variable reduces to the variable in classical statistics. According to Granados (2022), introducing the new measure of indeterminacy is termed as the measure of indeterminacy, which transforms neutrosophic logic into a generalization of fuzzy logic, where $I_N^2 = I_N, \ldots, I_N^n = I_N$. It is important to note that the proposed neutrosophic random variable consists of two components: the determinate part X_L and the indeterminate part $X_L I_N$, where $I_N \epsilon [I_L, I_U]$. When $I_L = 0$, this neutrosophic random variable simplifies to the classical random variable X_L. However, X_L is limited to scenarios where all observations in the data are either precise or accurate. In contrast, the proposed neutrosophic random variable is designed to manage varying degrees of uncertainty in the data effectively. Utilizing this information, we explore certain properties of expectation and variance for the introduced neutrosophic random variable. The mean of the neutrosophic random variable is expressed as

$$E(X_N) = E(X_L + X_L I_N) = (1 + I_N)\mu_X \tag{2.1}$$

The variance of the neutrosophic random variable is given as

$$Var(X_N) = Var(X_L + X_L I_N) = (1 + I_N)^2 \sigma_X^2 \tag{2.2}$$

Let $Y_N = Y_L + Y_L I_N$ be the another neutrosophic random having the mean μ_Y and variance σ_Y^2 the expectation and the variance of the two neutrosophic random variables are given by

$$E(X_N + Y_N) = (1 + I_N)\mu_X + (1 + I_N)\mu_Y \tag{2.3}$$

$$Var(X_N + Y_N) = Var(X_L + X_L I_N) = (1 + I_N)^2 \sigma_X^2 + (1 + I_N)^2 \sigma_Y^2 \tag{2.4}$$

Let a and b are the constant, using these constants, the expectation and variance of the neutrosophic random variables are given by

$$E(X_N + a) = E(X_L + X_L I_N + a) = (1 + I_N)\mu_X + a \tag{2.5}$$

$$Var(X_N + a) = Var(X_L + X_L I_N + a) = (1 + I_N)^2 \sigma_X^2 \tag{2.6}$$

$$E(aX_N + b) = a(1 + I_N)\mu_X + b \tag{2.7}$$

$$Var(aX_N + b) = a^2 (1 + I_N)^2 \sigma_X^2 \tag{2.8}$$

3. NEGATIVE BINOMIAL DISTRIBUTION

Let's consider a scenario where $X_N = X_L + X_L I_N$ is defined as the sum of X_L and X_L times I_N, with I_N belonging to the interval $[I_L, I_U]$, representing a neutrosophic random variable following the neutrosophic negative binomial distribution. Here, X_L is a random variable following the negative distribution in classical statistics, X_L times I_N represents the indeterminate part, and I_N is the indeterminacy within the interval $[I_L, I_U]$. The proposed neutrosophic negative binomial distribution transforms into the negative binomial distribution under classical statistics when there is no uncertainty recorded in the data (I_L=0). We represent this as X_N following the negative binomial distribution with parameters $NNB(r_N, p_N)$, where r_N is the number of successes and p_N is the probability of success. It's important to note that both r_N and p_N fall within certain intervals. In light of this information, the probability function for the proposed neutrosophic negative binomial distribution can be expressed as follows:

$$f(X_N) = \binom{r_N + (1+I_N) X_L - 1}{(1+I_N) X_L} p_N^{r_N} (1-p_N)^{(1+I_N)X_L}; X_N = 0,\ (1+I_N), 2(1+I_N), \ldots$$

Proof: We prove that the proposed distribution is the proper distribution. We have

$$f(X_N) = \binom{r_N + (1+I_N) X_L - 1}{(1+I_N) X_L} p_N^{r_N} (1-p_N)^{(1+I_N)X_L}$$

Let $\binom{r_N + (1+I_N) X_L - 1}{(1+I_N) X_L} = \frac{(r_N+(1+I_N)X_L-1)!}{(r-1)![(1+I_N)X_L]!} = (-1)^{(1+I_N)X_L} \binom{-r_N}{(1+I_N) X_L}$

$$\sum_{X_N=0}^{\infty} f(X_N) = \sum_{X_N=0}^{\infty} (-1)^{(1+I_N)X_L} \binom{-r_N}{(1+I_N) X_L} p_N^{r_N} (1-p_N)^{(1+I_N)X_L}$$

$$= p_N^{r_N} (-1 + p_N + 1)^{-r_N} = 1$$

□

Theorem 3.1. The expected value of the proposed distribution is $(1+I_N)(1-p_N) r_N / p_N$

Proof: By applying expectation

$$E((1+I_N) X_L) = (1+I_N) E(X_L) = (1+I_N) \sum_{X_L=o}^{\infty} \binom{r_N + X_L - 1}{X_L} p_N^{r_N} (1-p_N)^{X_L}$$

$$= (1+I_N) \sum_{X_L=1}^{\infty} \frac{(r_N + X_L - 1)!}{(X_L-1)!(r_N-1)!} p_N^{r_N} (1-p_N)^{X_L}$$

$$= (1+I_N) \sum_{X_L=1}^{\infty} \frac{r_N(1-p_N)}{p_N} \binom{r_N + X_L - 1}{X_L - 1} p_N^{r_N+1} (1-p_N)^{X_L-1}$$

$$E(X_N) = \frac{(1+I_N)(1-p_N) r_N}{p_N} \tag{3.1}$$

On the same lines, the variance of the proposed negative binomial distribution is given by

$$Var\left(X_N\right) = \frac{\left(1+I_N\right)^2\left(1-p_N\right)r_N}{p_N^2} \tag{3.2}$$

□

4. THE PROPOSED ALGORITHM-I

In this section, we will introduce modifications to the algorithm used for generating the negative Binomial distribution within the framework of classical statistics, as originally presented by . The algorithm proposed here, referred to as algorithm-I for generating the negative binomial distribution under neutrosophic statistics, serves as a generalization of the pre-existing algorithm designed for generating negative binomial distributed data under classical statistics. It is important to highlight that the algorithm presented by Thomopoulos (2012) is not suitable for generating imprecise data from the negative binomial distribution. The proposed algorithm-I, guided by Thomopoulos (2012), will be executed through the following sequential steps.

Step-1: Specify I_N

Step-2: set $x_L = 0$

Step-3: For $i = 1$ to r_N, generate y_N, a random geometric variate with the probability p_N, we have

$$x_N = y_N + x_L$$

Next i

Step-4: Return x_N

The procedure to implement the proposed algorithm-I is shown in Figure 1.

5. THE PROPOSED ALGORITHM-II

Rose and Smith (1997) presented another algorithm using the geometric distribution. It is well-known that the negative binomial distribution is the generalization of the geometric distribution. Therefore, the sum of r_N independent geometric random variables with parameter p_N follow the neutrosophic negative binomial distribution. Therefore, negative binomial distributed random variables can obtained by adding r_N independent geometric random variables. The existing algorithm given in Rose and Smith (1997) only provided the determinate data using the negative binomial distribution. We now present the extension of this algorithm using the neutrosophic negative binomial distribution. The necessary steps to generate the neutrosophic negative binomial data are given by

Step-1: Specify I_N

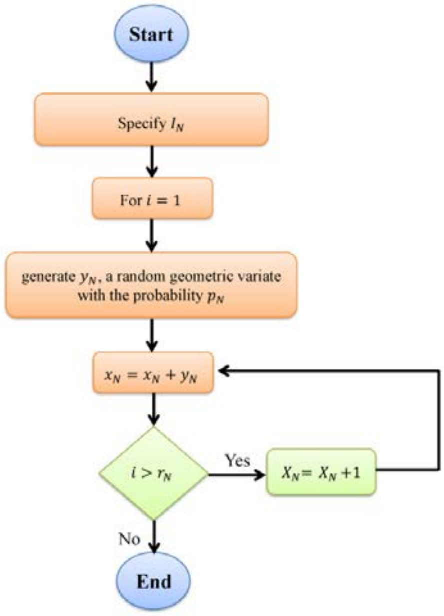

Figure 1: The process of algorithm-I.

Step-2: Generate r_N independent geometric random variables with probability p_N.

Step-3: Obtain $x_N = (x_{L1} + x_{L2} + \cdots + x_{Lr_L})\,(1 + I_N)$

Step-4: Repeat steps 1-3 to generate more negative binomial distributed data for another parameter.

The procedure to implement the proposed algorithm-II is shown in Figure 2.

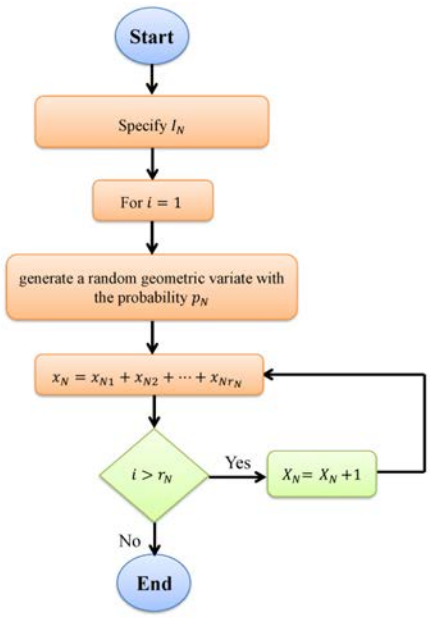

Figure 2: The process of algorithm-II.

6. SIMULATION STUDIES

Within this segment, we will delve into the simulation process of the suggested algorithms. First, we'll explore the data generation from the negative binomial distribution through algorithm-I. Subsequently, we will examine the simulation procedure designed to replicate data from the negative binomial distribution using algorithm-II.

6.1. SIMULATION USING ALGORITHM-I

In this section, we will explore the negative binomial data generated through algorithm-I. Following the steps outlined in algorithm-I, we have presented several tables displaying data from the negative binomial distribution, utilizing varying values of r_N. Table 1 corresponds to $r_N = 3$ and $p = 0.50$, Table 2 to $r_N = 5$ and $p = 0.50$, Table 3 to $r_N = 7$ and p=0.50, and Table 4 to $r_N = 9$ and $p = 0.50$. Tables 1-4 collectively demonstrate an overall increasing trend in the negative binomial distribution data as the value of I_N varies. For instance, in Table 1, the last row indicates that for $I_N = 0.1$, the corresponding value of x_N is 5, while for $I_N = 1$, the value of x_N increases to 8. The graphical representation of data behavior for different values of I_N is illustrated in Figure 3. Similarly, an overarching increasing trend is observed in the negative binomial distribution data as r_N increases from 3 to 9. This trend is visually presented in Figure 4.

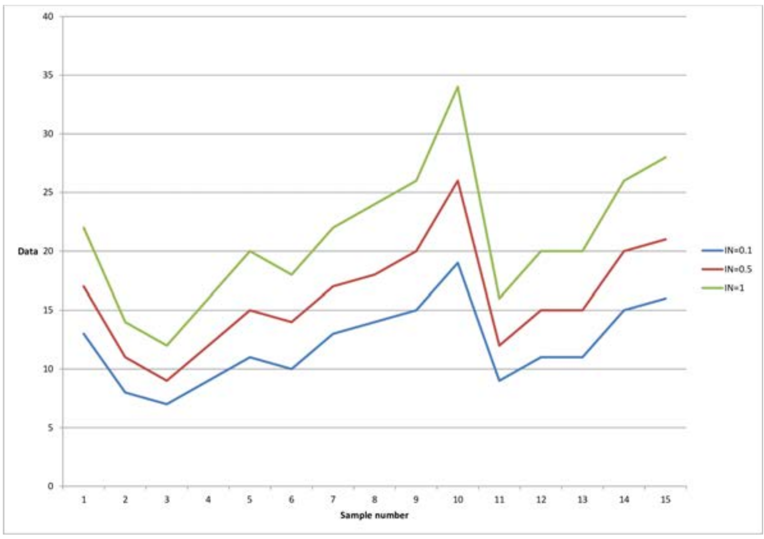

Figure 3: The curves for various values of I_N.

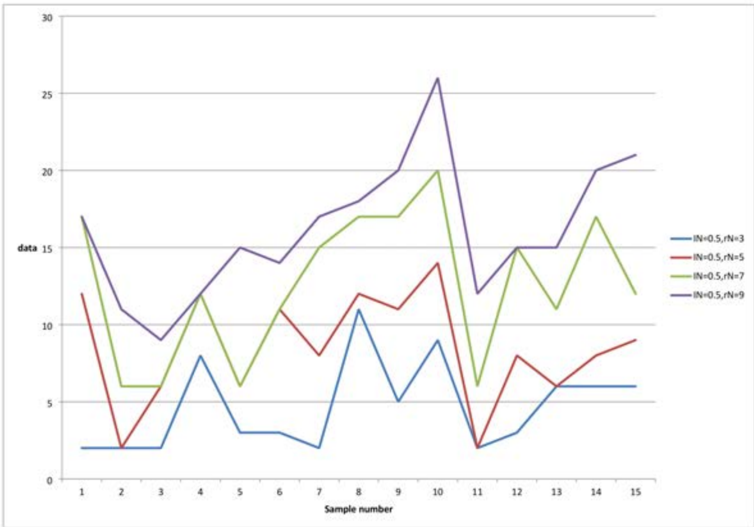

Figure 4: The curves for various values of I_N.

6.2. SIMULATION USING ALGORITHM-II

In this section, we will examine the negative binomial data generated through algorithm-II. Following the outlined steps in algorithm-II, we have presented several tables displaying data from the negative binomial distribution, utilizing varying values of r_N. Specifically, Table 5 corresponds to r_N=3 and p=0.50, while Table 6 corresponds to r_N=5 and $p = 0.50$. Tables 5-6 collectively illustrate a consistent upward trend in the negative binomial distribution data as the value of I_N varies. For instance, in Table 5, the last row indicates that for $I_N = 0.1$, the corresponding value of x_N is 14, and for $I_N = 1$, the value of x_N increases to 24. The graphical representation of data behavior for different values of I_N is depicted in Figure 5. Similarly, a general upward trend is observed in the negative binomial distribution data as r_N increases from 3 to 5.

From the simulation studies using Algorithm-I and Algorithm-II, it can be concluded that as the degree of uncertainty increases, the data exhibits an upward trend. Given that the data varies with the level of uncertainty, it is crucial for decision-makers to carefully quantify this uncertainty to ensure accurate results.

7. COMPARATIVE STUDIES

Within this section, we will contrast the data generated from the proposed algorithm for the negative binomial distribution with data derived from existing algorithms within classical statistics. Initially, we will juxtapose the outcomes of the proposed algorithm-I with those of the existing algorithm in classical statistics. Subsequently, we will undertake a comparison

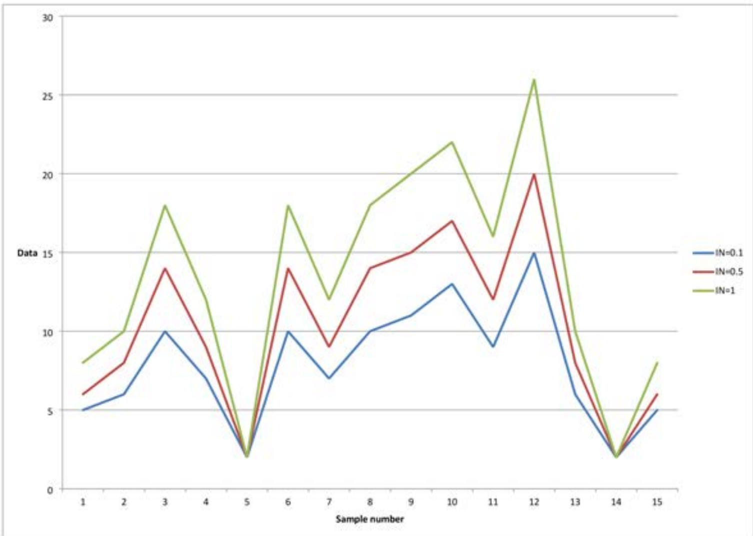

Figure 5: The curves for various values of I_N.

between the data generated from the proposed algorithm-II and the data of negative binomial distribution from the existing algorithm within classical statistics.

7.1. ALGORITHM-I VS. THE EXISTING ALGORITHM

As previously stated, the proposed algorithm-I serves as an extension of the algorithm outlined in Thomopoulos (2012). Notably, when the data is generated from the negative binomial distribution within an uncertain environment, the proposed algorithm-I converges to the existing algorithm within classical statistics. The data for the negative binomial distribution, specifically when $I_N = 0$, is generated using the existing algorithm and is positioned in the first column of Tables 1-4. Observing Tables 1-4 reveals that when $I_N = 0$, the values of X_N are comparatively smaller than those associated with other values of I_N. For instance, considering r_N=5 and $p = 0.50$ in Table 2 (first row), the value of X_N is 8 when $I_N = 0$, whereas it increases to 12 when $I_N = 0.5$. This investigation highlights notable distinctions between the data generated by the algorithm outlined in Thomopoulos (2012), and the data produced by the proposed algorithm-I. Further insight is provided by Figure 6, wherein the data curve from the existing algorithm consistently appears lower than the data curve obtained from the proposed algorithm. In summary, the generated data under uncertainty significantly differs from that produced under a certain environment. Consequently, caution is advised for decision-makers when applying the existing algorithm in situations characterized by a degree of uncertainty.

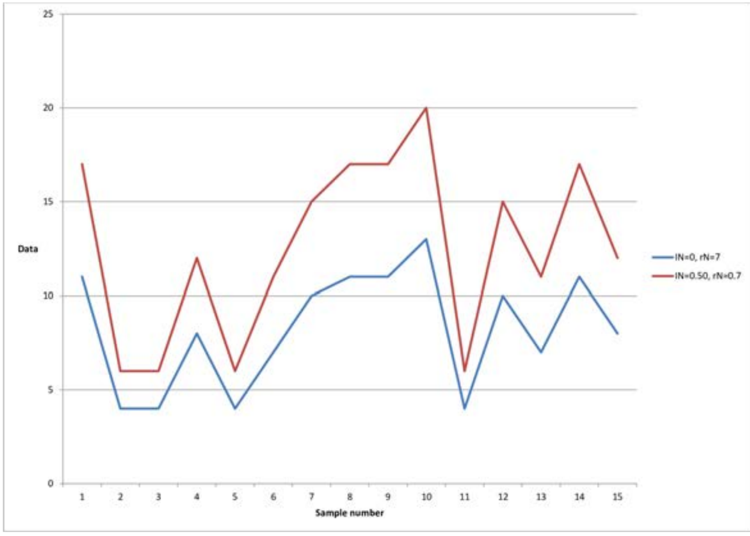

Figure 6: Data curves from the proposed algorithm-I and the existing algorithm.

7.2. ALGORITHM-II VS. THE EXISTING ALGORITHM

As previously mentioned, the proposed algorithm-II serves as an extension of the algorithm detailed in Rose and Smith (1997). Notably, when data is generated from the negative binomial distribution in an uncertain environment, the proposed algorithm-II aligns with the existing algorithm in classical statistics. Specifically, data for the negative binomial distribution, particularly when $I_N = 0$, is generated using the existing algorithm and is placed in the initial column of Tables 5-6. Upon examining Tables 5-6, it becomes apparent that when $I_N = 0$, the values of X_N are relatively smaller compared to those associated with other values of I_N. For example, considering $r_N = 5$ and $p = 0.50$ in Table 5 (last row), the value of X_N is 12 when $I_N = 0$, whereas it increases to 20 when $I_N = 0.5$. This exploration underscores significant disparities between the data generated by the algorithm outlined in Rose and Smith (1997) and that produced by the proposed algorithm-II. Further insights are gleaned from Figure 7, where the data curve from the existing algorithm consistently appears lower than the data curve obtained from the proposed algorithm. In summary, the generated data under uncertainty markedly differs from that produced under a certain environment. Consequently, decision-makers are advised to exercise caution when applying the existing algorithm in situations characterized by a degree of uncertainty.

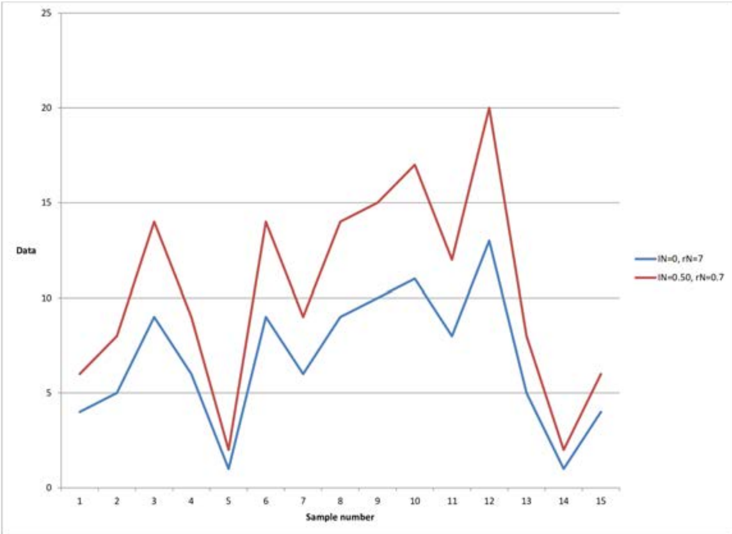

Figure 7: Data curves from the proposed algorithm-II and the existing algorithm.

8. APPLICATION IN OIL EXPLORATION

In this segment, we will explore the utilization of the suggested neutrosophic negative distribution within the context of an oil company. Let's consider a scenario where an oil company conducts a geological study suggesting that an exploratory oil well has a 50% probability of striking oil, with an indeterminacy of 10%. The question at hand is: What is the probability of striking oil on the 7th attempt out of the 20 drilled?

Sol: From Table 3 (first row), we have, $X_L = 20$, $I_N = 0.1$, $p_N = 0.50$, $r_N = 7$, using this data in our proposed negative distribution, we

$$f(X_N) = \binom{r_N + (1 + I_N) X_L - 1}{(1 + I_N) X_L} p_N^{r_N} (1 - p_N)^{(1+I_N)X_L}$$

$$= \binom{7 + (1 + 0.1)\, 20 - 1}{(1 + 0.1)\, 20} 0.50^7 0.50^{(1+0.1)20} = 0.0007$$

According to the data, there is approximately a 0.07% probability of experiencing the 7th strike on the 20^{th} attempt.

Now, we will use the existing negative binomial distribution to compute the probability using the same data as follows

$$f(X_L) = \binom{r_N + X_L - 1}{X_L} p_N^{r_N} (1 - p_N)^{X_L}$$

$$= \binom{7 + 20 - 1}{20} 0.50^7 0.50^{20} = 0.0017$$

Based on the data, there is an estimated 0.17% likelihood of encountering the 7th strike on the 20^{th} attempt. Upon comparing this probability derived from the proposed negative binomial distribution with that obtained from the conventional negative binomial distribution in classical statistics, it becomes evident that the probabilities differ in both certain and uncertain environments.

9. LIMITATIONS

In this section, we will examine the limitations of the traditional negative binomial distribution and its data generation algorithms within classical statistics. Existing methods, such as those outlined by Thomopoulos (2012), assume accurate and precise data, which is often not the case in real-world applications. When data or parameters are imprecise, these classical approaches become less effective. The proposed negative binomial distribution and its associated algorithms are designed to address these limitations by incorporating a measure of uncertainty, making them more suitable for scenarios where data is imprecise. Thus, the new method provides a more robust solution for handling data uncertainty in practical applications.

10. CONCLUDING REMARKS

In this paper, the negative binomial distribution was introduced using the new notion of the neutrosophic random variable. Some properties of the neutrosophic random variable were presented based on the expectation and the variance. The basic properties of the proposed negative binomial distribution were also outlined. Algorithms were introduced for generating neutrosophic data using the proposed distribution. The application of the proposed negative binomial distribution in oil exploration data was presented, and the results were compared. Extensive simulation and comparative studies were conducted. From the comparative studies, it was noted that the results using the proposed distribution and the existing negative binomial distribution under classical statistics yielded different results. The results led to the conclusion that the proposed negative binomial distribution could be applied under uncertain environments. The proposed methods can be applied in diverse areas, including reliability analysis, risk management, environmental studies, operations, machine learning, artificial intelligence, and political science. Suggestions for future research included exploring more statistical properties of the proposed negative binomial distribution, extending algorithms using accept-reject methods, investigating applications in various areas, and developing software utilizing the proposed distribution and algorithms.

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
1	2	2	2	2	2	2	2	2	2	2
1	2	2	2	2	2	2	2	2	2	2
1	2	2	2	2	2	2	2	2	2	2
5	6	6	7	7	8	8	9	9	10	10
2	3	3	3	3	3	4	4	4	4	4
2	3	3	3	3	3	4	4	4	4	4
1	2	2	2	2	2	2	2	2	2	2
7	8	9	10	10	11	12	12	13	14	14
3	4	4	4	5	5	5	6	6	6	6
3	7	8	8	9	9	10	11	11	12	12
1	2	2	2	2	2	2	2	2	2	2
2	3	3	3	3	3	4	4	4	4	4
4	5	5	6	6	6	7	7	8	8	8
4	5	5	6	6	6	7	7	8	8	8
4	5	5	6	6	6	7	7	8	8	8

Table 1: Random variates from Algorithm-I when $r_N = 3$ and $p = 0.50$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
8	9	10	11	12	12	13	14	15	16	16
1	2	2	2	2	2	2	2	2	2	2
4	5	5	6	6	6	7	7	8	8	8
8	9	10	11	12	12	13	14	15	16	16
4	5	5	6	6	6	7	7	8	8	8
7	8	9	10	10	11	12	12	13	14	14
5	6	6	7	7	8	8	9	9	10	10
8	9	10	11	12	12	13	14	15	16	16
7	8	9	10	10	11	12	12	13	14	14
9	10	11	12	13	14	15	16	17	18	18
1	2	2	2	2	2	2	2	2	2	2
5	6	6	7	7	8	8	9	9	10	10
4	5	5	6	6	6	7	7	8	8	8
5	6	6	7	7	8	8	9	9	10	10
6	7	8	8	9	9	10	11	11	12	12

Table 2: Random variates from Algorithm-I when $r_N = 5$ and $p = 0.50$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
11	13	14	15	16	17	18	19	20	21	22
4	5	5	6	6	6	7	7	8	8	8
4	5	5	6	6	6	7	7	8	8	8
8	9	10	11	12	12	13	14	15	16	16
4	5	5	6	6	6	7	7	8	8	8
7	8	9	10	10	11	12	12	13	14	14
10	11	12	13	14	15	16	17	18	19	20
11	13	14	15	16	17	18	19	20	21	22
11	13	14	15	16	17	18	19	20	21	22
13	15	16	17	19	20	21	23	24	25	26
4	5	5	6	6	6	7	7	8	8	8
10	11	12	13	14	15	16	17	18	19	20
7	8	9	10	10	11	12	12	13	14	14
11	13	14	15	16	17	18	19	20	21	22
8	9	10	11	12	12	13	14	15	16	16

Table 3: Random variates from Algorithm-I when $r_N = 7$ and $p = 0.50$

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$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
11	13	14	15	16	17	18	19	20	21	22
7	8	9	10	10	11	12	12	13	14	14
6	7	8	8	9	9	10	11	11	12	12
8	9	10	11	12	12	13	14	15	16	16
10	11	12	13	14	15	16	17	18	19	20
9	10	11	12	13	14	15	16	17	18	18
11	13	14	15	16	17	18	19	20	21	22
12	14	15	16	17	18	20	21	22	23	24
13	15	16	17	19	20	21	23	24	25	26
17	19	21	23	24	26	28	29	31	33	34
8	9	10	11	12	12	13	14	15	16	16
10	11	12	13	14	15	16	17	18	19	20
10	11	12	13	14	15	16	17	18	19	20
13	15	16	17	19	20	21	23	24	25	26
14	16	17	19	20	21	23	24	26	27	28

Table 4: Random variates from Algorithm-I when $r_N = 9$ and $p = 0.50$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
1	2	2	2	2	2	2	2	2	2	2
4	5	5	6	6	6	7	7	8	8	8
8	9	10	11	12	12	13	14	15	16	16
9	10	11	12	13	14	15	16	17	18	18
6	7	8	8	9	9	10	11	11	12	12
8	9	10	11	12	12	13	14	15	16	16
6	7	8	8	9	9	10	11	11	12	12
3	4	4	4	5	5	5	6	6	6	6
16	18	20	21	23	24	26	28	29	31	32
1	2	2	2	2	2	2	2	2	2	2
6	7	8	8	9	9	10	11	11	12	12
4	5	5	6	6	6	7	7	8	8	8
5	6	6	7	7	8	8	9	9	10	10
4	5	5	6	6	6	7	7	8	8	8
12	14	15	16	17	18	20	21	22	23	24

Table 5: Random variates from Algorithm-II when $r_N = 3$ and $p = 0.50$

$I_N = 0$	$I_N = 0.1$	$I_N = 0.2$	$I_N = 0.3$	$I_N = 0.4$	$I_N = 0.5$	$I_N = 0.6$	$I_N = 0.7$	$I_N = 0.8$	$I_N = 0.9$	$I_N = 1$
4	5	5	6	6	6	7	7	8	8	8
5	6	6	7	7	8	8	9	9	10	10
9	10	11	12	13	14	15	16	17	18	18
6	7	8	8	9	9	10	11	11	12	12
1	2	2	2	2	2	2	2	2	2	2
9	10	11	12	13	14	15	16	17	18	18
6	7	8	8	9	9	10	11	11	12	12
9	10	11	12	13	14	15	16	17	18	18
10	11	12	13	14	15	16	17	18	19	20
11	13	14	15	16	17	18	19	20	21	22
8	9	10	11	12	12	13	14	15	16	16
13	15	16	17	19	20	21	23	24	25	26
5	6	6	7	7	8	8	9	9	10	10
1	2	2	2	2	2	2	2	2	2	2
4	5	5	6	6	6	7	7	8	8	8

Table 6: Random variates from Algorithm-II when $r_N = 5$ and $p = 0.50$

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