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Supplementary material for "Information matrices for composite type distributions"

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Particular cases

Composite Weibull distribution

For this composite distribution due to Calderin-Ojeda [1], the body is described by the Weibull distribution while the distribution for the tail can be arbitrary. The probability density and cumulative distribution functions of the composite Weibull distribution are

$$f(x)=\begin{cases} r\dfrac{1}{1-e^{-\left(\frac{\theta}{\phi}\right)^{\tau}}}\left(\dfrac{\tau}{x}\right)\left(\dfrac{x}{\phi}\right)^{\tau}e^{-\left(\frac{x}{\phi}\right)^{\tau}}, & 0<x\leq\theta,\\[2ex] (1-r)\dfrac{f_0(x)}{1-F_0(\theta)}, & \theta<x<\infty \end{cases}$$

and

$$F(x)=\begin{cases} r\dfrac{1-e^{\left(\frac{x}{\phi}\right)^{\tau}}}{1-e^{\left(\frac{\theta}{\phi}\right)^{\tau}}}, & 0<x<\theta,\\[2ex] r+(1-r)\dfrac{F_0(x)-F_0(\theta)}{1-F_0(\theta)}, & \theta<x<\infty, \end{cases}$$

respectively, where $\phi>0$, $\tau>1$, $\theta>0$, $0\leq r\leq 1$ and

$$r=\frac{f_0(\theta)\left[1-e^{-\left(\frac{\theta}{\phi}\right)^{\tau}}\right]}{f_0(\theta)\left[1-e^{-\left(\frac{\theta}{\phi}\right)^{\tau}}\right]+\frac{\tau}{\theta}\left(\frac{\theta}{\phi}\right)^{\tau}e^{-\left(\frac{\theta}{\phi}\right)^{\tau}}}.$$

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Suppose f_0 and F_0 are parameterized by $\mathbf{\Omega}$ and let $\theta = \theta(\mathbf{\Omega})$ denote the mode of f_0. Using expressions in Section 2 of the paper, the observed information matrix is given by (2.1), where the second order partial derivatives are

$$\begin{aligned}\frac{\partial^2 \log L}{\partial \tau^2} =& \left(\frac{a}{r} - \frac{n-a}{1-r}\right)\frac{\partial^2 r}{\partial \tau^2} - \left[\frac{a}{r^2} + \frac{n-a}{(1-r)^2}\right]\left(\frac{\partial r}{\partial \tau}\right)^2 \\ & - \frac{a\left[\log\left(\frac{\theta}{\phi}\right)\right]^2\left(\frac{\theta}{\phi}\right)^\tau\left\{e^{\left(\frac{\theta}{\phi}\right)^\tau}\left[1-\left(\frac{\theta}{\phi}\right)^\tau\right]-1\right\}}{\left[e^{\left(\frac{\theta}{\phi}\right)^\tau}-1\right]^2} \\ & - \frac{a}{\tau^2} - \frac{(\log\phi)^2}{\phi^\tau}\sum_{x_i\le\theta} x_i^\tau + \frac{2\log\phi}{\phi^\tau}\sum_{x_i\le\theta} x_i^\tau \log x_i - \frac{1}{\phi^\tau}\sum_{x_i\le\theta} x_i^\tau\left(\log x_i\right)^2,\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 \log L}{\partial \tau \partial \phi} =& \left(\frac{a}{r} - \frac{n-a}{1-r}\right)\frac{\partial^2 r}{\partial \tau \partial \phi} - \left[\frac{a}{r^2} + \frac{n-a}{(1-r)^2}\right]\frac{\partial r}{\partial \tau}\frac{\partial r}{\partial \phi} - \frac{a}{\phi} \\ & - \frac{a\left(\frac{\theta}{\phi}\right)^\tau\left\{\left[\tau\log\left(\frac{\theta}{\phi}\right)\left(\frac{\theta}{\phi}\right)^\tau - \tau\log\left(\frac{\theta}{\phi}\right)-1\right]e^{\left(\frac{\theta}{\phi}\right)^\tau} + \tau\log\left(\frac{\theta}{\phi}\right)+1\right\}}{\phi\left[e^{\left(\frac{\theta}{\phi}\right)^\tau}-1\right]^2} \\ & + \frac{1-\tau\log\phi}{\phi^{\tau+1}}\sum_{x_i\le\theta} x_i^\tau + \frac{\tau}{\phi^{\tau+1}}\sum_{x_i\le\theta} x_i^\tau\log x_i,\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 \log L}{\partial \tau \partial \mathbf{\Omega}} =& \left(\frac{a}{r} - \frac{n-a}{1-r}\right)\frac{\partial^2 r}{\partial \tau \partial \mathbf{\Omega}} + \left[\frac{\frac{\partial a}{\partial \mathbf{\Omega}} r - \frac{\partial r}{\partial \mathbf{\Omega}} a}{r^2} + \frac{\frac{\partial a}{\partial \mathbf{\Omega}}(1-r) - \frac{\partial r}{\partial \mathbf{\Omega}} a}{(1-r)^2}\right]\frac{\partial r}{\partial \tau} \\ & + \left(\frac{1}{\tau} - \log\phi\right)\frac{\partial a}{\partial \mathbf{\Omega}} \\ & + \frac{\partial \theta}{\partial \mathbf{\Omega}}\sum_{i=1}^n \Delta\left(x_i-\theta\right)\left(\log x_i + \frac{\log\phi}{\phi^\tau}x_i^\tau - \frac{x_i^\tau\log x_i}{\phi^\tau}\right) \\ & - \frac{\frac{\partial a}{\partial \mathbf{\Omega}}\log\left(\frac{\theta}{\phi}\right)\left(\frac{\theta}{\phi}\right)^\tau + \frac{a\phi}{\theta}\frac{\partial \theta}{\partial \mathbf{\Omega}}\left(\frac{\theta}{\phi}\right)^\tau + \log\left(\frac{\theta}{\phi}\right)\frac{a\tau\theta^\tau}{\theta\phi^\tau}}{e^{\left(\frac{\theta}{\phi}\right)^\tau}-1} \\ & + \frac{\frac{a\tau}{\theta}\log\left(\frac{\theta}{\phi}\right)\left(\frac{\theta}{\phi}\right)^{2\tau}e^{\left(\frac{\theta}{\phi}\right)^\tau}\frac{\partial \theta}{\partial \mathbf{\Omega}}}{\left[e^{\left(\frac{\theta}{\phi}\right)^\tau}-1\right]^2},\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 \log L}{\partial \phi^2} =& \left(\frac{a}{r} - \frac{n-a}{1-r}\right)\frac{\partial^2 r}{\partial \phi^2} - \left[\frac{a}{r^2} + \frac{n-a}{(1-r)^2}\right]\left(\frac{\partial r}{\partial \phi}\right)^2 \\ & + \frac{a\tau\left(\frac{\theta}{\phi}\right)^\tau\left\{\left[\tau\left(\frac{\theta}{\phi}\right)^\tau - \tau - 1\right]e^{\left(\frac{\theta}{\phi}\right)^\tau} + \tau + 1\right\}}{\phi^2\left[e^{\left(\frac{\theta}{\phi}\right)^\tau}-1\right]^2} \\ & + \frac{a\tau}{\phi^2} - \tau(\tau+1)\phi^{-\tau-2}\sum_{x_i\le\theta} x_i^\tau,\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 \log L}{\partial \phi \partial \mathbf{\Omega}} =& \left(\frac{a}{r} - \frac{n-a}{1-r}\right)\frac{\partial^2 r}{\partial \phi \partial \mathbf{\Omega}} + \left[\frac{\frac{\partial a}{\partial \mathbf{\Omega}} r - \frac{\partial r}{\partial \mathbf{\Omega}} a}{r^2} + \frac{\frac{\partial a}{\partial \mathbf{\Omega}}(1-r) - \frac{\partial r}{\partial \mathbf{\Omega}} a}{(1-r)^2}\right]\frac{\partial r}{\partial \phi} \\ & \frac{\left[\frac{\partial a}{\partial \mathbf{\Omega}}\tau\left(\frac{\theta}{\phi}\right)^\tau + a\tau^2\frac{\theta^{\tau-1}}{\phi^\tau}\frac{\partial \theta}{\partial \mathbf{\Omega}}\right]\phi\left[e^{\left(\frac{\theta}{\phi}\right)^\tau}-1\right] - a\tau^2\left(\frac{\theta}{\phi}\right)^{2\tau-1}\phi e^{\left(\frac{\theta}{\phi}\right)^\tau}\frac{\partial \theta}{\partial \mathbf{\Omega}}}{\left\{\phi\left[e^{\left(\frac{\theta}{\phi}\right)^\tau}-1\right]\right\}^2} \\ & - \frac{\partial a}{\partial \mathbf{\Omega}}\frac{\tau}{\phi} + \frac{\tau}{\phi^{\tau+1}}\frac{\partial \theta}{\partial \mathbf{\Omega}}\sum_{i=1}^n \Delta\left(x_i-\theta\right)x_i^\tau\end{aligned}$$

and

$$
\begin{aligned}
\frac{\partial^2 \log L}{\partial \mathbf{\Omega}^2} &= \left\{ \log r - \log\left[1 - e^{-\left(\frac{\theta}{\phi}\right)^{\tau}}\right] + \log\tau - \tau\log\phi \right\} \frac{\partial^2 a}{\partial \mathbf{\Omega}^2} \\
&+ \left\{ \frac{\partial r}{r\partial \mathbf{\Omega}} - \frac{\tau\left(\frac{\theta}{\phi}\right)^{\tau}\frac{\partial\theta}{\partial\mathbf{\Omega}}}{\theta\left[e^{\left(\frac{\theta}{\phi}\right)^{\tau}} - 1\right]} \right\} \frac{\partial a}{\partial \mathbf{\Omega}} \\
&- \left\{\log(1-r) - \log\left[1 - F_0(\theta)\right]\right\} \frac{\partial^2 a}{\partial \mathbf{\Omega}^2} + \left[\frac{f_0(\theta)}{1 - F_0(\theta)} \frac{\partial \theta}{\partial \mathbf{\Omega}} + \frac{\partial r}{(1-r)\partial \mathbf{\Omega}} \right] \frac{\partial a}{\partial \mathbf{\Omega}} \\
&+ \left(\frac{a}{r} - \frac{n-a}{1-r} \right) \frac{\partial^2 r}{\partial \mathbf{\Omega}^2} + \frac{\frac{\partial a}{\partial \mathbf{\Omega}} r - a \frac{\partial r}{\partial \mathbf{\Omega}}}{r^2} \frac{\partial r}{\partial \mathbf{\Omega}} \\
&- \frac{\partial^2 \theta}{\partial \mathbf{\Omega}^2} \left\{ \frac{a\tau\left(\frac{\theta}{\phi}\right)^{\tau}}{\theta\left[e^{\left(\frac{\theta}{\phi}\right)^{\tau}} - 1\right]} - \frac{(n-a) f_0(\theta)}{1 - F_0(\theta)} \right\} \\
&+ \frac{\partial^2 \theta}{\partial \mathbf{\Omega}^2} \sum_{i=1}^{n} \Delta\left(x_i - \theta\right) \left[(\tau - 1)\log x_i - \frac{x_i^{\tau}}{\phi^{\tau}} - \log f_0\left(x_i\right) \right] \\
&- \frac{\partial \theta}{\partial \mathbf{\Omega}} \frac{\left[-\frac{\partial a}{\partial \mathbf{\Omega}} f_0(\theta) + (n-a) \frac{\partial f_0(\theta)}{\partial \mathbf{\Omega}} \right] \left[1 - F_0(\theta)\right] + (n-a)\left[f_0(\theta)\right]^2 \frac{\partial \theta}{\partial \mathbf{\Omega}}}{\left[1 - F_0(\theta)\right]^2} - \frac{\partial r}{\partial \mathbf{\Omega}} \frac{-\frac{\partial a}{\partial \mathbf{\Omega}}(1-r) + (n-a)\frac{\partial r}{\partial \mathbf{\Omega}}}{(1-r)^2} \\
&- \frac{a\tau\left(\frac{\theta}{\phi}\right)^{\tau}\left(\frac{\partial \theta}{\partial \mathbf{\Omega}}\right)^2 \left\{ (\tau-1)\left[e^{\left(\frac{\theta}{\phi}\right)^{\tau}} - 1\right] - \tau\left(\frac{\theta}{\phi}\right)^{\tau} e^{\left(\frac{\theta}{\phi}\right)^{\tau}} \right\}}{\theta^2 \left[e^{\left(\frac{\theta}{\phi}\right)^{\tau}} - 1\right]^2}.
\end{aligned}
$$

Composite exponential distribution

For this composite distribution due to Teodorescu and Vernic [2], the body is described by the exponential distribution while the distribution for the tail can be arbitrary. The probability density and cumulative distribution functions are

$$
f(x) = \begin{cases} r\dfrac{\lambda e^{-\lambda x}}{1 - e^{-\lambda\theta}}, & 0 < x \le \theta, \\[2ex] (1-r)\dfrac{f_0(x)}{1 - F_0(\theta)}, & \theta < x < \infty \end{cases}
$$

and

$$
F(x) = \begin{cases} r\dfrac{1 - e^{-\lambda x}}{1 - e^{-\lambda\theta}}, & 0 < x \le \theta, \\[2ex] r + (1-r)\dfrac{F_0(x) - F_0(\theta)}{1 - F_0(\theta)}, & \theta < x < \infty, \end{cases}
$$

respectively, where $\lambda > 0$, $\theta > 0$ and $0 \le r \le 1$. Suppose f_0 and F_0 are parameterized by $\mathbf{\Omega}$. The following conditions ensure continuity and differentiability of f at θ

$$
r = \frac{f_0(\theta)\left(1 - e^{-\lambda\theta}\right)}{\lambda e^{-\lambda\theta}\left[1 - F_0(\theta)\right] + f_0(\theta)\left(1 - e^{-\lambda\theta}\right)}
$$

and

$$
\frac{-r\lambda^2 e^{-\lambda\theta}}{e^{2\lambda\theta}} = \frac{(1-r) f_0^{\prime}(\theta)\left[1 - F_0(\theta)\right] + \left[f_0(\theta)\right]^2}{\left[1 - F_0(\theta)\right]^2}.
$$

Let $\theta = \theta\left(\lambda, \mathbf{\Omega}\right)$ and $r = r\left(\lambda, \mathbf{\Omega}\right)$ be the solutions of these two conditions.

Using expressions in Section 2 of the paper, the observed information matrix is given by (2.1), where the second order partial derivatives are

$$
\begin{aligned}
\frac{\partial^2 \log L}{\partial \lambda^2} = {} & \frac{\partial^2 a}{\partial \lambda^2}\left[\log r + \log \lambda - \log\left(1 - e^{-\lambda\theta}\right)\right] + \frac{2\partial a}{\partial \lambda}\left(\frac{\partial r}{r\partial \lambda} + \frac{1}{\lambda} - \frac{\theta + \lambda \frac{\partial \theta}{\partial \lambda}}{e^{\lambda\theta} - 1}\right) \\
& + a\left[\frac{\partial^2 r}{r\partial \lambda^2} - \frac{1}{r^2}\left(\frac{\partial r}{\partial \lambda}\right)^2 - \frac{1}{\lambda^2} - \frac{2\frac{\partial \theta}{\partial \lambda} + \frac{\lambda \partial^2 \theta}{\partial \lambda^2}}{e^{\lambda\theta} - 1} + \frac{\left(\theta + \lambda\frac{\partial \theta}{\partial \lambda}\right)^2 e^{\lambda\theta}}{\left(e^{\lambda\theta} - 1\right)^2}\right] \\
& - \frac{\partial^2 a}{\partial \lambda^2}\left\{\log(1-r) - \log\left[1 - F_0(\theta)\right]\right\} - \frac{2\partial a}{\partial \lambda}\left[\frac{\partial r}{(r-1)\partial \lambda} + \frac{f_0(\theta)}{1 - F_0(\theta)}\frac{\partial \theta}{\partial \lambda}\right] \\
& + (n-a)\left[\frac{\partial^2 r}{(r-1)\partial \lambda^2} - \frac{1}{(r-1)^2}\left(\frac{\partial r}{\partial \lambda}\right)^2 + \frac{f_0(\theta)}{1 - F_0(\theta)}\frac{\partial^2 \theta}{\partial \lambda^2}\right] \\
& + (n-a)\frac{\partial \theta}{\partial \lambda}\frac{\frac{\partial f_0(\theta)}{\partial \lambda}\left[1 - F_0(\theta)\right] + \left[f_0(\theta)\right]^2 \frac{\partial \theta}{\partial \lambda}}{\left[1 - F_0(\theta)\right]^2} \\
& - \frac{\partial^2 \theta}{\partial \lambda^2}\sum_{i=1}^{n} \Delta\left(x_i - \theta\right)\left[f_0\left(x_i\right) + \lambda x_i\right] + \sum_{x_i > \theta} \frac{\partial^2 f_0\left(x_i\right)}{\partial \lambda^2} \\
& - \frac{\partial \theta}{\partial \lambda}\sum_{i=1}^{n} \Delta\left(x_i - \theta\right)\left[\frac{2\partial f_0\left(x_i\right)}{\partial \lambda} + 2x_i\right],
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial^2 \log L}{\partial \lambda \partial \mathbf{\Omega}} = {} & \frac{\partial^2 a}{\partial \lambda \partial \mathbf{\Omega}}\left[\log r + \log \lambda - \log\left(1 - e^{-\lambda\theta}\right)\right] + \frac{\partial a}{\partial \lambda}\left(\frac{\partial r}{r\partial \mathbf{\Omega}} - \frac{\lambda\frac{\partial \theta}{\partial \mathbf{\Omega}}}{e^{\lambda\theta} - 1}\right) \\
& + \frac{\partial a}{\partial \mathbf{\Omega}}\left(\frac{\partial r}{r\partial \lambda} + \frac{1}{\lambda} - \frac{\theta + \lambda\frac{\partial \theta}{\partial \lambda}}{e^{\lambda\theta} - 1}\right) \\
& + m\left\{\frac{\partial^2 r}{r\partial \lambda \partial \mathbf{\Omega}} - \frac{1}{r^2}\frac{\partial r}{\partial \lambda}\frac{\partial r}{\partial \mathbf{\Omega}} - \left[\frac{\frac{\partial \theta}{\partial \mathbf{\Omega}} + \lambda\frac{\partial^2 \theta}{\partial \lambda \partial \mathbf{\Omega}}}{e^{\lambda\theta} - 1} - \frac{\left(\theta + \lambda\frac{\partial \theta}{\partial \mathbf{\Omega}}\right)\lambda\frac{\partial \theta}{\partial \mathbf{\Omega}} e^{\lambda\theta}}{\left(e^{\lambda\theta} - 1\right)^2}\right]\right\} \\
& - \frac{\partial \theta}{\partial \mathbf{\Omega}}\sum_{i=1}^{n} \Delta\left(x_i - \theta\right)\left[x_i + \frac{\partial f_0\left(x_i\right)}{\partial \lambda}\right] - \frac{\partial^2 a}{\partial \lambda \partial \mathbf{\Omega}}\left\{\log(1-r) - \log\left[1 - F_0(\theta)\right]\right\} \\
& - \frac{\partial a}{\partial \mathbf{\Omega}}\left[\frac{\partial r}{(r-1)\partial \lambda} + \frac{f_0(\theta)}{1 - F_0(\theta)}\frac{\partial \theta}{\partial \lambda}\right] - \frac{\partial a}{\partial \lambda}\left[\frac{\partial r}{(r-1)\partial \mathbf{\Omega}} + \frac{f_0(\theta)}{1 - F_0(\theta)}\frac{\partial \theta}{\partial \mathbf{\Omega}}\right] \\
& + (n-a)\left[\frac{\partial^2 r}{(r-1)\partial \lambda \partial \mathbf{\Omega}} - \frac{1}{(r-1)^2}\frac{\partial r}{\partial \lambda}\frac{\partial r}{\partial \mathbf{\Omega}} + \frac{\partial^2 \theta}{\partial \lambda \partial \mathbf{\Omega}}\frac{f_0(\theta)}{1 - F_0(\theta)}\right] \\
& + (n-a)\frac{\partial \theta}{\partial \lambda}\frac{\frac{\partial f_0(\theta)}{\partial \mathbf{\Omega}}\left[1 - F_0(\theta)\right] + \left[f_0(\theta)\right]^2 \frac{\partial \theta}{\partial \mathbf{\Omega}}}{\left[1 - F_0(\theta)\right]^2} \\
& - \frac{\partial^2 \theta}{\partial \lambda \partial \mathbf{\Omega}}\sum_{i=1}^{n} \Delta\left(x_i - \theta\right)\left[f_0\left(x_i\right) + \lambda x_i\right] + \sum_{x_i > \theta} \frac{\partial^2 f_0\left(x_i\right)}{\partial \lambda \partial \mathbf{\Omega}} \\
& - \frac{\partial \theta}{\partial \lambda}\sum_{i=1}^{n} \Delta\left(x_i - \theta\right)\frac{\partial f_0\left(x_i\right)}{\partial \mathbf{\Omega}}
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial^2 \log L}{\partial \mathbf{\Omega}^2} &= \frac{\partial^2 a}{\partial \mathbf{\Omega}^2}\left[\log r + \log\lambda - \log\left(1 - e^{-\lambda\theta}\right)\right] + \frac{2\partial a}{\partial \mathbf{\Omega}}\left(\frac{\partial r}{r\partial \mathbf{\Omega}} - \frac{\lambda \frac{\partial \theta}{\partial \mathbf{\Omega}}}{e^{\lambda\theta} - 1}\right) \\
&+ a\left[\frac{\partial^2 r}{r\partial \mathbf{\Omega}^2} - \frac{1}{r^2}\left(\frac{\partial r}{\partial \mathbf{\Omega}}\right)^2 - \frac{\lambda \frac{\partial^2\theta}{\partial \mathbf{\Omega}^2}\left(e^{\lambda\theta} - 1\right) - \left(\lambda \frac{\partial \theta}{\partial \mathbf{\Omega}}\right)^2 e^{\lambda\theta}}{\left(e^{\lambda\theta} - 1\right)^2}\right] \\
&- \frac{\partial^2\theta}{\partial \mathbf{\Omega}^2}\sum_{i=1}^{n}\Delta\left(x_i - \theta\right)\left[f_0\left(x_i\right) + \lambda x_i\right] \\
&- \frac{2\partial\theta}{\partial \mathbf{\Omega}}\sum_{i=1}^{n}\Delta\left(x_i - \theta\right)\frac{\partial f_0\left(x_i\right)}{\partial \mathbf{\Omega}} + \sum_{x_i > \theta}\frac{\partial^2 f_0\left(x_i\right)}{\partial \mathbf{\Omega}^2} - \frac{\partial^2 a}{\partial \mathbf{\Omega}^2}\left\{\log(1 - r) - \log\left[1 - F_0(\theta)\right]\right\} \\
&- \frac{2\partial a}{\partial \mathbf{\Omega}}\left[\frac{\partial r}{(r - 1)\partial \mathbf{\Omega}} + \frac{f_0(\theta)}{1 - F_0(\theta)}\frac{\partial \theta}{\partial \mathbf{\Omega}}\right] \\
&+ (n - a)\left[\frac{\partial^2 r}{(r - 1)\partial \mathbf{\Omega}^2} - \frac{1}{(r - 1)^2}\left(\frac{\partial r}{\partial \mathbf{\Omega}}\right)^2 + \frac{\partial^2\theta}{\partial \mathbf{\Omega}^2}\frac{f_0(\theta)}{1 - F_0(\theta)}\right] \\
&+ (n - a)\frac{\partial \theta}{\partial \mathbf{\Omega}}\frac{\frac{\partial f_0(\theta)}{\partial \mathbf{\Omega}}\left[1 - F_0(\theta)\right] + \left[f_0(\theta)\right]^2 \frac{\partial \theta}{\partial \mathbf{\Omega}}}{\left[1 - F_0(\theta)\right]^2}.
\end{aligned}
$$

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