REVSTAT – Statistical Journal
Volume 0, Number 0, Month 0000, 000-000
https://doi.org/00.00000/revstat.v00i0.000

Efficiency Evaluation in Practice: a Comparison of Parametric and Non-parametric Approaches

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Received: Month 0000 Revised: Month 0000 Accepted: Month 0000

Abstract:

- In the case of efficiency evaluation, most authors typically choose only one model for evaluation. It is possible to use different models built on the basis of more or less varying approaches. However, even the choice of specific model may lead to divergent results. Using economic data, this article demonstrates the effects of model choice on efficiency results derived through the parametric method of stochastic frontier analysis and the non-parametric method of data envelopment analysis. The results show that in practice underestimated assumptions about the probability distribution or the type of estimation chosen have a major impact on efficiency.

Keywords:

- *Correlation; Data envelopment analysis; Efficiency; Stochastic frontier analysis.*

AMS Subject Classification:

- 90C08, 62P20, 91B38.

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1. INTRODUCTION

Measuring the efficiency of a company has been and always will be an important task in the field of management, as it not only indicates the past successes of a company, but also indicates the direction for its future development. The company's management, but also the government itself, are entities that would like to find out what companies should do to become fully efficient. Globalization, a growing world population and the development of emerging economies are all leading to an intensified struggle around the world, especially for natural resources. The problem arising from the limited resources available has already been pointed out by classical economists. We are currently working to drive European economies to greater resource efficiency (along with environmental protection) through various national and EU regulations.

All companies are under pressure to make the most of the resources available, as inefficient units are strongly threatened by competition. According to an OECD (2001) [25] report, competition between companies is a very important factor, as it forces them to increase efficiency and seek new business opportunities, along with the implementation of new technologies. The competitive environment, amplified by the constant development of technology, is forcing companies to look for mechanisms that will allow them not to lag behind and to achieve sustainable growth. Evaluation of the efficiency of companies is a current topic not only at the microeconomic, but also at the macroeconomic level, as the competitiveness of individual countries is largely determined by differences in the efficiency of companies within these countries.

The evaluation of efficiency is based on the idea of dividing the units examined into efficient and inefficient ones. Efficiency can then be understood in terms of productive (technical) efficiency. Productive efficiency occurs when a company cannot produce more of one good without producing less of another good. There are many methods that allow the division of units into the two groups, but only a few can be used to evaluate efficiency. Unlike the simple calculation of the ratio (for example, when assessing productivity), the evaluation of efficiency is performed having regard to the performance of other entities. Hollingsworth (2003) [16] summarizes a total of 188 published articles that deal with measuring efficiency. He found that this area is dominated by methods in which a frontier is set which determines the state of full efficiency. Using this efficiency frontier, a unit's (in)efficiency level is then determined. The properties of this frontier, as well as the exact calculation of efficiency, are specific to each approach.

In the Hollingsworth (2003) [16] study, models based on the non-parametric and deterministic data envelopment analysis (DEA) methods were represented in particular. The DEA method itself was used in 50% of the studies. In another 12% of cases, parametric methods were used, especially the stochastic frontier analysis (SFA) method. The SFA model is a stochastic model and is therefore able to distinguish between inefficiency and noise. On the other hand, DEA is a non-parametric method and therefore does not depend on econometric assumptions, the shape of the efficiency frontier or the probability distribution. The SFA method is often criticized mainly due to the assumption that the production (or cost) function should have the same functional form for all units and the fact that the SFA efficiency estimate may lead to an inconsistent parameter estimate. The DEA method is often criticized not only for its deterministic nature, but also for its high sensitivity to the omission of essential variables. Odeck and Brathen (2012) [24] consider the DEA method as the most commonly used method for evaluating technical (production) efficiency regardless of the selected industry. Of the 40 studies which they report from 1999 to 2008, almost 70% were solved by non-parametric approaches. Silva *et al.* (2016) [29] say that regardless of the choice of specific model, it is very important to check the reliability of the results using an estimate from another approach.

Lampe and Hilgers (2015) [23] focused on a closer comparison of the frequency of use of the DEA and SFA methods. Based on the Clarivate Web of Science (WoS) database, they found 4,021 publications using the DEA method and 761 articles using the SFA method. They also found that the implementation of both of these methods shows a growing trend over time. Among the current studies using the DEA method, for example, Hosseinzadeh *et al.* (2016) [18], Pérez-López *et al.* (2016) [27], Haque and Brown (2017) [15], Fei and Li (2018) [11] and Staňková and Hampel (2018) [31] may be mentioned. Current studies based on the SFA method are, for example, by Horta *et al.* (2016) [17], Chen *et al.* (2017) [8], Anaya and Pollitt (2017) [1], Balliauw *et al.* (2018) [2] and Staňková and Hampel (2019) [33].

According to the internal categorization in WoS, Lampe and Hilgers (2015) [23] also found that 51.77% of publications with the SFA method are classified in the field of "Economics". In the case of the DEA method, 39.27% of publications were in the category "Operations Research Management Science". The claim that economists use only parametric methods can therefore be considered rather historical (Poirier (1977) [28], Hallam (1992) [14]). At present, the DEA method is also widely used in the field of economics. This raises the question of the appropriate use of these methods given the problem. The choice of a specific model is critical for the calculation of technical efficiency. Researchers therefore began to look for links between these different approaches to efficiency evaluations.

Silva *et al.* (2016) [29] compared the results of DEA and SFA, based on data on Chinese banking institutions. According to their results, it can be said that both models provide similar information on the efficiency of the banking system as a whole, but become divergent at the individual level. It is banking institutions that are often mentioned as the application area for efficiency evaluation, as these are typical homogeneous units that provide the same or a similar product and thus allow good conditions for comparison. Kuosmanen *et al.* (2013) [22] measured the performance of DEA and SFA methods using a Monte Carlo simulation and observed different root mean square error (RMSE) values for DEA and SFA models working with small and large data sets. However, when comparing the results from the DEA and SFA methods, [10], Silva *et al.* (2016) [29], and Staňková and Hampel (2019) [33] recorded a statistically significant rank correlation.

2. MOTIVATION AND CONTRIBUTION

Both SFA and DEA methods allow different model settings for efficiency evaluation. However, there is only a general instruction in the literature to determine which model can be used for a particular task. The choice of a specific DEA and SFA model with a given setting is critical for the efficiency calculation, because the choice of an unsuitable model does not lead to correct results. In contrast to the "classical" cases of binary classification, where the output directly includes a two-state evaluation of the company, the evaluation and finding of a suitable model for efficiency evaluation is considerably more complicated. Comparison of DEA with SFA results can be done using correlation coefficients.

Unlike studies focused only on the calculation of technical efficiency in order to comment on the current situation of the unit being examined, in this article is attention also paid to the impact of a specific setting of the SFA and DEA model on the resulting efficiency. The main aim of this article is to compare the differences in technical efficiency at the level of individual companies as well as at the national level, through DEA and SFA models with different settings. Analyses will therefore be based on two types of data – the micro level perspective is based on individual company data and the macro level perspective is based on aggregated data for individual EU countries. Although the DEA method, unlike the SFA method, allows more variables to be used on the output side, the same variables will be used for both methods in order to more accurately compare these two different approaches.

The results of the technical efficiency will be examined using the Spearman rank correlation coefficient and the Pearson correlation coefficient. The results of the efficiency of individual companies will be further examined taking into account the link to the size of companies and also to the country in which the company is registered. In the case of analyses using aggregated data, due to the nature of the data, it is necessary to use panel models in the case of the SFA method. Within these models, the influence of different settings of SFA panel models will also be investigated.

3. DATA AND METHODS

As the analyses will be performed at the level of individual companies, as well as aggregated data for entire countries, two data sources will be used.

Aggregated data for EU countries are obtained from the EU KLEMS database. To calculate technical efficiency, a relatively homogeneous area called Basic metals and fabricated metal products, except machinery and equipment, was chosen. This specific sector includes, according to NACE code, two sectors: Manufacture of basic metals (NACE code 24) and Manufacture of fabricated metal products, except machinery and equipment (NACE code 25). However, the EU KLEMS database provides data only in aggregate for sectors 24 and 25. Financial data on individual companies are available in the Orbis database. Data on individual companies can be divided according to the prevailing NACE code and sector 24 and sector 25 thus examined separately. Although it would be possible to combine the data into one data set, with regard to the results of Silva *et al.* (2016) [29] two sub-sets will be used here. The use of two sub-sets will allow cross-sectoral comparison of the results of both methods, which will help verify the robustness of the results obtained. All statistical analyses regarding differences (or, conversely, correlations) between the results of different models in this article consider a 5% significance level.

The SFA as a parametric method requires several assumptions. The first one is the specification of the function by which the efficiency limit will be derived. Due to the availability of data, the analysis will be based on the production function. For this purpose, a two-factor translog production function was chosen to guarantee sufficient flexibility. This functional form makes it possible to model both constant and variable returns to scale. The translog functional form is non-linear, but it is possible to work with its linearized version. Although the DEA method, unlike the SFA method, allows for multiple output variables, the same variables will be used for both methods. This step will allow a more accurate comparison of these two different approaches.

3.1. Data and Models for Evaluation at the Level of Companies

The output variable in both approaches will be represented by the added value in thousands of EUR. The input variables (representing the labour and capital factors) are the value of capital in thousands of EUR and the value of costs of employees in thousands of EUR. The models will be estimated separately for 2015, 2014, 2013 and 2012 (it was not possible to obtain more up-to-date company data in the quantities requested). As already discussed in the Staňková and Hampel (2019) [33] study, in the case of some companies the value added (or capital) may attain negative values, which is an obstacle to estimating the linearized version of the production function. There are various ways to solve this problem. According to Zhu and Cook (2007) [38], the elementary method is to add a sufficiently large positive constant, which is added to the required variable which contains a negative value. However, this practice may be considered outdated. In the field of economic data research, the inverse hyperbolic sine (or arcsinh) transformation, which has similar properties to the logarithm, has gained great popularity. In addition, this transformation (unlike the logarithmic transformation) remains defined even for negative and zero values, see for example Bellemare and Wichman (2020) [6]. When evaluating efficiency at the company level, it is common to encounter negative values in practice and therefore the focus of this article is on the inverse hyperbolic sine transformation.

Within the SFA method, another topic discussed is the distribution of inefficiency. Therefore, when estimating the SFA model, the three most frequently mentioned probability distributions will be considered here – exponential, half-normal and truncated-normal. In addition to the specification of the frontier function, and to the assumption regarding the distribution of inefficiency, the last requirement is to choose the efficiency estimator. For this estimates it is possible to use two procedures. It is possible to derive technical efficiency directly through a conditional mean of efficiency as in Battese and Coelli (1988) [3]. This will henceforth be referred to as the BC estimate. The second option is to follow the ideas of Jondrow *et al.* (1982) [19] and use the conditional mean of inefficiency and convert inefficiency to efficiency in the second step. This procedure will henceforth be referred to as the JLMS estimate. A more detailed description of SFA models can be found in Kumbhakar *et al.* (2015) [21]. Given the different settings, six efficiency estimates will be obtained in each year through the SFA method, see Table 7.

Table 1: Overview of used SFA cross-sectional models due to different settings.

Distribution type	Estimator by	Label
Exponencial	JLMS	S1 JLMS
Exponencial	BC	S1 BC
Half-normal	JLMS	S2 JLMS
Half-normal	BC	S2 JLMS
Truncated-normal	JLMS	S3 JLMS
Truncated-normal	BC	S3 BC

In the case of the DEA method, several models with different settings will also be estimated. Due to the fact that small and large companies are examined in the data set, DEA models will be constructed in the variants of both constant returns to scale (CRS) and variable returns to scale (VRS). Both radial (CCR and BCC) models and SBM (slack based measure) models will be taken into account. The last change in the DEA model settings will be the model orientation. All the selected models allow you to select both input and output orientation. In addition, SBM models will be estimated in the variant of the non-oriented model. More details about DEA models can be found in Cooper *et al.* (2007) [9]. An overview of these DEA models, including their specific settings, can be found in Table 2.

There are 80 DEA models and 48 SFA models (i.e. 96 SFA efficiency estimates) for both sectors and the four reference periods. Similarly to Staňková and Hampel (2019) [33], the correlation coefficients already mentioned will be used to determine the possible link between the results of the parametric and non-parametric approaches. Through the Kruskal-Wallis analysis, the impact of the size of the company will be examined, as well as the impact of the regional affiliation of the company (i.e. the impact of the nationality of the company) on the efficiency achieved.

Table 2: Overview of used DEA models due to different settings.

Model type	**Returns**	**Orientation**	**Label**	**Model type**	**Returns**	**Orientation**	**Label**
Radial (CCR)	Constant	Input	D1	SBM	Variable	Non-oriented	D6
Radial (CCR)	Constant	Output	D2	SBM	Constant	Input	D7
Radial (BCC)	Variable	Input	D3	SBM	Constant	Output	D8
Radial (BCC)	Variable	Output	D4	SBM	Variable	Input	D9
SBM	Constant	Non-oriented	D5	SBM	Variable	Output	D10

3.2. Data and Models for Evaluation at the Country Level

In the case of the SFA method, it will be necessary to use panel models to evaluate the efficiency of individual EU countries. Even within the SFA panel models, it is necessary to introduce an assumption regarding the frontier function, the distribution of inefficiency and the efficiency estimator. At present, there are only a few studies dealing with a more comprehensive evaluation of SFA panel models due to different settings. Sun *et al.* (2017) [37] used the model proposed by Battese and Coelli (1992) [4] and its later modification in Battese and Coelli (1995) [5] together with the SFA model based on random effects according to Greene (2005) [13]. Garcia-Diaz *et al.* (2016) [12], on the other hand, used the SFA model with fixed effects according to Greene (2005) [13] together with two models according to Battese and Coelli (1988) [3] and Battese and Coelli (1995) [5]. In contrast to these studies, this article pays attention to the influence of all three mentioned assumptions. In addition to the influence of model type, probability distribution, and efficiency estimator, the robustness of the estimates when changing the output variable and the variable representing the shift of the efficiency limit over time will also be investigated.

The calculations will be performed on the basis of aggregated annual data from 1995 to 2015. The length of this panel has been selected with regard to other studies and can therefore be considered sufficiently representative. Due to the length of the panel, it is not possible to assume that the efficiency will not change over time, so time-independent models will not be estimated. As in the works of Sun *et al.* (2017) [37], Garcia-Diaz *et al.* (2016) [12] and Staňková and Hampel (2021) [35] we will use three types of models: a model called the "true" SFA model with fixed effects (TFE) and random effects (TRE) according to Greene (2005) [13], together with the model where the change in efficiency is formed on the basis of subsequent time decomposition (hereinafter only TD models) according to Battese and Coelli (1995) [5].

The SFA model according to Battese and Coelli (1995) [5] assumes a truncated-normal (TN) distribution of inefficiency. The Greene (2005) [13] models allow the application of an another probability distribution. Therefore, the exponential (EX), half-normal (HN) and truncated-normal (TN) inefficiency distributions will be compared. If SFA panel models are considered, it is necessary to make an assumption about possible changes in the case of the efficiency frontier itself. Contrary to the original ideas of Solow (1956) [30], exponential growth (E), linear growth (L) but also no change (N) within the production possibility frontier (i.e. no frontier shift) will be considered here. An overview of SFA panel models due to different settings is shown in Table 3.

Due to the availability of data, it is possible to use two variables representing the output – gross value added and gross output. Both of these variables are expressed in current basic prices in millions of national currencies within the EU KLEMS database. The labour factor is represented in the EU KLEMS database by the variable number of employees in thousands. However, thanks to the information on average national wages/salaries in the EUROSTAT database, it is possible to convert this variable into the form of employee costs. Thus, as in the case of accounting data of companies, the labour factor is represented by the same variable in the aggregated data. The capital factor will represent the value of nominal gross fixed capital formation, which in the EU KLEMS is expressed in millions of national currencies. Due to the fact, that the data are only available in national currency, it is necessary to convert financial variables into common units for comparison purposes. Therefore, the values are converted into Euros using the average annual exchange rates.

The results of the SFA panel analysis will be compared with the results of the DEA method (through correlation coefficients). For these purposes, 10 types of the most frequently mentioned DEA models are selected. These are the same types of models as in Table 2. If a large number of units (countries) are identified as fully efficient, the models from Table 2 will be constructed in the form of super-efficiency models. This is because super-efficiency models allow the classification (ranking) of fully efficient units, which will allow a more accurate comparison through correlation analysis.

All the procedures described above will be performed using the Stata 15.1 computer system (functions developed according to Belotti *et al.* (2013) [7]), the DEA SolverPro program (version 15) and the MATLAB R2020a computer system.

4. EFFICIENCY EVALUATION FOR SECTOR 24

In order to make SFA estimates, it was necessary to remove the group of companies that have very large designations in the Orbis database from the data set. After their exclusion, only companies marked as large, medium-sized, and small remained in the data set. After this adjustment (and the removal of extreme values), the composite error term had the required positive skewness and the SFA models could be estimated. The DEA and SFA models are estimated

Table 3: Overview of SFA panel models used due to different settings. Model type: time decay (TD), “true” fixed effects (TFE), “true” random effects (TRE). Output variable: value added (VA), gross output (GO). Distribution type: exponential (EX), half-normal (HN), truncated-normal (TN). Frontier shift: none (N), exponential (E), linear (L). M01 to M42 is the model designation.

Model type	Output variable	Frontier shift	Distr. type	Model name	Model type	Output variable	Frontier shift	Distr. type	Model name
TD	VA	N	TN	M01	TD	GO	N	TN	M22
TD	VA	E	TN	M02	TD	GO	N	TN	M23
TD	VA	L	TN	M03	TD	GO	N	TN	M24
TRE	VA	N	EX	M04	TRE	GO	N	EX	M25
TRE	VA	E	EX	M05	TRE	GO	E	EX	M26
TRE	VA	L	EX	M06	TRE	GO	L	EX	M27
TRE	VA	N	HN	M07	TRE	GO	N	HN	M28
TRE	VA	E	HN	M08	TRE	GO	E	HN	M29
TRE	VA	L	HN	M09	TRE	GO	L	HN	M30
TRE	VA	N	TN	M10	TRE	GO	N	TN	M31
TRE	VA	E	TN	M11	TRE	GO	E	TN	M32
TRE	VA	L	TN	M12	TRE	GO	L	TN	M33
TFE	VA	N	EX	M13	TFE	GO	N	EX	M34
TFE	VA	E	EX	M14	TFE	GO	E	EX	M35
TFE	VA	L	EX	M15	TFE	GO	L	EX	M36
TFE	VA	N	HN	M16	TFE	GO	N	HN	M37
TFE	VA	E	HN	M17	TFE	GO	E	HN	M38
TFE	VA	L	HN	M18	TFE	GO	L	HN	M39
TFE	VA	N	TN	M19	TFE	GO	N	TN	M40
TFE	VA	E	TN	M20	TFE	GO	E	TN	M41
TFE	VA	L	TN	M21	TFE	GO	L	TN	M42

on the basis of 3,116 companies in 2012, 3,180 in 2013, 2,784 in 2014, and 2,810 in 2015. These are companies representing 20 EU countries. Denmark, Estonia, Croatia, Cyprus, Lithuania, Malta, the Netherlands, and Greece could not be analyzed as data on local companies operating in sector 24 were not available. An overview of the basic characteristics of the data set is given in Table 4.

Table 4: Basic characteristics of variables used in thousands of EUR for all periods within sector 24.

Year	**2012**			**2013**		
Variable	**Value added**	**Capital**	**Costs of employees**	**Value added**	**Capital**	**Costs of employees**
Minimum	-13,247.31	-4,709.95	0.05	-3,553.30	-5,015.00	0.13
Average	3,443.07	1,449.67	2,571.05	3,405.31	1,323.75	2,497.85
Maximum	61,505.00	16,4376.70	39,956.30	59,740.41	64,018.35	37,612.43
Year	**2014**			**2015**		
Variable	**Value added**	**Capital**	**Costs of employees**	**Value added**	**Capital**	**Costs of employees**
Minimum	-2,475.97	-292.32	0.01	-4,251.54	-180.00	0.01
Average	3,107.57	1,337.68	2,195.83	3,154.49	1,445.12	2,230.78
Maximum	51,881.23	16,4376.70	41,778.54	38,750.33	16,4376.70	41,556.05

Figure 1 shows the median efficiencies (circle) in % and the average efficiencies (square) in %, including the standard deviation for the SFA models (S1 to S3) and DEA models (D1 to D10) in all periods. If we focus on the evaluation of the whole sector, according to the results of the DEA models, most of the companies in the sector are inefficient. The average and median level of efficiency for the whole sector is typically around 15%. Even in the case of the SFA models, most companies are labeled as inefficient, but the overall efficiency score (whether in the form of an average or median) for the whole sector is higher here.

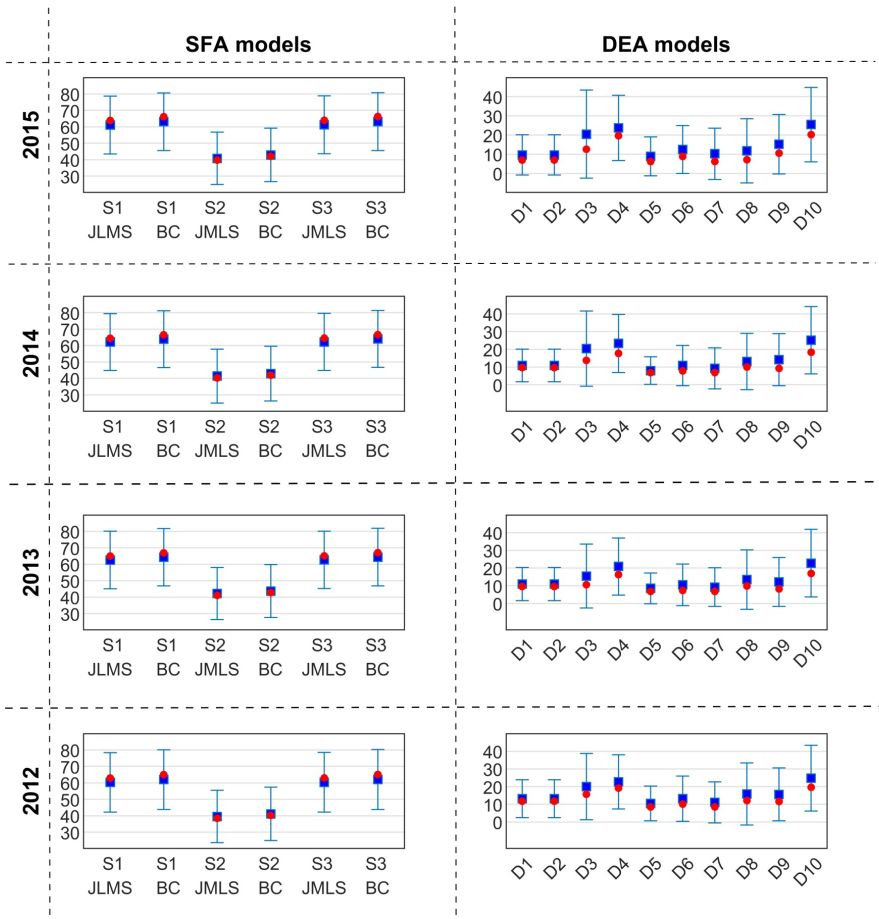

Figure 1: Median efficiency (circle) in % and average efficiency (square) in % including standard deviation for the SFA models (S1 to S3) and DEA models (D1 to D10) in all periods in sector 24

Based on the results, it is possible to conclude that the choice of a particular estimate (i.e., JLMS or BC) has no appreciable effect on the resulting efficiency. In general, the BC estimate leads to an efficiency about 0.02 higher than the JLMS estimate (given the number of observations, this is a statistically significant difference). In terms of absolute values, within the group of SFA models, the S2 model is the most different, showing systematically lower efficiency values (but the same variability) throughout the period compared to the other SFA models. However, the results of the S2 model are closest to the results of the DEA models. Similar average and median values to the S2 model are obtained for the D3, D4, and D10 models. These DEA models are different from the other DEA models not only in their average (and median) efficiency scores, but also in their variability. The D3 model may be identified as the model with the largest variability in the estimated efficiency scores.

In addition to examining the absolute values of the estimated efficiency scores, it is possible to examine the results of parametric and non-parametric methods through correlation coefficients. Most researchers focus on the order derived from the level of efficiency rather than on its absolute value, so here we primarily focus on the Spearman correlation coefficient. Nevertheless, the relationships found are similar when the Pearson correlation coefficient is used. Figure 2 shows the values of the Spearman rank correlation coefficient in the form of a colormap for the SFA models (S1 to S3) and DEA models (D1 to D10) in all periods.

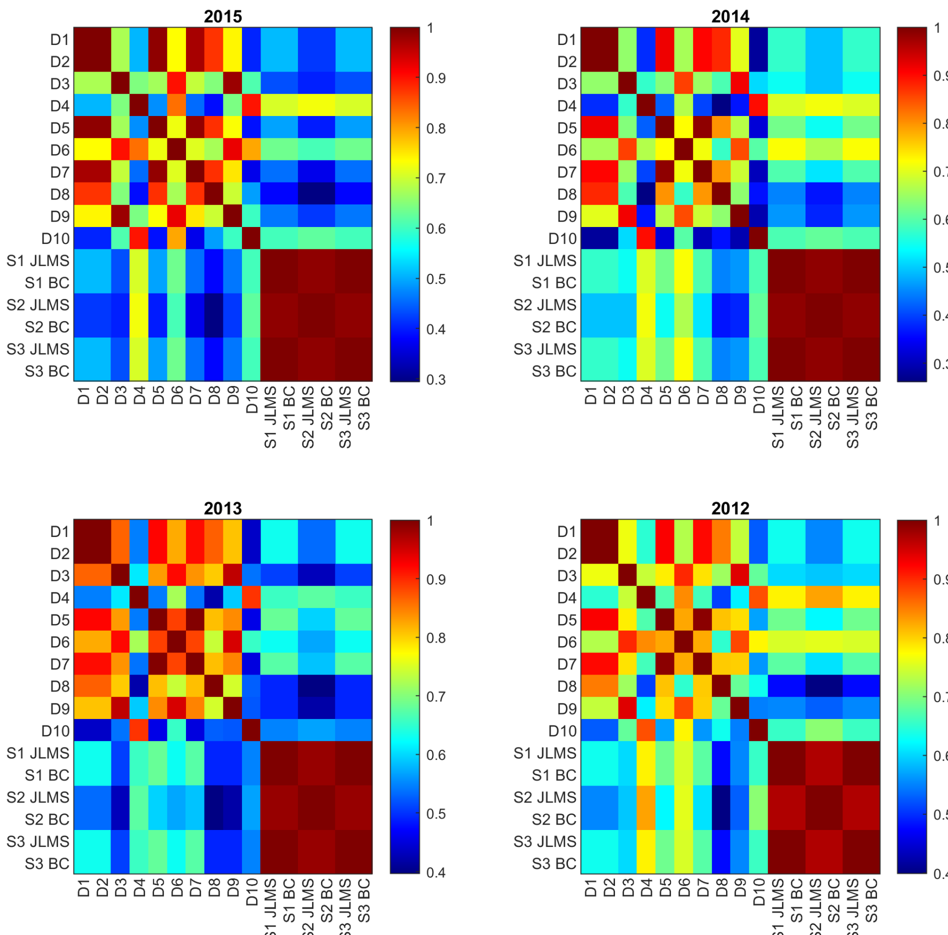

Figure 2: Colormap of the Spearman rank correlation coefficient between the SFA models (S1 to S3) and DEA models (D1 to D10) in all periods in sector 24

In terms of ranking, the results of all the SFA models are very similar. The values of the Spearman correlation coefficients between these models never fell below 0.97. The effect of the estimation itself (i.e., the JLMS and BC estimates) is not evident here. Within the group of DEA models, the influence of the specific model setting is noticeable. On average, there are correlations around 0.75, which may be described as a moderately strong dependence. A stronger link is found for models that assume the same returns to scale (e.g., the D1 model with the D5 model). The orientation of the model also has an influence here. In terms of correlation, the D10 model is the most different, with correlations with other DEA models ranging from 0.27 (in 2014) to 0.89 (in 2015). According to the results, the D4 model also differs, having the lowest correlations 0.1 points higher than the D10 model (the D4 and D10 models have the highest measured correlations at the same level).

Based on the analysis of the results between the SFA and DEA models, it is possible to conclude that the S2 model has lower correlation coefficient values than the S1 and S3 models in eight out of 10 cases. The S2 model has higher correlations only with the D4 and D10 models. Although in terms of absolute values, the S2 model is close to the D3 model, in terms of correlation the relationship between the two models is weaker than with the other DEA models (typically 0.1 points lower). However, even with this decline, it is still a moderate rank correlation (values around 0.5). Systematically, the SFA models show the highest correlations with the D4, D6, and D5 models (ranked by strength of relationship). In the case of the three DEA models and the SFA models, the correlation coefficients range from 0.55 to 0.83. Due to the high number of units, all correlation coefficients greater than 0.03 in absolute value are statistically significant.

The results of the technical efficiency score are also examined with respect to possible division factors. First, the efficiency results are divided according to the geographical nationality (country) of the companies. Due to the non-fulfillment of the assumption found in respect of the normal distribution in the efficiency results, a non-parametric analysis was chosen. Based on the Kruskal-Wallis analysis, it is possible to state that in all the SFA and DEA models there are significant differences between some countries in the median of their technical efficiency. Based on the medians, countries may be ranked separately for each model.

If we focus only on the results of the group of SFA models, we find that the S1 and S3 models (regardless of the type of estimate chosen) have similar efficiency results for each country. However, the S2 model gives systematically lower median efficiency values compared to the other SFA models. Under the SFA method, Portugal achieves an imaginary victory throughout the period. Other countries that ranked highest using the SFA method include Spain and Hungary. On the other hand, Latvia had the worst position throughout the period in all the SFA models. Leaving aside the best and worst positions, in the rest of the rankings the order of countries varies slightly depending on the point in time. Considering the median values obtained, there are also statistically insignificant differences (especially for countries with lower representation such as Ireland). For this reason, even minor changes in the order are insignificant for us.

In the case of the DEA models, the results are more influenced by the specific model settings. The individual DEA models do not agree with each other even in the case of the best and worst positions. For example, in the D7 or D8 model, similar median efficiencies were obtained for both Portugal and Latvia. The most similar models to the SFA models in terms of country median values are D4, D6, and D10. In these models, Portugal has several times higher median efficiency than Latvia. Just as there are lower median values for the whole industry in the DEA method compared to the SFA method (see Figure 1), there are also lower median values for individual countries in the DEA models.

Analysis of the results with respect to company size reveals that for all the SFA models, an increase in efficiency may be seen as company size increases. The median efficiency of large companies in the S1 and S3 models converges to 0.7. On the other hand, for the smallest companies it is around 0.5. The S2 model shows the same trend, but the median efficiency values are about 0.2 points lower than in the other SFA models. Given the number of observations, these are statistically significant differences.

In the case of the individual DEA methods, there is not as much agreement between the results of the individual models as in the case of the SFA method. For some models (i.e., D4, D6, and D10), the same trend as in the SFA models is identified (i.e., as the company grows, efficiency increases). In the second group of models, which consists of the D5, D7, D8, and D9 models, there are similar efficiencies between large and small companies. Medium-sized companies in these models have the highest median efficiency. In the third group of models (consisting of the D1, D2, D3, and D9 models), on the other hand, medium-sized companies have the lowest efficiency.

5. EFFICIENCY EVALUATION FOR SECTOR 25

Within sector 25, there are many more companies than in the previous sector 24. Unfortunately, this increase also brings with it greater heterogeneity and problems with estimating the SFA model. Removing very large companies was not enough to solve this problem, as the data set still did not lead to a model where the composed error term would have the positive skew that is necessary for SFA estimation based on production functions. In order to achieve a more homogeneous data set and to solve the problem with the composed error component, it was necessary to filter only medium-sized and small companies. After this adjustment (and the elimination of extreme observations), 33,860 companies remained in the data set with available data from 2012, 33,348 from 2013, 34,944 from 2014, and 33,393 from 2015. These are companies representing 17 EU countries. Representatives of Denmark, Estonia, Croatia, Cyprus, Lithuania, Malta, the Netherlands, Portugal, Greece, Spain, and Sweden could not be analyzed as data on local companies in sector 25 were not available. The basic characteristics of the data set are given in Table 5.

Table 5: Basic characteristics of variables used in thousands of EUR for all periods within sector 25.

Year	**2012**			**2013**		
Variable	**Value added**	**Capital**	**Costs of employees**	**Value added**	**Capital**	**Costs of employees**
Minimum	-4,016.22	-5,730.41	0.01	-102.20	-1,018.23	0.00
Average	710.20	115.60	539.78	653.02	90.73	471.70
Maximum	16,081.32	56,248.00	14,195.97	14,470.87	56,248.00	9,192.61
Year	**2014**			**2015**		
Variable	**Value added**	**Capital**	**Costs of employees**	**Value added**	**Capital**	**Costs of employees**
Minimum	-8,608.86	-404.93	0.34	-574.99	-620.83	0.08
Average	718.38	114.12	540.55	669.60	102.34	492.13
Maximum	15,660.04	19,061.28	10,312.72	11,403.01	19,061.28	9,004.49

According to the estimated SFA models, efficiency in this sector is generally at a high level. On the contrary, the DEA method shows that most companies operate very inefficiently, see Figure 3. Figure 3 plots the median efficiencies (circle) and average efficiencies (blue), including the standard deviation for the SFA models (S1 to S3) and the DEA models (D1 to D10). Similar to the previous sector, all the SFA models have similar levels of variability, but the S2 model shows lower average and median efficiencies in sector 25. Also, in this sector it may be found that the BC estimate leads to systematically higher efficiency (about 0.01 higher) than the JLMS estimate. Also, on the side of the DEA models, similar symptoms with sector 24 may be seen. In terms of variability, but also average and median values, the D3, D4, and D10 models differ. However, even in these models, where the values of average efficiency for the whole sector are higher, the average efficiency does not exceed 25%. In 2012, the D8 model is also different in terms of variability.

As in sector 24, attention is paid to correlation analysis in sector 25. Figure 4 shows the results of the Spearman rank correlation coefficient in the form of a colormap of the Spearman rank correlation coefficient for the SFA models (S1 to S3) and the DEA models (D1 to D10) in all periods.

As in sector 24, there are strong bonds between all SFA models as the correlation coefficients here are very close to one regardless of the selected period. In terms of correlation within the group of DEA models, the models may be divided into two groups. The D4 and D10 models form one group and all the other models form a second group. Within each group of models there is a strong correlation (ranging from 0.70 to 0.99). However, there is a weak relationship across the two groups (correlation typically around 0.4).

If we focus on the correlations between the group of DEA models and SFA models, it is possible to talk about a moderate rank correlation. Throughout the period, the values ranged from 0.29 to 0.75. Systematically, the highest correlations were achieved between the SFA models and the D4, D6, and D10 models (ranked by highest correlation coefficients). As in sector 24, the values

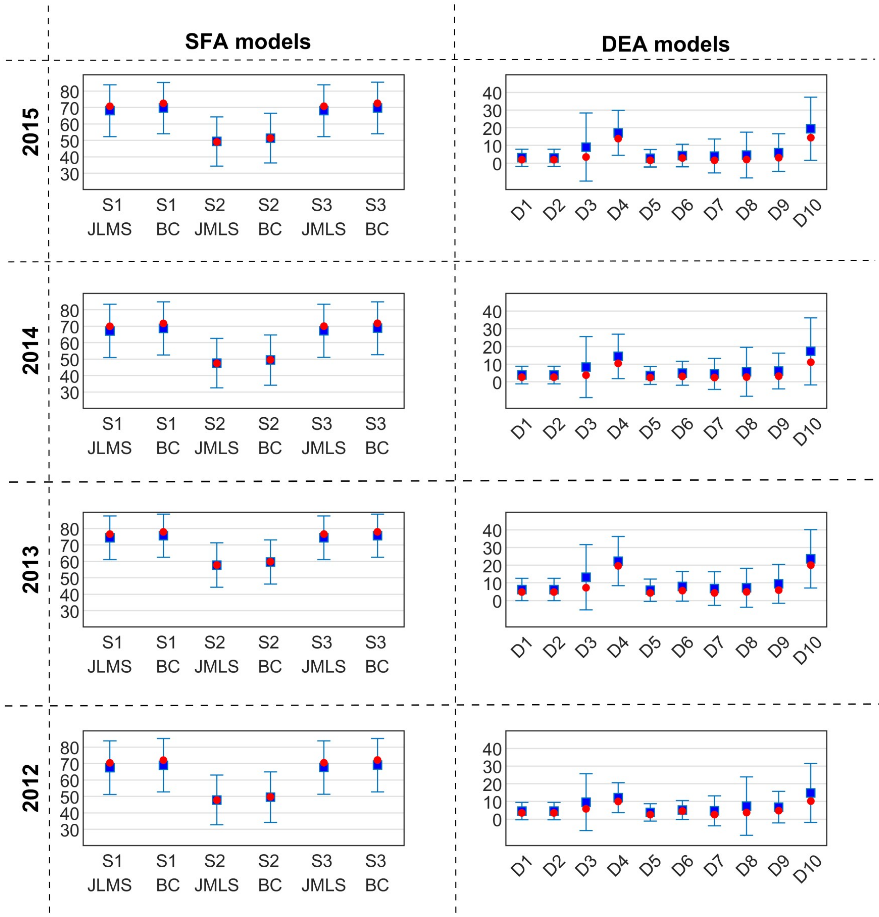

Figure 3: Median efficiency (circle) in % and average efficiency (square) in % including standard deviation for the SFA models (S1 to S3) and DEA models (D1 to D10) in all periods in sector 25.

of the correlation coefficients for the S1 model are similar to those for the S3 model. The S2 model correlations are typically 0.02 points lower than for other SFA models. Due to the large number of observations, the presented correlations are always statistically significant.

As in the previous sector, according to the Kruskal-Wallis analysis, there are significant differences in the median efficiency of individual countries. In the case of the SFA models, the decrease in the median efficiency of the S2 model is again visible in the results. In terms of ranking, however, the country rankings are the same for all the SFA models. The top three countries in terms of median values over the whole period are Hungary, Poland, and Belgium. The worst position in this sector varies with the time period. For example, in 2015, the worst position is occupied by Latvia; however, in 2012, the worst position is held by Austria (Latvia is the penultimate country).

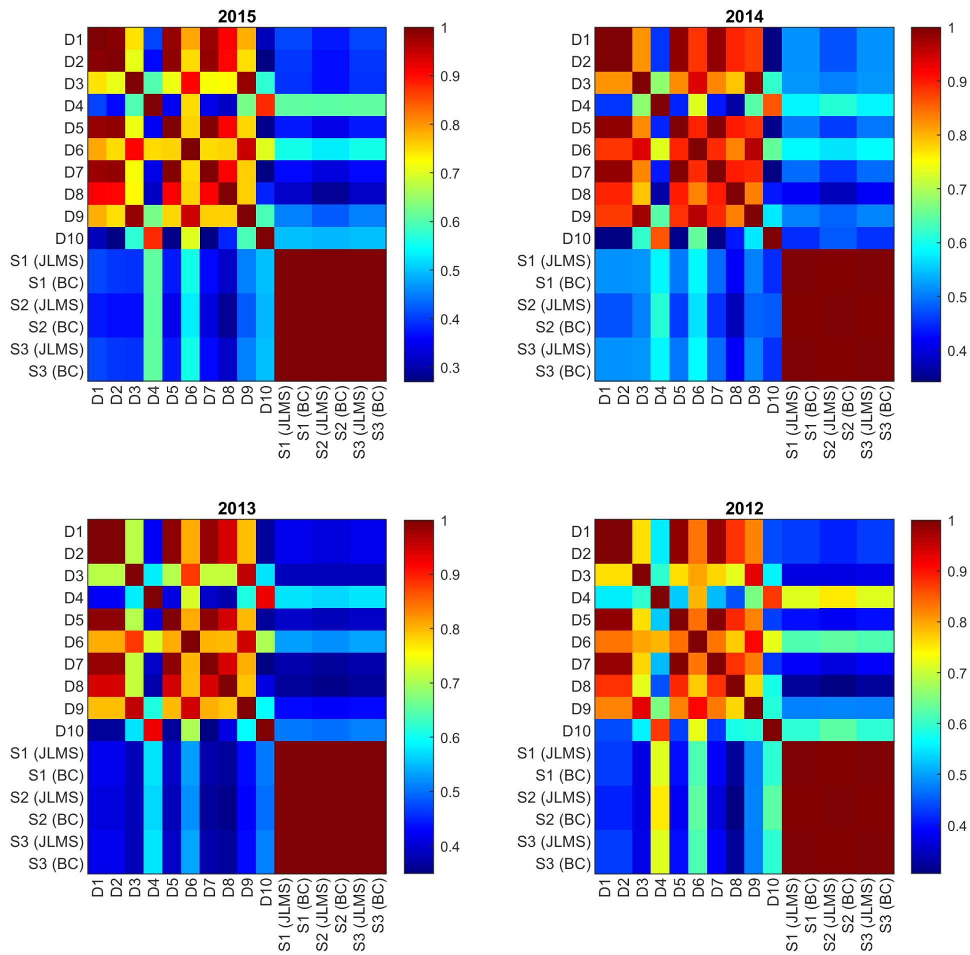

Figure 4: Colormap of the Spearman rank correlation coefficient between the SFA models (S1 to S3) and DEA models (D1 to D10) in all periods in sector 25.

In the case of the DEA models, as in the case of the previous sector, it is more difficult to find a clear pattern between the models. The difference is particularly noticeable for the D3 model, which is the only one with Romania in first place. The other models have this country roughly in the middle of the rankings. The influence of the specific settings of the DEA models is still very evident in the results. For example, the D1, D2, D5, and D7 models have Belgium as the worst ranked country among the top three countries in the SFA models.

In addition, efficiency is examined in terms of company size. Here, the Kruskal-Wallis analysis is replaced by the Wilcoxon test, as only two groups of medians remained. Although only two categories of companies remain in sector 25, medium-sized companies have a higher median efficiency than small companies in all the SFA models. As mentioned above, due to the large number of observations, these are always statistically different results. Systematically, the

S2 models show about 0.2 points lower median efficiency values than the other SFA models. The properties of the SFA estimates are therefore the same in this respect in both sectors.

Even within the group of DEA models, common features may be found. In all the models, medium-sized companies have a higher median efficiency than small companies. Therefore, in this respect, the result of the DEA and SFA methods is the same. The differences are evident in the absolute values of the medians. In the D4, D6, and D10 models, the differences in the median efficiency reach up to 0.1 points. For the other models, the differences are typically 0.01 points. Given the number of observations, even this slight difference is statistically significant.

6. EFFICIENCY EVALUATION AT THE C OUNTRY LEVEL

Unfortunately, four countries did not have any values available at all within the NACE C sector (Belgium, Croatia, Ireland, and Cyprus). Some countries (Bulgaria, Estonia, Lithuania, Latvia, Luxembourg, Hungary, Malta, Poland, Portugal, Romania, and Slovenia) had data available only for the whole of sector C and not for the sectors 24 and 25 under observation. Slovakia had data available only from 2004; therefore, it was deleted from the data file. The Netherlands had data on capital only available from 1999, but as there are only four missing observations, the Netherlands was kept in the file. In total, it was possible to make estimates for 12 EU countries. Table 6 represents the average annual values of the variables used in individual years (costs of employees in millions of EUR, other variables in billions of EUR).

Table 6: Average values of variables used for efficiency evaluation according to aggregated data in individual years (costs of employees in millions of EUR, other variables in billions of EUR).

Year	Gross output	Value added	Costs of employees	Fixed capital	Year	Gross output	Value added	Costs of employees	Fixed capital
1995	35.91	12.91	549.06	17.44	2006	60.83	18.29	748.65	27.44
1996	35.42	12.97	570.05	17.91	2007	67.68	19.74	785.03	28.94
1997	37.77	13.37	583.80	18.35	2008	67.02	19.38	825.57	29.93
1998	39.56	14.11	609.18	19.60	2009	48.10	14.87	779.23	28.61
1999	38.49	13.82	633.01	20.33	2010	56.50	16.32	769.86	28.30
2000	44.12	15.54	644.44	22.79	2011	62.90	17.43	801.70	28.98
2001	44.70	15.60	671.36	23.52	2012	60.38	17.46	815.67	29.27
2002	44.24	15.33	684.73	23.74	2013	58.29	17.52	817.64	28.77
2003	44.60	15.27	703.85	23.40	2014	58.81	18.14	836.63	28.52
2004	49.44	15.98	712.81	24.00	2015	59.61	18.97	866.94	29.15
2005	53.54	16.94	728.78	26.46					

Of the originally planned 42 SFA models (i.e., 84 efficiency estimates), only 36 SFA models (i.e., 72 efficiency estimates) could be considered for subsequent analyses. Due to convergence issues, TFE models with truncated-normal probability distributions (i.e., models M19–M21 and M40–M42) are not included in the analyses. In the case of the DEA method, the analyses contain all 10 planned

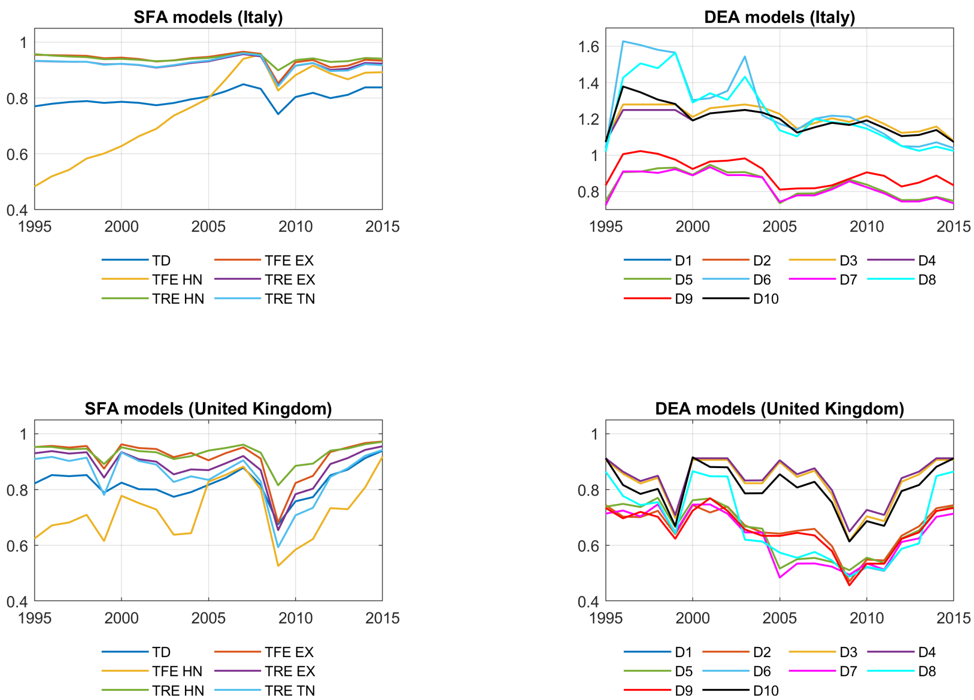

Figure 5: Development of the efficiency of the individual SFA and DEA models

model types.

When the panel data are processed by SFA panel models, the effect of all three factors may be seen in the results of the individual SFA models, i.e., the type of model, the chosen probability distribution, and the procedure for estimating the efficiency itself. The relationships found are the same for both models with the output presented via a value added variable and gross output variable.

To visually demonstrate the difference in results, Figure 5 shows the evolution of efficiency for the two countries for the value added output variable. In the case of the SFA models, these are the aggregate results of the BC estimate.

In the case of the SFA models, the TD models and TFE models with half-normal distribution show the largest differences. These models have systematically lower absolute values of calculated efficiency (see Table 7) and also show slight differences in trend. Typically, these two models provided efficiency results about 20 percentage points lower than the remaining SFA models. Even in the case of DEA models, differences in absolute efficiency are identified. The implementation of the super-efficiency models here allowed countries to obtain efficiencies even higher than 1, resulting in sector-wide efficiencies on average at a higher level than in the SFA models. The systematically lowest results are achieved by the D5, D7, and D9 models. On average, these values are 20 to 30 percentage points lower than the other DEA models (see Table 8).

Table 7: Median and average efficiency values according to the SFA panel models (by model type with the value added output variable) for the whole sector

Estimator	**JLMS**					
Model	**TD**	**TFE EX**	**TFE HN**	**TRE EX**	**TRE HN**	**TRE TN**
Median	0.7620	0.9389	0.9348	0.9351	0.9386	0.7903
Average	0.7631	0.9195	0.9370	0.9139	0.9165	0.7237
Estimator	**BC**					
Model	**TD**	**TFE EX**	**TFE HN**	**TRE EX**	**TRE HN**	**TRE TN**
Median	0.7648	0.8924	0.9301	0.8676	0.9177	0.7263
Average	0.7650	0.9521	0.9665	0.9342	0.9397	0.7952

Table 8: Median and average efficiency values according to the DEA super-efficiency models (by model type with the value added output variable) for the whole sector

Model	**D1**	**D2**	**D3**	**D4**	**D5**
Median	0.8402	0.8402	1.0000	1.0000	0.7652
Average	0.8150	0.8150	1.0047	1.0661	0.7607
Model	**D6**	**D7**	**D8**	**D9**	**D10**
Median	1.0707	0.7636	1.0000	0.8370	1.0000
Average	1.0048	0.7579	0.9649	0.8123	1.0538

A detailed analysis of technical efficiency according to the individual SFA panel models is performed using correlation coefficients. Since the output variable turned out to be the least significant factor, we focused on the efficiency changes caused by the chosen type of model and the chosen estimate. The main differences found are evident in both types of correlation coefficients. Here, special attention is paid to the derived ranks of companies. Therefore, Table 9 shows the Spearman correlation coefficients. A detailed graphical depiction of the correlation coefficients of each model is shown as a colormap in Figure 6 in the Appendix.

Table 9: Spearman correlation coefficients for the individual SFA panel models (by model type with the value added output variable) for the whole sector

Model /Estimator		TD JLMS	TRE EX JLMS	TRE HN JLMS	TRE TN JLMS	TFE EX JLMS	TFE HN JLMS	TD BC	TRE EX BC	TRE HN BC	TRE TN BC	TFE EX BC	TFE HN BC
TD	JLMS	0.9258	0.4939	0.5917	0.5267	0.4851	0.4188	0.9505	0.6716	0.7962	0.5267	0.4851	0.4188
TRE EX	JLMS	0.4939	0.7778	0.8423	0.8301	0.8132	0.4823	0.4939	0.3104	0.4113	0.3328	0.8132	0.4823
TRE HN	JLMS	0.5917	0.8423	0.8842	0.8254	0.7990	0.5248	0.5917	0.4076	0.5323	0.4349	0.7990	0.5248
TRE TN	JLMS	0.5267	0.8301	0.8254	0.9436	0.9438	0.4633	0.5267	0.3859	0.4784	0.4509	0.9438	0.4633
TFE EX	JLMS	0.4851	0.8132	0.7990	0.9438	0.9619	0.3378	0.4851	0.3638	0.4479	0.4338	0.9746	0.3378
TFE HN	JLMS	0.4188	0.4823	0.5248	0.4633	0.3378	0.6533	0.4188	0.2224	0.3058	0.2163	0.3378	0.7688
TD	BC	0.9505	0.4939	0.5917	0.6686	0.4851	0.4188	0.9258	0.6716	0.7962	0.6686	0.4851	0.4188
TRE EX	BC	0.6716	0.3104	0.4076	0.3859	0.3638	0.2224	0.6716	0.7683	0.8512	0.8440	0.3638	0.2224
TRE HN	BC	0.7962	0.4113	0.5323	0.4784	0.4479	0.3058	0.7962	0.8512	0.8818	0.8411	0.4479	0.3058
TRE TN	BC	0.6686	0.3328	0.4349	0.4509	0.4338	0.2163	0.6686	0.8440	0.8411	0.8639	0.4338	0.2163
TFE EX	BC	0.4851	0.8132	0.7990	0.9438	0.9746	0.3378	0.4851	0.3638	0.4479	0.4338	0.9619	0.3378
TFE HN	BC	0.4188	0.4823	0.5248	0.4633	0.3378	0.7688	0.4188	0.2224	0.3058	0.2163	0.3378	0.7028

Within the group of TD models, regardless of the type of estimation chosen, there is always a strong correlation between the models. However, within the group of TRE models, the influence of the chosen estimate is apparent. In the case of the JLMS estimate, there is a strong relationship between the results of the TRE models (correlation between 0.78 and 0.94). A similar situation occurs if we focus on the relationships within the group of TRE models with the BC estimate (correlation between 0.77 and 0.88). However, when comparing the results of the same TRE models using both the JLMS and BC estimates, we find major differences. Correlations here range from 0.31 to 0.53. If we examine the strength of the link between the TD and TRE models, the highest correlations are found between the TD models where the influence of the estimate is negligible, and the TRE models where efficiency is calculated via the BC estimate. In this case, the correlations range from 0.67 to 0.80. When applying the JLMS estimate, the correlation coefficients decrease by almost 0.2 points.

Finding regularities in the results of TFE models is more complicated. Indeed, it was not possible to apply a truncated-normal distribution to these models and, as may be seen in Figure 5 above, the results with a half-normal distribution show significant difference. Therefore, in the case of the TFE models, the chosen probability distribution assumption has the largest effect on the results, overriding the effect induced by a different estimate. If we focus on the highest correlation coefficients, the results of the TRE models are most consistent with the TFE model with an exponential distribution. In the case of the JLMS estimate, the correlations are above 0.8. In the case of the BC estimate, the correlations are around 0.4. A moderate correlation is also measured when comparing the results of the TD and TFE models.

Last but not least, attention is paid to a comparison of the results of panel SFA models and DEA models. Since the DEA models are estimated individually, due to the scope of the analyses, Table 10 presents the results of the correlation analysis only for the first and last observation periods. A more detailed view of the individual panel SFA models is shown in the colormap of Spearman correlation coefficients in Figure 7 in the Appendix. The values of the correlation coefficients in Table 10 are calculated for a specific year; therefore, only correlations greater than 0.576 are statistically significant. A statistically significant correlation is therefore measured in 2015 between the DEA and DT models and between the DEA and TRE models with the BC estimate. In 1995, however, the results are slightly different. There is still a strong link between the DEA and TRE models with the BC estimate; however, there is no longer a link between the DEA and TD models. On the contrary, there is a connection between the DEA and TFE models with an exponential distribution. In this correlation analysis, time emerged as a factor influencing the efficiency results. Only the associations between the DEA and TRE models are consistent throughout the study period.

Table 10: Spearman correlation coefficients between the DEA and SFA models (by model type with the value added output variable) for the first and last monitored periods

2015	TD JLMS	TRE EX JLMS	TRE HN JLMS	TRE TN JLMS	TFE EX JLMS	TFE HN JLMS	TD BC	TRE EX BC	TRE HN BC	TRE TN BC	TFE EX BC	TFE HN BC
D1	0.6000	-0.0212	0.0576	-0.1091	-0.1697	0.3404	0.6000	0.8636	0.8394	0.7758	-0.1697	0.3404
D2	0.6000	-0.0212	0.0576	-0.1091	-0.1697	0.3404	0.6000	0.8636	0.8394	0.7758	-0.1697	0.3404
D3	0.3667	0.0818	0.1394	-0.0061	0.0152	0.0410	0.3667	0.7636	0.7697	0.7424	0.0152	0.0410
D4	0.7183	0.1746	0.2946	0.1017	0.0942	0.4361	0.7183	0.7593	0.8550	0.7411	0.0942	0.4361
D5	0.6091	-0.0758	-0.0091	-0.1515	-0.2242	0.3192	0.6091	0.8606	0.8121	0.7879	-0.2242	0.3192
D6	0.5606	0.1758	0.2939	0.1970	0.1909	0.1127	0.5606	0.8364	0.8121	0.8758	0.1909	0.1127
D7	0.6091	-0.0758	-0.0091	-0.1515	-0.2242	0.3192	0.6091	0.8606	0.8121	0.7879	-0.2242	0.3192
D8	0.3788	0.1424	0.2000	0.0394	0.0636	0.0099	0.3788	0.7606	0.7636	0.7303	0.0636	0.0099
D9	0.6000	-0.0212	0.0576	-0.1091	-0.1697	0.3404	0.6000	0.8636	0.8394	0.7758	-0.1697	0.3404
D10	0.7183	0.1746	0.2946	0.1017	0.0942	0.4361	0.7183	0.7593	0.8550	0.7411	0.0942	0.4361
1995	TD JLMS	TRE EX JLMS	TRE HN JLMS	TRE TN JLMS	TFE EX JLMS	TFE HN JLMS	TD BC	TRE EX BC	TRE HN BC	TRE TN BC	TFE EX BC	TFE HN BC
D1	0.3303	0.3303	0.3970	0.6394	0.6121	-0.0061	0.3303	0.8545	0.7697	0.8091	0.6121	-0.0061
D2	0.3303	0.3303	0.3970	0.6394	0.6121	-0.0061	0.3303	0.8545	0.7697	0.8091	0.6121	-0.0061
D3	0.3212	0.3697	0.5394	0.6788	0.7909	-0.1303	0.3212	0.7061	0.6727	0.8000	0.7909	-0.1303
D4	0.0636	0.2182	0.2364	0.4818	0.5030	-0.1152	0.0636	0.7212	0.5061	0.6636	0.5030	-0.1152
D5	0.1909	0.3061	0.2333	0.5697	0.5182	-0.0273	0.1909	0.7455	0.6121	0.6121	0.5182	-0.0273
D6	0.2606	0.1636	0.3212	0.4364	0.4697	-0.0879	0.2606	0.7303	0.6061	0.7576	0.4697	-0.0879
D7	0.1909	0.3061	0.2333	0.5697	0.5182	-0.0273	0.1909	0.7455	0.6121	0.6121	0.5182	-0.0273
D8	0.4030	0.4091	0.5697	0.7000	0.8000	-0.0818	0.4030	0.7273	0.7121	0.8212	0.8000	-0.0818
D9	0.3303	0.3303	0.3970	0.6394	0.6121	-0.0061	0.3303	0.8545	0.7697	0.8091	0.6121	-0.0061
D10	0.0636	0.2182	0.2364	0.4818	0.5030	-0.1152	0.0636	0.7212	0.5061	0.6636	0.5030	-0.1152

7. SUMMARY AND DISCUSSION

In the case of the analyses based on cross-sectional data, the results of differently set SFA models (i.e., S1 to S3) are almost identical in terms of correlation. However, the assumption of a half-normal distribution in the model led to systematically lower absolute efficiency values. Greater differences are found between DEA models than within SFA models. Due to the characteristics found, the D3, D4, and D10 models may be identified as the most distinct as they are models with a higher variability of results and also with a higher level of efficiency than the other DEA models. If we accept the idea that the most highly correlated results of the different approaches are the most plausible results, then the D4, D6, and D10 models would be chosen in terms of correlation. These models are connected through the assumption of variable returns to scale. The D4 and D10 models are output oriented, whereas the D6 model does not have a strictly defined orientation. Therefore, if the application of the research requires that both the inputs of the company are reduced and the outputs of the model are at same time increased, then the D6 model may be recommended. These findings are valid for both (micro)sectors analyzed, with the exception that in sector 25 the links are strengthened by the fact that the selection of data here produced a more homogeneous data set. In general, there is a moderate to strong correlation between the DEA and SFA models. Similar results in terms of correlation analysis were also achieved in the article by Staňková and Hampel (2019) [33], where values of the Spearman correlation coefficient between 0.08 and 0.66 were measured between SFA and radial DEA models within the construction industry. Strong correlations (values around 0.7) between DEA and SFA models were also found in Oh and Shin (2015) [26].

In both sectors, the results of SFA models show a higher level of efficiency than DEA models. Similar differences in average efficiency between these methods may also be found in other studies from other sectors. For example, in the article of Silva *et al.* (2016) [29] SFA models for the banking sector also led to significantly higher average efficiency results compared to the DEA method. For their SFA models, the average efficiency was typically around 90%. On the other hand, for the BCC input-oriented model (the only DEA model in their work) the efficiency was up to a third lower. In addition, their DEA models were characterized by higher variability compared to the SFA models. All these findings are consistent with the results obtained in this article. The BCC input-oriented model in sectors 24 and 25 has the highest measured variability in the results. The difference between the average efficiencies in the SFA and DEA models is more pronounced here than in the study by Silva *et al.* (2016) [29], but it is a consequence of the problem with negative values and also of the fact that both smaller and larger companies were present in the data set and the level of efficiency is significantly affected by the size of the company.

Company size and geographic area are two factors affecting the level of efficiency in cross-sectional data. However, a detailed analysis showed that in terms of country rankings, there are differences in the SFA models and the individual DEA models. Here, the results agree with Silva *et al.* (2016) [29], because even here the SFA and DEA methods become divergent at the individual level. Pairwise analysis reveals that differences in country efficiency levels are not always statistically significant (especially for countries that had only dozens of representatives in the data set), making small differences in country rankings in the models unimportant. If we only focus on statistically significant differences between countries, we are able to identify DEA models that are close to SFA models in their results. As no studies of this magnitude are currently available for the selected sectors 24 and 25, it is not possible to make a direct comparison of the results of technical efficiency to assess which method has yielded more accurate results. Studies focusing on the efficiency of a selected sector at the level of countries from across the EU usually use aggregated data and, unlike this article, SFA analyses are conducted through panel models. In this article, SFA panel models are also used and are commented on separately later in this chapter.

In the case of the distribution of companies according to their size (where there are enough observations for statistically significant differences in all categories), three DEA models may be identified whose results are consistent with the results of the SFA method. These are the aforementioned D4, D6, and D10 models. For these three DEA models as well as the SFA models, it is true that as a company grows, so does its efficiency. Therefore, the effect of economies of scale is shown here. A similar relationship has been observed in other economic sectors such as banking (Haque and Brown (2017) [15]), forestry (Staňková *et al.* (2022) [36]), and the automotive industry (Kovárník and Staňková [20]). However, a question for future research (both for cross-sectional or panel data analysis) is whether differences at the country level are simply a reflection of the different structure of companies in a given country (i.e., company size). The EU common policy may have already leveled the playing field between countries to such an extent that the market structure and size of companies will be the main factor in the future.

The use of SFA models working with cross-sectional data may be criticized because of omitted variables bias. The missing variable (and misspecification of the model) is then erroneously translated into the efficiency value. To minimize this risk, our analyses cover two micro views as well as one macro view (panel analysis based on aggregate data). In the case of SFA models, it is possible to use "true" models with effects (fixed or random) or models without effects (time decay models). Sun *et al.* (2017) [37] state in their study, inter alia, that TD models generally lead to situations where the model overestimates the value of inefficiency, so as a result the values of technical efficiency according to TD models are lower than in the case of other models with time-varying efficiency. The results in this article are in accordance with this statement, because on average the resulting values of technical efficiency for TD models are actually lower than for both "true" models.

Sun *et al.* (2017) [37] observed a weak correlation between TFE and TD models according to Battese and Coelli (1992) [4] and Battese and Coelli (1995) [5]. The resulting values of their correlation coefficients were only 0.0925 and 0.2836. Garcia-Diaz *et al.* (2016) [12] observed a moderate or strong correlation (from 0.3345 to 0.6827) between TRE and TD models according to Battese and Coelli (1988) [3] and Battese and Coelli (1995) [5]. Within this article, it was proved that the correlation between "true" models and the TD model is greatly influenced by the chosen efficiency estimate. If a JLMS estimate is applied, then the correlations are generally low. This finding is consistent with a study by Sun *et al.* (2017) [37], because in their study the JLMS estimate for TFE and TD models was used and the correlations found were weak. For the TRE models examined in this article, a large variability of results was found. However, even here, the results are consistent with the study by Garcia-Diaz *et al.* [12]. In their study, they used the TRE model with a truncated-normal distribution of inefficiency and a JLMS estimate. The values of correlation coefficients between the M10 to M12 models (which are the only estimated TRE models with the same type of distribution as Garcia-Diaz *et al.* (2016) [12]) with the TD model are from 0.46 to 0.64 for the Pearson correlation coefficient and from 0.50 to 0.61 for the Spearman correlation coefficient.

Within studies such as Sun *et al.* (2017) [37] or Garcia-Diaz *et al.* (2016) [12], where attention is paid to the effect of the type of SFA model, the influence of another factor is hidden, namely the assumption regarding the probability distribution for inefficiency. This factor has so far been greatly underestimated when choosing a model. This factor affects the efficiency results both when using cross-sectional data and panel data. Considering the results of this article and also the results in Staňková and Hampel (2021) [35], it is possible to conclude that changing the output variable or changing the functional form of the production function is not as important as choosing the probability distribution and the type of SFA model.

When comparing the results of SFA panel models and DEA models, it was found that at a certain model setting, very strong correlations may be found, i.e., correlations higher than 0.8 in both the Spearman and Pearson correlation coefficients. These are considerably higher values than in sectors 24 and 25. At this point, it is possible to refer to the findings in Oh and Shin (2015) [26], where the effect of measurement errors on the correlation between these approaches was demonstrated. It is possible to assume that aggregate data for the whole country will contain fewer errors than may be found in the financial statements of individual companies. The study by Oh and Shin (2015) [26] examined the correlation between SFA annual models and SFA panel models, and the rank correlation between SFA and TRE results ranged from 0.15 to 0.90 (these results were based on a Monte Carlo simulation for different levels of the specified error rate in the data).

Silva *et al.* (2016) [29] point out the different results in a micro- and macro-perspective analysis. Silva *et al.* (2016) [29] observed a weak rank correlation between DEA and TFE models (0.026). They found that the differences between the approaches are not caused by heterogeneity in the data set and assumed that this was a data or sector problem. However, as already mentioned above, the possible correlation between DEA models and SFA panel models is significantly influenced by the type of model or the chosen type of efficiency estimate. It is this finding that may explain the weak correlations (and, moreover, the statistically insignificant correlations) between the TFE and DEA models in the work of Silva *et al.* (2016) [29]. Although their study covered a different area (banking, where cost functions were estimated), their results may be explained by the findings in this article. This is because the choice of a TFE model with a half-normal probability distribution with a JLMS estimate makes correlations with the DEA method weak and often statistically insignificant.

However, the question for future research still remains whether the highly correlated results of two different approaches really correspond to reality. In this respect, the analyses could be extended to include information from the opposite condition for efficiency, i.e., bankruptcy. Companies heading for bankruptcy should systematically achieve lower efficiency values, as they are unable to keep up with their competitors in the long run. As already demonstrated in Staňková and Hampel (2019) [32], the DEA method may also be used as a good tool for predicting bankruptcy. In the article by Staňková and Hampel (2020) [34], based on data from the travel industry, it was proved that it is possible to link the prediction of bankruptcy to the assessment of inefficiency via "classical" DEA models. It is therefore possible to focus on finding models that best reflect the inefficiency of bankruptcy companies.

8. CONCLUSION

The main aim of this article is to evaluate the approaches for determining the efficiency of companies in specific sectors and in selected EU countries. Unlike most studies, the evaluation of efficiency is performed both at the level of the accounting data of companies in two sectors (from 2012 to 2015), and on the basis of the aggregated data of individual EU countries for both sectors together (from 1995 to 2015). Two different approaches are used to calculate efficiency, the parametric stochastic frontier analysis method and the non-parametric data envelopment analysis method. Within both of these methods, many models with different settings are estimated.

The empirical results show that the assumption regarding the probability distribution must not be underestimated in the efficiency analysis, as the absolute efficiency values are systematically lower when using the half-normal distribution than when using the exponential and truncated-normal distributions. Similarly, the form of the estimate should be carefully chosen. Even in the case of non-parametric models, the choice of a particular setting is crucial for the calculation of efficiency. According to the correlation analysis, models that allow working with variable returns to scale that are output-oriented (or allow adjustments of both output and input variables) seem to be advisable.

ACKNOWLEDGMENTS

We acknowledge the valuable suggestions from the referees. The authors declare that they have no conflict of interest.

APPENDIX

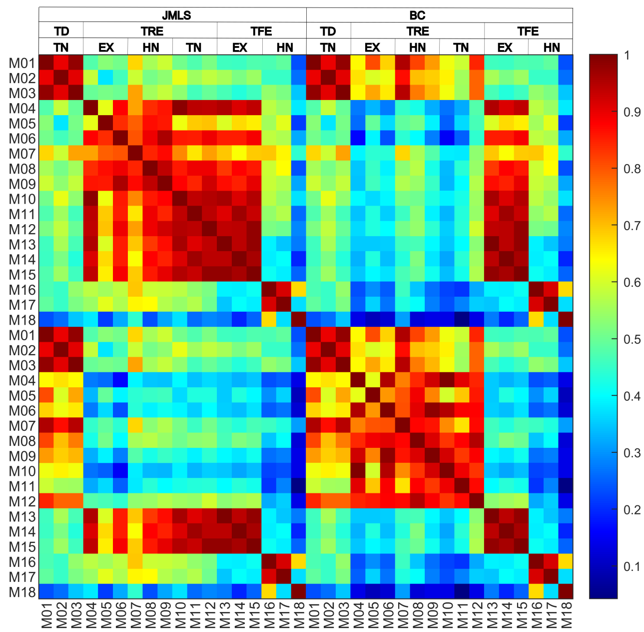

Figure 6: Colormap of the Spearman correlation coefficient for all SFA panel models.

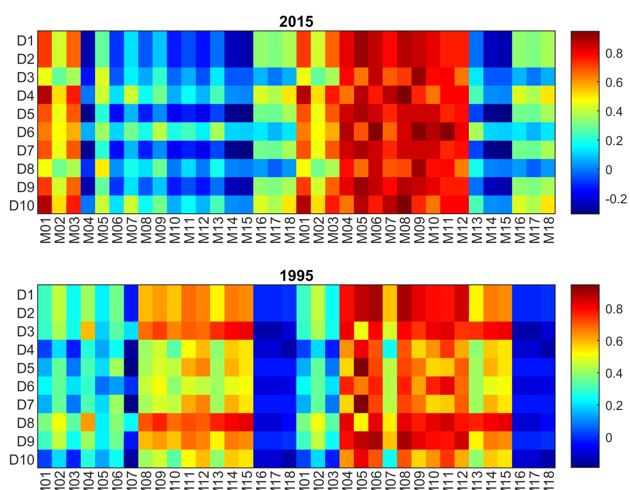

Figure 7: Colormap of the Spearman correlation coefficient for all SFA panel models.

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