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Supplementary file for: Optimal imputation methods under stratified ranked set sampling

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This supplement to the main paper includes Appendixes A-B-C mentioned in the text.

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Supplementary file

Appendix A

This section considers the MSE of the resultant estimators of the existing combined imputation methods for ready reference.

(A.1) $$V(T_m^c) = \bar{Y}^2 I_1^*$$

(A.2) $$MSE(T_{R_1}^c) = \bar{Y}^2 [I_0^* + I_1 - 2I_{01}]$$

(A.3) $$MSE(T_{R_2}^c) = \bar{Y}^2 [I_0^* + I_1^* - 2I_{01}^*]$$

(A.4) $$MSE(T_{R_3}^c) = \bar{Y}^2 [I_0^* + I_1^* - I_1 - 2(I_{01}^* - I_{01})]$$

(A.5) $$MSE(T_{DP_1}^c) = \bar{Y}^2 I_0^* + b_1^2 \bar{X}^2 I_1 - 2b_1 \bar{X}\bar{Y} I_{01}$$

(A.6) $$MSE(T_{DP_2}^c) = \bar{Y}^2 I_0^* + b_2^2 \bar{X}^2 I_1^* - 2b_2 \bar{X}\bar{Y} I_{01}^*$$

(A.7) $$MSE(T_{DP_3}^c) = \bar{Y}^2 I_0^* + b_3^2 \bar{X}^2 (I_1^* - I_1) - 2b_3 \bar{X}\bar{Y} (I_{01}^* - I_{01})$$

(A.8) $$minMSE(T_{DP_1}^c) = \bar{Y}^2 \left[I_0^* - \frac{I_{01}^2}{I_1} \right]$$

(A.9) $$minMSE(T_{DP_2}^c) = \bar{Y}^2 \left[I_0^* - \frac{I_{01}^{*^2}}{I_1^*} \right]$$

(A.10) $$minMSE(T_{DP_3}^c) = \bar{Y}^2 \left[I_0^* - \frac{\left(I_{01}^* - I_{01}\right)^2}{\left(I_1^* - I_1\right)} \right]$$

(A.11) $$MSE(T_{s_i}^c) = \bar{Y}^2 \left[I_0^* + \beta_i^2 I_1 - 2\beta_i I_{01} \right], \ i = 1, 4$$

(A.12) $$MSE(T_{S_i}^c) = \bar{Y}^2 \left[I_0^* + \beta_i^2 I_1^* - 2\beta_i I_{01}^* \right], \ i = 2, 5$$

(A.13) $$MSE(T_{S_i}^c) = \bar{Y}^2 \left[\begin{array}{l} I_0^* + \beta_i^2 \left\{ I_1^* - I_1 \right\} \\ -2\beta_i \left\{ I_{01}^* - I_{01} \right\} \end{array} \right], \ i = 3, 6$$

(A.14) $$minMSE(T_{S_i}^c) = \bar{Y}^2 \left[I_0^* - \frac{I_{01}^2}{I_1} \right]; \quad i = 1, 4$$

(A.15) $$minMSE(T_{S_i}^c) = \bar{Y}^2 \left[I_0^* - \frac{I_{01}^{*^2}}{I_1^*} \right]; \quad i = 2, 5$$

(A.16) $$minMSE(T_{S_i}^c) = \bar{Y}^2 \left[I_0^* - \frac{\left(I_{01}^* - I_{01}\right)^2}{\left(I_1^* - I_1\right)} \right]; \quad i = 3, 6$$

The optimum values of the scalars associated with the estimators discussed in section 3 are given below.
$b_1 = R\frac{I_{01}}{I_1}$, $b_2 = R\frac{I_{01}^*}{I_1^*}$, $b_3 = R\frac{(I_{01}^* - I_{01})}{(I_1^* - I_1)}$, $\beta_{1(opt)} = \beta_{4(opt)} = \frac{I_{01}}{I_1}$, $\beta_{2(opt)} = \beta_{5(opt)} = \frac{I_{01}^*}{I_1^*}$, $\beta_{3(opt)} = \beta_{6(opt)} = \frac{\left(I_{01}^* - I_{01}\right)}{(I_1^* - I_1)}$.

Appendix B

This section considers the MSE of the resultant estimators of the existing separate imputation methods for ready reference.

(B.17) $$V(T_m^s) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 J_1^*$$

(B.18) $$MSE(T_{R_1}^s) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[J_0^* + J_1 - 2J_{01}\right]$$

(B.19) $$MSE(T_{R_2}^s) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[J_0^* + J_1^* - 2J_{01}^*\right]$$

(B.20) $$MSE(T_{R_3}^s) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[J_0^* + J_1^* - J_1 - 2(J_{01}^* - J_{01})\right]$$

(B.21) $$MSE(T_{DP_1}^s) = \sum_{h=1}^{L} W_h^2 [\bar{Y}_h^2 J_0^* + b_{1_h}^2 \bar{X}_h^2 J_1 - 2b_{1_h} \bar{X}_h \bar{Y}_h J_{01}]$$

(B.22) $$MSE(T_{DP_2}^s) = \sum_{h=1}^{L} W_h^2 [\bar{Y}_h^2 J_0^* + b_{2_h}^2 \bar{X}^2 J_1^* - 2b_{2_h} \bar{X}_h \bar{Y}_h J_{01}^*]$$

(B.23) $$MSE(T_{DP_3}^s) = \sum_{h=1}^{L} W_h^2 [\bar{Y}_h^2 J_0^* + b_{3_h}^2 \bar{X}_h^2 (J_1^* - J_1) - 2b_{3_h} \bar{X}_h \bar{Y}_h (J_{01}^* - J_{01})]$$

(B.24)

$$minMSE(T_{DP_1}^s) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[J_0^* - \frac{J_{01}^2}{J_1}\right]$$

(B.25)

$$minMSE(T_{DP_2}^s) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[J_0^* - \frac{{J_{01}^*}^2}{J_1^*}\right]$$

(B.26)

$$minMSE(T_{DP_3}^s) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[J_0^* - \frac{\left(J_{01}^* - J_{01}\right)^2}{\left(J_1^* - J_1\right)}\right]$$

(B.27) $$MSE(T_{S_i}^s) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[J_0^* + \beta_{i_h}^2 J_1 - 2\beta_{i_h} J_{01}\right], \ i = 1, 4$$

(B.28) $$MSE(T_{S_i}^s) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[J_0^* + \beta_{i_h}^2 J_1^* - 2\beta_{i_h} J_{01}^*\right], \ i = 2, 5$$

(B.29)

$$MSE(T^{s}_{S_i}) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[\ J_0^* + \beta_{i_h}^2 \left\{\ J_1^* - J_1\ \right\} - 2\beta_{i_h} \left\{J_{01}^* - J_{01}\right\}\right],\ i = 3, 6$$

(B.30)

$$minMSE(T^{s}_{S_i}) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[J_0^* - \frac{J_{01}^2}{J_1}\right];\quad i = 1, 4$$

(B.31)

$$minMSE(T^{s}_{S_i}) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[J_0^* - \frac{{J_{01}^*}^2}{J_1^*}\right];\quad i = 2, 5$$

(B.32)

$$minMSE(T^{s}_{S_i}) = \sum_{h=1}^{L} W_h^2 \bar{Y}_h^2 \left[\ J_0^* - \frac{\left(J_{01}^* - J_{01}\right)^2}{\left(\ J_1^* - J_1\ \right)}\ \right];\quad i = 3, 6$$

The optimum values of the constant involved in the estimators are given hereunder.
$b_{1_h} = R_h \frac{J_{01}}{J_1}$, $b_{2_h} = R_h \frac{J_{01}^*}{J_1^*}$, $b_{3_h} = R_h \frac{(J_{01}^* - J_{01})}{(J_1^* - J_1)}$, $\beta_{1_h(opt)} = \beta_{4_h(opt)} = \frac{J_{01}}{J_1}$, $\beta_{2_h(opt)} = \beta_{5_h(opt)} = \frac{J_{01}^*}{J_1^*}$, $\beta_{3_h(opt)} = \beta_{6_h(opt)} = \frac{\left(J_{01}^* - J_{01}\right)}{J_1^* - J_1}$.

Appendix C

This section contains the proof of Theorem 4.1 to Theorem 4.4.
Under Strategy I, consider the estimator $T^c_{SA_1}$ as

$$T^c_{SA_1} = \alpha_1 \bar{y}_{r_{st}} + \theta_1(\bar{x}_{n_{st}} - \bar{X})$$

Employing the notations discussed in the earlier section, we get

(C.33) $$T^c_{SA_1} - \bar{Y} = (\alpha_1 - 1)\bar{Y} + \alpha_1 \bar{Y}\epsilon_0 + \theta_1 \bar{X}\epsilon_1$$

On squaring both the sides of ($C.33$) and taking expectation, we will get the MSE of the estimator as

(C.34) $$MSE(T^c_{SA_1}) = (\alpha_1 - 1)^2 \bar{Y}^2 + \alpha_1^2 \bar{Y}^2 I_0^* + \theta_1^2 I_1 + 2\alpha_1\theta_1 \bar{X}\bar{Y} I_{01}$$

By minimizing ($C.34$) w.r.t α_1 and θ_1, we get the optimum values of α_1 and θ_1 as

(C.35) $$\alpha_{1(opt)} = \frac{1}{\left(1 + I_0^* - \frac{I_{12}^2}{I_1}\right)} = \frac{A_1}{B_1}$$

(C.36) $$\theta_{1(opt)} = -\frac{\bar{Y}}{\bar{X}} \frac{I_{01}}{I_1} \alpha_{1(opt)}$$

where $A_1 = 1$ and $B_1 = 1 + I_0^* - \{I_{12}^2/I_1\}$.
Putting the value of $\alpha_{1(opt)}$ and $\theta_{1(opt)}$ in ($C.34$), we get the minimum MSE as

$$minMSE(T^c_{SA_1}) = \bar{Y}^2(1-\alpha_{1(opt)}) = \bar{Y}^2\left(1-\frac{A_1^2}{B_1}\right) \tag{C.37}$$

The minimum MSE of other resultant estimators of the proposed imputation methods can be obtained in a similar fashion using Taylor series expansion. The optimum values of the constants are given below for ready reference.

$$\alpha_{2(opt)} = \frac{1}{\left(1+I_0^* - \frac{{I_{01}^*}^2}{I_1^*}\right)} = \alpha_{8(opt)} \tag{C.38}$$

$$\theta_{2(opt)} = -\frac{\bar{Y}}{\bar{X}}\frac{I_{01}^*}{I_1^*}\alpha_{2(opt)} \tag{C.39}$$

$$\alpha_{3(opt)} = \frac{1}{\left(1+I_0^* - \frac{\left(I_{01}^*-I_{01}\right)^2}{\left(I_1^*-I_1\right)}\right)} \tag{C.40}$$

$$\theta_{3(opt)} = -\frac{\bar{Y}}{\bar{X}}\left(\frac{I_{01}^*-I_{01}}{I_1^*-I_1}\right)\alpha_{3(opt)} \tag{C.41}$$

$$\alpha_{j(opt)} = \frac{A_j}{B_j};\quad j=4,5,6,7,8,9 \tag{C.42}$$

$$\theta_{j(opt)} = \frac{I_{01}}{I_1};\quad j=4,7. \tag{C.43}$$

$$\theta_{j(opt)} = \frac{I_{01}^*}{I_1^*};\quad j=5,8. \tag{C.44}$$

$$\theta_{j(opt)} = \frac{(I_{01}^*-I_{01})}{(I_1^*-I_1)};\quad j=6,9 \tag{C.45}$$

where

$$A_4 = \left(1+\frac{I_{01}}{2}-\frac{I_{12}^2}{2I_1}\right);\ B_4 = \left(1+I_0^*+I_{01}-\frac{2I_{12}^2}{I_1}\right);\ A_5 = \left(1+\frac{I_{01}^*}{2}-\frac{{I_{01}^*}^2}{2I_1^*}\right)$$
$$B_5 = \left(1+I_0^*+I_{01}^*-\frac{2{I_{01}^*}^2}{I_1^*}\right);\ A_6 = \left[1-\frac{1}{2}\frac{\left\{I_{01}^*-I_{01}\right\}^2}{I_1^*-I_1}+\frac{1}{2}(I_{01}^*-I_{01})\right]$$
$$B_6 = \left[1+I_0^*-2\frac{\left\{I_{01}^*-I_{01}\right\}^2}{I_1^*-I_1}+(I_{01}^*-I_{01})\right];\ A_7=1;\ B_7 = \left(1+I_0^*-\frac{I_{12}^2}{I_1}\right);$$
$$A_8=1;\ B_8 = \left(1+I_0^*-\frac{{I_{01}^*}^2}{I_1^*}\right);\ A_9 = 1\ and\ B_9 = \left[1+I_0^*-\frac{(I_{01}^*-I_{01})^2}{(I_1^*-I_1)}\right].$$

Similarly, the outlines of the derivations of the other MSE expressions can easily be obtained.