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Supplementary Materials of Bayesian Variable Selection for Zero-inflated Longitudinal Count Data

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SUPPLEMENTARY MATERIAL A: The conditional posterior distribution of parameters for Continuous spike approach

The full conditional posterior distributions of parameters of the ZIPS random effects model are as follows:
For $\beta_k, k = 1, \cdots, p$

$$\begin{aligned}
P(\beta_k \neq 0|\boldsymbol{Y},\ \tau^2_{\beta_k},\ \zeta_k) &= \prod_{i=1}^{n}\prod_{j=1}^{T}\{(1-\pi_{ij})p(Y_{ij}=y_{ij}|\mu_{ij})\}^{1-I(y_{ij})}\{\pi_{ij}+(1-\pi_{ij}) \\
&\times\ p(Y_{ij}=0|\mu_{ij})\}^{I(y_{ij})}p(\beta_k\neq 0|\tau^2_{\beta_k}), \\
&\propto \prod_{i=1}^{n}\prod_{j=1}^{T}\{\frac{a_{y_{ij}}\mu_{ij}^{y_{ij}}}{f(\mu_{ij})}\}^{1-I(y_{ij})} \times\ \{\pi_{ij}+(1-\pi_{ij})\frac{a_0}{f(\mu_{ij})}\}^{I(y_{ij})} \\
&\times \phi(\beta_k; 0, \tau^2_{\beta_k})
\end{aligned}$$

Let $f_2(\beta_k)$ be the normalized density of $p(\beta_k \neq 0|\tau^2_{\beta_k})$, then

$$\beta_k|rest \sim l_k\phi(\beta_k; 0,\ \sigma^2_{\beta_k}) + (1-l_k)f_2(\beta_k),$$

where l_k, the posterior probability of $\beta_k = 0$, is given by

$$\begin{aligned}
l_k &= p(\beta_k = 0|rest) \\
&= \frac{L(\boldsymbol{\theta}_{-\beta_k}, \beta_k = 0|\boldsymbol{y},\boldsymbol{x})p(\beta_k=0)}{\int L(\boldsymbol{\theta}|\boldsymbol{y},\boldsymbol{x})\pi(\beta_k)d\beta_k} \\
&= \frac{p(\boldsymbol{Y}=\boldsymbol{y}|\beta_k=0)p(\beta_k=0)}{\int p(\boldsymbol{Y}=\boldsymbol{y}|\beta_k)\pi(\beta_k)d\beta_k} \\
&= \frac{p(\boldsymbol{Y}=\boldsymbol{y}|\beta_k=0)p(\beta_k=0)}{\int p(\boldsymbol{Y}=\boldsymbol{y}|\beta_k)\{\zeta_k\phi(\beta_k;0,\sigma^2_{\beta_k})+(1-\zeta_k)\phi(\beta_k;0,\tau^2_{\beta_k})\}d\beta_k}, \qquad (0.1)
\end{aligned}$$

where

$$\begin{aligned}
&\int p(\boldsymbol{Y}=\boldsymbol{y}|\beta_k)\{\zeta_k\phi(\beta_k; 0,\ \sigma^2_{\beta_k}) + (1-\zeta_k)\phi(\beta_k; 0,\ \tau^2_{\beta_k})\}d\beta_k \\
&= \int\prod_{i=1}^{n}\prod_{j=1}^{T}\{(1-\pi_{ij})p(Y_{ij}=y_{ij}|\mu_{ij})\}^{1-I(y_{ij})}\{\pi_{i_ij}+(1-\pi_{ij})p(Y_{ij}=0|\mu_{ij})\}^{I(y_{ij})} \\
&\times\ \{\zeta_k\phi(\beta_k; 0,\ \sigma^2_{\beta_k}) + (1-\zeta_k)\phi(\beta_k; 0,\ \tau^2_{\beta_k})\}d\beta_k \\
&= \zeta_k\prod_{i=1}^{n}\prod_{j=1}^{T}\{(1-\pi_{ij})\frac{a_{y_{ij}}(\boldsymbol{x}'_{ij}\boldsymbol{\beta}+b_{i1})^{y_{ij}}}{f(\boldsymbol{x}'_{ij}\boldsymbol{\beta}+b_{i1})}\}^{1-I(y_{ij})}\times\{\pi_{ij}+(1-\pi_{ij})\frac{a_0}{f(\boldsymbol{x}'_{ij}\boldsymbol{\beta}+b_{i1})}\}^{I(y_{ij}=0)} \\
&\times\ \phi(\beta_k; 0,\ \sigma^2_{\beta_k}) \\
&+\ (1-\zeta_k)\int_{\beta_k\neq 0}\prod_{i=1}^{n}\prod_{j=1}^{T}\{(1-\pi_{ij})\frac{a_{y_{ij}}(\boldsymbol{x}'_{ij}\boldsymbol{\beta}+b_{i1})^{y_{ij}}}{f(\boldsymbol{x}'_{ij}\boldsymbol{\beta}+b_{i1})}\}^{1-I(y_{ij})} \\
&\times\ \{\pi_{ij}+(1-\pi_{ij})\frac{a_0}{f(\boldsymbol{x}'_{ij}\boldsymbol{\beta}+b_{i1})}\}^{I(y_{ij})} \\
&\times\ \phi(\beta_k; 0,\ \tau^2_{\beta_k})d\beta_k
\end{aligned}$$

If $\alpha_l \neq 0$

$$
\begin{aligned}
p(\alpha_l|\boldsymbol{Y},\ \tau^2_{\alpha_l},\ \omega_l) &= \prod_{i=1}^{n}\prod_{j=1}^{T}\{(1-\pi_{ij})p(Y_{ij}=y_{ij}|\mu_{ij})\}^{1-I(y_{ij})}\{\pi_{ij}+(1-\pi_{ij}) \\
&\times\ p(Y_{ij}=0|\mu_{ij})\}^{I(y_{ij})}p(\alpha_l\neq 0|\tau^2_{\alpha_l}), \\
&\propto \prod_{i=1}^{n}\prod_{j=1}^{T}\{(1-\pi_{ij})\frac{a_{y_{ij}}\mu_{ij}^{y_{ij}}}{f(\mu_{ij})}\}^{1-I(y_{ij})}\{\pi_{ij}+(1-\pi_{ij})\frac{a_0}{f(\mu_{ij})}\}^{I(y_{ij})} \\
&\times\ \phi(\alpha_l;0,\ \tau_{\alpha_l}).
\end{aligned}
$$

Let $g_2(\alpha_l)$ be the normalized density of $p(\alpha_l \neq 0|\tau^2_{\alpha_l})$, then

$$\alpha_l|rest \sim h_l\phi(\alpha_l;0,\ \sigma^2_{\alpha_l}) + (1-h_l)g_2(\alpha_l),$$

where h_l, the posterior probability of $\alpha_l = 0$, is given by

$$
\begin{aligned}
h_l &= p(\beta_k=0|rest) \\
&= \frac{L(\boldsymbol{\theta}_{-\alpha_l},\alpha_l=0|\boldsymbol{y},\boldsymbol{x})p(\alpha_\iota=0)}{\int L(\boldsymbol{\theta}|\boldsymbol{y},\boldsymbol{x})\pi(\alpha_l)d\alpha_l} \\
&= \frac{p(\boldsymbol{Y}=\boldsymbol{y}|\alpha_l=0)p(\alpha_l=0)}{\int p(\boldsymbol{Y}=\boldsymbol{y}|\alpha_l)\pi(\alpha_\iota)d\alpha_l} \\
&= \frac{p(\boldsymbol{Y}=\boldsymbol{y}|\alpha_l=0)p(\alpha_l=0)}{\int p(\boldsymbol{Y}=\boldsymbol{y}|\alpha_l)\{\omega_l\phi(\alpha_l;0,\sigma^2_{\alpha_l})+(1-\omega_l)\phi(\alpha_l;0,\tau^2_{\alpha_l})\}d\alpha_l},
\end{aligned}
\tag{0.2}
$$

where

$$
\begin{aligned}
&\int p(\boldsymbol{Y}=\boldsymbol{y}|\alpha_l)\{\omega_l\phi(\alpha_l;0,\ \sigma^2_{\alpha_l})+(1-\omega_l)\phi(\alpha_l;0,\ \tau^2_{\alpha_l})\}d\alpha_l \\
&= \int\prod_{i=1}^{n}\prod_{j=1}^{T}\{(\frac{1}{1+\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})})p(Y_{ij}=y_{ij}|\mu_{ij})\}^{1-I(y_{ij})} \\
&\times\ \{\frac{\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})}{1+\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})}+(\frac{1}{1+\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})})p(Y_{ij}=0|\mu_{ij})\}^{I(y_{ij})} \\
&\times\ \{\omega_l\phi(\alpha_l;0,\ \sigma^2_{\alpha_l})+(1-\omega_l)\phi(\alpha_l;0,\ \tau^2_{\alpha_l})\}d\alpha_l \\
&= \omega_l\prod_{i=1}^{n}\prod_{j=1}^{T}\{(\frac{1}{1+\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})})\frac{a_{y_{ij}}\mu_{ij}^{y_{ij}}}{f(\mu_{ij})}\}^{1-I(y_{ij})} \\
&\times\ \{\frac{\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})}{1+\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})}+(\frac{1}{1+\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})})\frac{a_0}{f(\mu_{ij})}\}^{I(y_{ij}=0)}\phi(\alpha_l;0,\ \sigma^2_{\alpha_l}) \\
&+\ (1-\omega_l)\int_{\alpha_l\neq 0}\prod_{i=1}^{n}\prod_{j=1}^{T}\{(\frac{1}{1+\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})})\frac{a_{y_{ij}}\mu_{ij}^{y_{ij}}}{f(\mu_{ij})}\}^{1-I(y_{ij})} \\
&\times\ \{\frac{\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})}{1+\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})}+(\frac{1}{1+\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})})\frac{a_0}{f(\mu_{ij})}\}^{I(y_{ij})}\phi(\alpha_l;0,\ \tau^2_{\alpha_l})d\alpha_l,
\end{aligned}
$$

$$p(\boldsymbol{b}_i|\boldsymbol{Y},\boldsymbol{D}) = \prod_{j=1}^{T}\{(1-\pi_{ij})p(Y_{ij}=y_{ij}|\mu_{ij})\}^{1-I(y_{ij})}\{\pi_{ij}+(1-\pi_{ij})$$

$$
\begin{aligned}
& \times \ \ p(Y_{ij}=0|\mu_{ij})\}^{I(y_{ij}=0)} p(b_i|\boldsymbol{D}) \\
& \propto \ \prod_{j=1}^{T}\{\frac{a_{y_{ij}}\mu_{ij}^{y_{ij}}}{f(\mu_{ij})}\}^{1-I(y_{ij})}\{\pi_{ij}+(1-\pi_{ij})\frac{a_0}{f(\mu_{ij})}\}^{I(y_{ij})} \\
& \times \ \phi(b_i;0,\boldsymbol{D}),\ i=1,\ \cdots,\ n,
\end{aligned} \tag{0.3}
$$

$$
\begin{aligned}
p(\boldsymbol{D}|\boldsymbol{Y},\ \boldsymbol{b}_i) &\propto p(\boldsymbol{b}_i|\boldsymbol{D})p(\boldsymbol{D}) \\
&\propto \ \exp(-\frac{1}{2}(\boldsymbol{b}_i'\boldsymbol{b}_i+\boldsymbol{\Psi})\boldsymbol{D}^{-1})|\boldsymbol{D}|^{-\frac{(r+4+1)}{2}}, \\
\boldsymbol{D}|rest &\sim IWishart(r+2,\boldsymbol{b}_i'\boldsymbol{b}_i+\boldsymbol{\Psi}),
\end{aligned} \tag{0.4}
$$

$$
\begin{aligned}
p(\tau_{\beta_k}^2|\boldsymbol{Y},\ \beta_k) &\propto p(\beta_k|\tau_{\beta_k}^2)p(\tau_{\beta_k}^2) \\
&\propto \ \tau_{\beta_k}^{2c_{1k}-\frac{1}{2}-1}\exp(-\frac{\beta_k^2+2c_{2k}}{2\tau_{\beta_k}^2}), \\
\tau_{\beta_k}^2|rest &\sim \ IG(c_{1k}-\frac{1}{2},\ \frac{\beta_k^2}{2}+c_{2k}),
\end{aligned} \tag{0.5}
$$

$$
\begin{aligned}
p(\tau_{\alpha_l}^2|\boldsymbol{Y},\ \alpha_l) &\propto p(\alpha_l|\tau_{\alpha_l}^2)p(\tau_{\alpha_l}^2) \\
&\propto \ \tau_{\alpha_l}^{2d_{1l}-\frac{1}{2}-1}\exp(-\frac{\alpha_l^2+2d_{2l}}{2\tau_{\alpha_l}^2}), \\
\tau_{\alpha_l}^2|rest &\sim \ IG(d_{1l}+\frac{1}{2},\ \frac{\alpha_l^2}{2}+d_{2l}),
\end{aligned} \tag{0.6}
$$

$$
\begin{aligned}
p(\zeta_k=1|\boldsymbol{Y},\ \beta_k,\ \lambda_{\beta_k}) &\propto \ p(\beta_k|\tau_{\beta_k}^2,\ \zeta_k=1,\ \lambda_{\beta_k})p(\zeta_k=1|\lambda_{\beta_k}) \\
&\propto \ \exp(-\frac{\beta_k}{2\tau_{\beta_k}^2})\lambda_{\beta_k},
\end{aligned} \tag{0.7}
$$

$$
\begin{aligned}
p(\zeta_k=0|\boldsymbol{Y},\ \beta_k,\ \lambda_{\beta_k}) &\propto p(\beta_k|\tau_{\beta_k}^2,\zeta_k=0,\ \lambda_{\beta_k})p(\zeta_k=0|\lambda_{\beta_k}) \\
&\propto \ \exp(-\frac{\beta_k}{2\sigma_{\beta_k}^2})(1-\lambda_{\beta_k}), \\
\zeta_k|rest &\sim \ Ber(\frac{\exp(-\frac{\beta_k}{2\tau_{\beta_k}^2})\lambda_{\beta_k}}{\exp(-\frac{\beta_k}{2\sigma_{\beta_k}^2})(1-\lambda_{\beta_k})+\exp(-\frac{\beta_k}{2\tau_{\beta_k}^2})\lambda_{\beta_k}}),
\end{aligned} \tag{0.8}
$$

$$
\begin{aligned}
p(\lambda_{\beta_k}|\boldsymbol{Y},\ \zeta_k) &\propto \ p(\zeta_k|\lambda_{\beta_k})p(\lambda_{\beta_k}) \\
&\propto \ (\lambda_{\beta_k})^{\zeta_k}(1-\lambda_{\beta_k})^{1-\zeta_k}(\lambda_{\beta_k})^{f_{1k}-1}(1-\lambda_{\beta_k})^{f_{2k}-1}), \\
\lambda_{\beta_k}|rest &\sim Beta(\zeta_k+f_{1k},\ 1-\zeta_k+f_{2k}),
\end{aligned} \tag{0.9}
$$

$$
\begin{aligned}
p(\omega_l=1|\boldsymbol{Y},\ \alpha_l,\ \lambda_{\alpha_l}) &\propto \ p((y_l|\tau_{\alpha_l}^2)\omega_l=1,\ \lambda_{\alpha_l})p(\omega_l=1|\lambda_{\beta_k}) \\
&\propto \ \exp(-\frac{\alpha_l}{2\tau_{\alpha_l}^2})\lambda_{\alpha_l},
\end{aligned} \tag{0.10}
$$

$$
p(\omega_l=0|\boldsymbol{Y},\ \alpha_l,\ \lambda_{\alpha_l}) \ \propto \ p(\alpha_l|\tau_{\alpha_l}^2,\ \omega_l=0,\ \lambda_{\alpha_l})p(\omega_l=0|\lambda_{\alpha_l})
$$

$$
\begin{aligned}
&\propto \exp(-\frac{\alpha_l}{2\sigma^2_{\alpha_l}})(1-\lambda_{\alpha_l}),\\
\omega_l|rest &\sim Ber(\frac{\exp(-\frac{\alpha_l}{2\tau^2_{\alpha_l}})\lambda_{\alpha_l}}{\exp(-\frac{\alpha_l}{2\sigma^2_{\alpha_l}})(1-\lambda_{\alpha_l})+\exp(-\frac{\alpha_l}{2\tau^2_{\alpha_l}})\lambda_{\alpha_l}}), \qquad (0.11)
\end{aligned}
$$

$$
\begin{aligned}
p(\lambda_{\alpha_l}|\boldsymbol{Y},\ \omega_l) &\propto p(\omega_l|\lambda_{\alpha_l})p(\lambda_{\alpha_l})\\
&\propto (\lambda_{\alpha_l})^{\omega_l}(1-\lambda_{\alpha_l})^{1-\omega_l}(\lambda_{\alpha_l})^{m_{1k}-1}(1-\lambda_{\alpha_l})^{m_{2k}-1}),\\
\lambda_{\alpha_l}|rest &\sim Beta(\omega_l+m_{1k},\ 1-\omega_l+m_{2k}). \qquad (0.12)
\end{aligned}
$$

SUPPLEMENTARY MATERIAL B: The conditional posterior distribution of parameters for Dirac spike approach

The full conditional posterior distributions of parameters of the ZIPS random effects model are as follows:
For $\beta_k, k=1,\cdots,p$

$$
\begin{aligned}
P(\beta_k\neq 0|\boldsymbol{Y},\sigma^2_{\beta_k},\gamma_k) &= \prod_{i=1}^{n}\prod_{j=1}^{T}\{(1-\pi_{ij})p(Y_{ij}=y_{ij}|\mu_{ij})\}^{1-I(y_{ij})}\{\pi_{ij}+(1-\pi_{ij})\\
&\times\ p(Y_{ij}=0|\mu_{ij})\}^{I(y_{ij})}p(\beta_k\neq 0|\sigma^2_{\beta_k}),\\
&\propto \prod_{i=1}^{n}\prod_{j=1}^{T}\{\frac{a_{y_{ij}}\mu_{ij}^{y_{ij}}}{f(\mu_{ij})}\}^{1-I(y_{ij})}\\
&\times\ \{\pi_{ij}+\ (1-\pi_{ij})\frac{a_0}{f(\mu_{ij})}\}^{I(y_{ij})}\times\exp(-\frac{\beta_k^2}{2\sigma^2_{\beta_k}}), \qquad (0.13)
\end{aligned}
$$

Let $f(\beta_k)$ be the normalized density of $p(\beta_k\neq 0|\sigma^2_{\beta_k})$, then The posterior distribution of β_k conditioning on everything else is as follows:

$$\beta_k|rest\sim l_k\delta_0(\beta_k)+(1-l_k)f(\beta_k),$$

where l_k, the posterior probability of $\beta_k=0$, is given by

$$
\begin{aligned}
l_k &= p(\beta_k=0|rest)\\
&= \frac{L(\boldsymbol{\theta}_{-\beta_k},\beta_k=0|\boldsymbol{y},\boldsymbol{x})p(\beta_k=0)}{\int L(\boldsymbol{\theta}|\boldsymbol{y},\boldsymbol{x})\pi(\beta_k)d\beta_k}\\
&= \frac{p(\boldsymbol{Y}=\boldsymbol{y}|\beta_k=0)p(\beta_k=0)}{\int p(\boldsymbol{Y}=\boldsymbol{y}|\beta_k)\pi(\beta_k)d\beta_k}\\
&= \frac{p(\boldsymbol{Y}=\boldsymbol{y}|\beta_k=0)p(\beta_k=0)}{\int p(\boldsymbol{Y}=\boldsymbol{y}|\beta_k)\{\gamma_k\phi(\beta_k;0,\ \sigma^2_{\beta_k})+(1-\gamma_k)\delta_0(\beta_k)\}d\beta_k}, \qquad (0.14)
\end{aligned}
$$

where

$$\int p(\boldsymbol{Y}=\boldsymbol{y}|\beta_k)\{\gamma_k\delta_0(\beta_k)+(1-\gamma_k)\phi(\beta_k;0,\ \sigma^2_{\beta_k})\}d\beta_k$$
$$=\int\prod_{i=1}^{n}\prod_{j=1}^{T}\{(1-\pi_{ij})p(Y_{ij}=y_{ij}|\mu_{ij})\}^{1-I(y_{ij})}\{\pi_{ij}+(1-\pi_{ij})p(Y_{ij}=0|\mu_{ij})\}^{I(y_{ij})}$$
$$\times\{\gamma_k\delta_0(\beta_k)+(1-\gamma_k)\phi(\beta_k;0,\ \sigma^2_{\beta_k})\}d\beta_k$$
$$=\gamma_k\prod_{i=1}^{n}\prod_{j=1}^{T}\{(1-\pi_{ij})\frac{a_{y_{ij}}(\boldsymbol{x}'_{ij}\boldsymbol{\beta}+b_{i1})^{y_{ij}}}{f(\boldsymbol{x}'_{ij}\boldsymbol{\beta}+b_{i1})}\}^{1-I(y_{ij})}\times\{\pi_{ij}+(1-\pi_{ij})\{\frac{a_0}{f(\boldsymbol{x}'_{ij}\boldsymbol{\beta}+b_{i1})}\}^{I(y_{ij})}$$
$$+(1-\gamma_k)\int_{\beta_k\neq 0}\prod_{i=1}^{n}\prod_{j=1}^{T}\{\frac{a_{y_{ij}}(\boldsymbol{x}'_{ij}\boldsymbol{\beta}+b_{i1})^{y_{ij}}}{f(\boldsymbol{x}'_{ij}\boldsymbol{\beta}+b_{i1})}\}^{1-I(y_{ij})}$$
$$\times\{\pi_{ij}+(1-\pi_{ij})\{\frac{a_0}{f(\boldsymbol{x}'_{ij}\boldsymbol{\beta}+b_{i1})}\}^{I(y_{ij})}\phi(\beta_k;0,\ \sigma^2_{\beta_k})d\beta_k.$$

For $\alpha_l,\ l=1,\cdots,q$

$$\begin{aligned}p(\alpha_l|\boldsymbol{Y},\sigma^2_{\alpha_l},\nu_l) &= \prod_{i=1}^{n}\prod_{j=1}^{T}\{(1-\pi_{ij})p(Y_{ij}=y_{ij}|\mu_{ij})\}^{1-I(y_{ij})}\{\pi_{ij}+(1-\pi_{ij})\\ &\times\ p(Y_{ij}=0|\mu_{ij})\}^{I(y_{ij})}p(\alpha_l\neq 0|\sigma^2_{\alpha_l}),\\ &\propto \prod_{i=1}^{n}\prod_{j=1}^{T}\{\ (1-\ \pi_{ij})\{\frac{a_{y_{ij}}\mu_{ij}^{y_{ij}}}{f(\mu_{ij})}\}^{1-I(y_{ij})}\{\pi_{ij}\\ &\quad +(1-\ \pi_{ij})\exp\ (-\exp(\mu_{ij}))\}^{I(y_{ij})}\\ &\times\ \phi(\alpha_l;0,\sigma^2_{\alpha_l}).\end{aligned}\tag{0.15}$$

Let $f(\alpha_l)$ be the normalized density of $p(\alpha_l\neq 0|\sigma^2_{\alpha_l})$, then the full conditional of α_l is as follows:

$$\alpha_l|rest\sim h_l\delta_0(\alpha_l)+(1-h_l)f(\alpha_l),$$

where h_l, the posterior probability of $\alpha_l=0$, is given by

$$\begin{aligned}h_l &= p(\alpha_l=0|\boldsymbol{Y},\ \tau^2_{\alpha_l},\ \nu_l)\\ &= \frac{L(\boldsymbol{\theta}_{-\alpha_l},\alpha_l=0|\boldsymbol{y},\boldsymbol{x})p(\alpha_l=0)}{\int L(\boldsymbol{\theta}|\boldsymbol{y},\boldsymbol{x})\pi(\alpha_l=0)d\alpha_l}\\ &= \frac{p(\boldsymbol{Y}=\boldsymbol{y}|\alpha_l=0)p(\alpha_l=0)}{\int p(\boldsymbol{Y}=\boldsymbol{y}|\alpha_l)\pi(\alpha_l)d\alpha_l}\\ &= \frac{p(\boldsymbol{Y}=\boldsymbol{y}|\alpha_l=0)p(\alpha_l=0)}{\int p(\boldsymbol{Y}=\boldsymbol{y}|\alpha_l)\{\nu_l\delta_0(\alpha_l)+(1-\nu_l)\phi(\alpha_l;0,\sigma^2_{\alpha_l})\}d\alpha_l},\end{aligned}\tag{0.16}$$

where

$$\int p(\boldsymbol{Y}=\boldsymbol{y}|\alpha_l)\{\nu_l\delta_0(\alpha_l)+(1-\nu_l)\phi(\alpha_l;0,\sigma^2_{\alpha_l})\}d\alpha_l$$
$$=\int\prod_{i=1}^{n}\prod_{j=1}^{T}\{(1-\pi_{ij})p(Y_{ij}=y_{ij}|\mu_{ij})\}^{1-I(y_{ij})}\{\pi_{ij}+(1-\pi_{ij})p(Y_{ij}=0|\mu_{ij})\}^{I(y_{ij})}$$
$$\times\{\nu_l\delta_0(\alpha_l)+(1-\nu_l)\phi(\alpha_l;\ 0,\ \sigma^2_{\alpha_l})\}d\alpha_l$$
$$=\nu_l\prod_{i=1}^{n}\prod_{j=1}^{T}\{\frac{1}{1+\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})}\{\frac{a_{y_{ij}}\mu_{ij}^{y_{ij}}}{f(\mu_{ij})}\}^{1-I(y_{ij})}$$
$$\times\{\frac{\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})}{1+\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})}+(\frac{1}{1+\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})})\{\frac{a_0}{f(\mu_{ij})}\}^{I(y_{ij})}$$
$$+(1-\nu_l)\int_{\alpha_l\neq 0}\prod_{i=1}^{n}\prod_{j=1}^{T}\{\frac{1}{1+\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})}\{\frac{a_{y_{ij}}\mu_{ij}^{y_{ij}}}{f(\mu_{ij})}\}^{1-I(y_{ij})}$$
$$\times\{\frac{\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})}{1+\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})}+(\frac{1}{1+\exp(\boldsymbol{z}'_{ij}\boldsymbol{\alpha}+b_{i2})})\exp(-\exp(\mu_{ij}))\}^{I(y_{ij})}\phi(\alpha_l;0,\sigma^2_{\alpha_l})d\alpha_l,$$

$$\begin{aligned}p(\boldsymbol{b}_i|\boldsymbol{Y},\boldsymbol{D}) &= \prod_{j=1}^{T}\{(1-\pi_{ij})p(Y_{ij}=y_{ij}|\mu_{ij})\}^{1-I(y_{ij})}\{\pi_{ij}+(1-\pi_{ij})\\ &\times\ p(Y_{ij}=0|\mu_{ij})\}^{I(y_{ij})}p(\boldsymbol{b}_i|\boldsymbol{D})\\ &\propto\ \prod_{j=1}^{T}\{\frac{a_{y_{ij}}\mu_{ij}^{y_{ij}}}{f(\mu_{ij})}\}^{1-I(y_{ij})}\{\pi_{ij}+(1-\pi_{ij})\{\frac{a_0}{f(\mu_{ij})}\}^{I(y_{ij})}\\ &\times\ \exp(-\frac{\boldsymbol{b}'_i\boldsymbol{b}_i}{\boldsymbol{D}}),\ i=1,\cdots,n,\end{aligned}\tag{0.17}$$

$$\begin{aligned}p(\boldsymbol{D}|\boldsymbol{Y},\ b_i) &\propto\ p(\boldsymbol{b}_i|\boldsymbol{D})p(\boldsymbol{D})\\ &\propto\ \exp(-\frac{1}{2}(\boldsymbol{b}'_i\boldsymbol{b}_i+\boldsymbol{\Psi})\boldsymbol{D}^{-1})|\boldsymbol{D}|^{-\frac{(r+4+1)}{2}},\\ \boldsymbol{D}|rest &\sim\ IWishart(r+2,\boldsymbol{b}'_i\boldsymbol{b}_i+\boldsymbol{\Psi}),\end{aligned}\tag{0.18}$$

$$\begin{aligned}p(\sigma^2_{\beta_k}|\boldsymbol{Y},\beta_k) &\propto\ p(\beta_k|\sigma^2_{\beta_k})p(\sigma^2_{\beta_k})\\ &\propto\ \sigma_{\beta_k}^{2-c_{1k}-\frac{1}{2}-1}\exp(-\frac{\beta_k^2+2c_{2k}}{2\sigma^2_{\beta_k}}),\\ \sigma^2_{\beta_k}|rest &\sim\ IG(c_{1k}+\frac{1}{2},\ \frac{\beta_k^2}{2}+c_{k2}),\end{aligned}\tag{0.19}$$

$$\begin{aligned}p(\sigma^2_{\alpha_l}|\boldsymbol{Y},\alpha_l) &\propto\ p(\alpha_l|\sigma^2_{\alpha_l})p(\sigma^2_{\alpha_l})\\ &\propto\ \sigma_{\alpha_l}^{2-d_{1l}-\frac{1}{2}-1}\exp(-\frac{\alpha_l^2+2d_{2l}}{2\sigma^2_{\alpha_l}}),\\ \sigma^2_{\alpha_l}|rest &\sim\ IG(d_1+\frac{1}{2},\ \frac{\alpha_l^2}{2}+d_2),\end{aligned}\tag{0.20}$$

$$
\begin{aligned}
p(\gamma_k|\boldsymbol{Y},\beta_k) &\propto p(\beta_k|\sigma^2_{\beta_k},\gamma_k)p(\gamma_k)\\
&\propto (\gamma_k)^{\neq(\beta_k=0)}(1-\gamma_k)^{\neq(\beta_k\neq 0)}(\gamma_k)^{f_1-1}(1-\gamma)^{f_2-1},\\
\gamma_k|rest &\sim Beta(\#(\beta_k=0)+f_{1k},\ \#(\beta_k\neq 0)+f_{2k}),
\end{aligned} \tag{0.21}
$$

$$
\begin{aligned}
p(\nu_l|\boldsymbol{Y},\alpha_l) &\propto p(\alpha_l|\sigma^2_{\alpha_l},\nu_l)p(\nu_l)\\
&\propto (\nu_l)^{\#(\alpha_l=0)}(1-\nu_l)^{\#(\alpha_l\neq 0)}\nu_l^{m_{1l}-1}(1-\nu_l)^{m_{2l}-1},\\
\nu_l|rest &\sim Beta(\#(\alpha_1=0)+m_{1l},\ \#(\alpha_l\neq 0)+m_{2l}),
\end{aligned} \tag{0.22}
$$

where $\#(x)$ denotes count number of x.

SUPPLEMENTARY MATERIAL C: Results of the simulation study of DS and CS for generated data under the ZIP random effects model for Scenarioes 1 and 2 and results of the simulation study of DS and CS for generated data under the ZINB random effects model for Scenarios 1 and 2 with $\phi = 2$

Table C.1: Results of the simulation study of CS for generated data under the ZIP random effects model for scenario 2. The posterior mean, the standard deviation of estimators, the posterior median, the root of the mean squared error (RMSE), and relative bias (Rbias) for each of the parameter estimates for M= 100 simulated data with sample sizes of 500 and 1000. The generated data are analyzed with the ZIP model (*: the relative bias cannot be calculated since the real value of the related parameter is zero).

Parameter	True	mean	sd	median	RMSE	Rbias
			n=500			
β_1	0.500	0.491	0.044	0.493	0.226	-0.045
β_2	0.500	0.559	0.046	0.559	0.051	0.226
β_3	0.500	0.466	0.043	0.468	0.001	-0.033
β_4	0.500	0.523	0.044	0.520	0.005	-0.018
β_5	0.500	0.489	0.044	0.490	0.001	-0.013
β_6	0.000	-0.005	0.002	-0.002	0.000	*
β_7	0.000	-0.003	0.002	-0.001	0.000	*
β_8	0.000	-0.043	0.003	-0.044	0.000	*
β_9	0.000	0.003	0.002	0.001	0.000	*
β_{10}	0.000	0.011	0.004	0.003	0.000	*
β_{11}	0.000	-0.005	0.003	-0.003	0.000	*
β_{12}	0.000	0.020	0.004	0.018	0.000	*
β_{13}	0.000	-0.031	0.004	-0.038	0.000	*
β_{14}	0.000	-0.003	0.003	-0.001	0.000	*
β_{15}	0.000	-0.002	0.002	-0.001	0.000	*
α_1	0.500	0.455	0.063	0.454	0.007	-0.082
α_2	0.500	0.518	0.065	0.519	0.002	-0.039
α_3	0.500	0.543	0.065	0.542	0.001	0.032
α_4	0.500	0.492	0.065	0.492	0.003	-0.051
α_5	0.500	0.477	0.065	0.476	0.000	-0.013
α_6	0.000	0.000	0.005	0.000	0.000	*
α_7	0.000	-0.005	0.005	-0.001	0.000	*
α_8	0.000	0.002	0.006	0.000	0.000	*
α_9	0.000	0.000	0.006	0.000	0.000	*
α_{10}	0.000	0.016	0.009	0.002	0.000	*
			n=1000			
β_1	0.500	0.497	0.043	0.496	0.001	-0.052
β_2	0.500	0.506	0.045	0.506	0.001	0.055
β_3	0.500	0.473	0.043	0.474	0.001	-0.057
β_4	0.500	0.495	0.042	0.494	0.003	-0.059
β_5	0.500	0.494	0.043	0.495	0.000	-0.018
β_6	0.000	-0.007	0.002	-0.003	0.000	*
β_7	0.000	-0.003	0.002	-0.001	0.000	*
β_8	0.000	0.001	0.002	0.001	0.000	*
β_9	0.000	-0.020	0.003	-0.022	0.000	*
β_{10}	0.000	-0.001	0.002	0.000	0.000	*
β_{11}	0.000	0.002	0.003	0.001	0.000	*
β_{12}	0.000	0.001	0.002	0.001	0.000	*
β_{13}	0.000	0.002	0.003	0.001	0.000	*
β_{14}	0.000	0.000	0.002	0.000	0.000	*
β_{15}	0.000	0.000	0.002	0.000	0.000	*
α_1	0.500	0.487	0.061	0.486	0.010	-0.098
α_2	0.500	0.507	0.063	0.506	0.000	0.043
α_3	0.500	0.528	0.062	0.527	0.001	0.030
α_4	0.500	0.508	0.063	0.508	0.002	0.052
α_5	0.500	0.470	0.063	0.469	0.000	-0.009
α_6	0.000	0.003	0.005	0.001	0.000	*
α_7	0.000	0.018	0.006	0.005	0.000	*
α_8	0.000	-0.004	0.006	-0.001	0.000	*

Table C.2: Mean (SD) of true/false positive rate (TPR/FPR) and Matthews correlation coefficient of BF and LBFDR for ZINB and ZIP random effects models of Scenario 2 for CS with M = 100 simulated data with a sample size of 500 and 1000.

		ZIP		ZINB	
n		BF	LBFDR	BF	LBFDR
	TPR	0.903(0.032)	0.911(0.012)	0.921(0.012)	0.931(0.002)
500	FPR	0.275(0.028)	0.025(0.028)	0.262(0.025)	0.000(0.000)
	MCC	0.588(0.027)	0.944(0.063)	0.600(0.024)	1.000(0.000)
	TPR	1.000(0.000)	1.000(0.000)	1.000(0.000)	1.000(0.000)
1000	FPR	0.251(0.021)	0.000(0.000)	0.000(0.000)	0.000(0.000)
	MCC	0.612(0.029)	1.000(0.000)	1.000(0.000)	1.000(0.000)

Table C.3: Results of the simulation study of DS for generated data under the ZIP random effects model for scenario 1. The posterior mean, the standard deviation of estimators, the posterior median, the root of the mean squared error (RMSE), and relative bias (Rbias) for each of the parameter estimates for M= 100 simulated data with sample sizes of 500 and 1000. The generated data are analyzed with the ZIP model (*: the relative bias cannot be calculated since the real value of the related parameter is zero).

Parameter	True	mean	sd	median	RMSE	Rbias
			n=500			
β_1	1.000	0.992	0.044	0.991	0.000	- 0.018
β_2	1.000	0.959	0.079	0.960	0.004	- 0.020
β_3	1.000	0.960	0.064	0.960	0.002	- 0.080
β_4	1.000	0.918	0.066	0.919	0.000	- 0.038
β_5	1.000	0.980	0.022	0.981	0.000	- 0.038
β_6	0.000	- 0.002	0.005	0.000	0.000	*
β_7	0.000	0.000	0.000	0.000	0.000	*
β_8	0.000	- 0.023	0.052	- 0.028	0.001	*
β_9	0.000	0.003	0.006	0.000	0.000	*
β_{10}	0.000	0.004	0.003	0.000	0.000	*
β_{11}	0.000	- 0.001	0.001	0.000	0.000	*
β_{12}	0.000	0.003	0.038	0.013	0.000	*
β_{13}	0.000	- 0.040	0.084	- 0.041	0.000	*
β_{14}	0.000	0.008	0.016	0.000	0.000	*
β_{15}	0.000	- 0.003	0.002	0.000	0.000	*
α_1	1.000	0.958	0.041	0.956	0.002	- 0.088
α_2	1.000	0.917	0.119	0.915	0.000	- 0.030
α_3	1.000	0.950	0.033	0.947	0.002	- 0.094
α_4	1.000	0.988	0.043	0.986	0.000	- 0.028
α_5	1.000	0.984	0.117	0.984	0.000	- 0.032
α_6	0.000	- 0.002	0.003	0.000	0.000	*
α_7	0.000	- 0.004	0.008	0.000	0.000	*
α_8	0.000	0.008	0.015	0.000	0.000	*
α_9	0.000	0.001	0.001	0.000	0.000	*
α_{10}	0.000	0.009	0.025	0.000	0.000	*
			n=1000			
β_1	1.000	0.992	0.044	0.991	0.000	- 0.018
β_2	1.000	0.959	0.079	0.960	0.004	- 0.020
β_3	1.000	0.960	0.064	0.960	0.002	- 0.080
β_4	1.000	0.918	0.066	0.919	0.000	- 0.038
β_5	1.000	0.980	0.022	0.981	0.000	- 0.038
β_6	0.000	- 0.002	0.005	0.000	0.000	*
β_7	0.000	0.000	0.000	0.000	0.000	*
β_8	0.000	- 0.023	0.052	- 0.028	0.001	*
β_9	0.000	0.003	0.006	0.000	0.000	*
β_{10}	0.000	0.004	0.003	0.000	0.000	*
β_{11}	0.000	- 0.001	0.001	0.000	0.000	*
β_{12}	0.000	0.003	0.038	0.013	0.000	*
β_{13}	0.000	- 0.040	0.084	- 0.041	0.000	*
β_{14}	0.000	0.008	0.016	0.000	0.000	*
β_{15}	0.000	- 0.003	0.002	0.000	0.000	*
α_1	1.000	0.958	0.041	0.956	0.002	- 0.088
α_2	1.000	0.917	0.119	0.915	0.000	- 0.030
α_3	1.000	0.950	0.033	0.947	0.002	- 0.094
α_4	1.000	0.988	0.043	0.986	0.000	- 0.028
α_5	1.000	0.984	0.117	0.984	0.000	- 0.032
α_6	0.000	- 0.002	0.003	0.000	0.000	*
α_7	0.000	- 0.004	0.008	0.000	0.000	*
α_8	0.000	0.008	0.015	0.000	0.000	*
α_9	0.000	0.001	0.001	0.000	0.000	*
α_{10}	0.000	0.009	0.025	0.000	0.000	*

Table C.4: Means (sds) of True/False Positive Rate (TPR/FPR) and Matthews correlation coefficient of BF and LBFDR for ZINB and ZIP random effects models of Scenario 1 for DS with M = 100 simulated data with sample sizes 500 and 1000.

		ZIP			ZINB		
n		BF	LBFDR	median	BF	LBFDR	median
	TPR	0.975(0.051)	0.978(0.055)	1.000(0.000)	0.978(0.055)	1.000(0.000)	1.000(0.000)
500	FPR	0.033(0.045)	0.031(0.032)	0.090(0.027)	0.031(0.121)	0.090(0.012)	0.085(0.017)
	MCC	0.938(0.079)	0.939(0.069)	0.921(0.021)	0.939(0.069)	0.921(0.032)	0.925(0.071)
	TPR	1.000(0.000)	1.000(0.000)	0.023(0.045)	1.000(0.000)	1.000(0.000)	1.000(0.000)
1000	FPR	0.017(0.033)	0.000(0.000)	1.000(0.000)	0.000(0.000)	0.000(0.000)	0.000(0.000)
	MCC	0.980(0.039)	1.000(0.000)	0.981(0.039)	1.000(0.000)	1.000(0.000)	1.000(0.000)

Table C.5: Results of the simulation study of DS for generated data under the ZIP random effects model for scenario 2. The posterior mean, the standard deviation of estimators, the posterior median, the root of the mean squared error (RMSE), and relative bias (Rbias) for each of the parameter estimates for M= 100 simulated data with sample sizes of 500 and 1000. The generated data are analyzed with the ZIP model (*: the relative bias cannot be calculated since the real value of the related parameter is zero).

Parameter	True	mean	sd	median	RMSE	Rbias
			n=500			
β_1	0.500	0.499	0.043	0.498	0.000	-0.004
β_2	0.500	0.535	0.044	0.534	0.001	0.068
β_3	0.500	0.438	0.043	0.438	0.004	-0.052
β_4	0.500	0.506	0.042	0.504	0.000	0.004
β_5	0.500	0.430	0.042	0.429	0.005	-0.142
β_6	0.000	0.000	0.001	0.000	0.000	*
β_7	0.000	-0.001	0.002	0.000	0.000	*
β_8	0.000	-0.032	0.002	-0.035	0.001	*
β_9	0.000	0.006	0.003	0.000	0.000	*
β_{10}	0.000	0.019	0.002	0.000	0.000	*
β_{11}	0.000	-0.003	0.003	0.000	0.000	*
β_{12}	0.000	0.000	0.002	-0.006	0.000	*
β_{13}	0.000	-0.001	0.002	0.000	0.000	*
β_{14}	0.000	-0.001	0.002	0.000	0.000	*
β_{15}	0.000	-0.003	0.003	0.000	0.000	*
α_1	0.500	0.420	0.002	0.457	0.001	-0.043
α_2	0.500	0.352	0.062	0.353	0.022	-0.147
α_3	0.500	0.542	0.064	0.540	0.002	0.080
α_4	0.500	0.546	0.064	0.544	0.002	0.088
α_5	0.500	0.488	0.064	0.487	0.000	-0.036
α_6	0.500	0.375	0.000	-0.026	0.000	*
α_7	0.000	0.001	0.006	0.000	0.000	*
α_8	0.000	0.000	0.005	0.000	0.000	*
α_9	0.000	0.000	0.005	0.000	0.000	*
α_{10}	0.000	0.002	0.007	0.000	0.000	*
			n=1000			
β_1	0.500	0.483	0.043	0.483	0.000	-0.034
β_2	0.500	0.502	0.044	0.502	0.000	0.004
β_3	0.500	0.448	0.043	0.449	0.003	0.102
β_4	0.500	0.522	0.042	0.522	0.000	0.022
β_5	0.500	0.499	0.042	0.499	0.000	-0.002
β_6	0.000	-0.003	0.001	0.000	0.000	*
β_7	0.000	0.001	0.002	0.000	0.000	*
β_8	0.000	0.001	0.002	0.000	0.000	*
β_9	0.000	-0.002	0.003	0.000	0.000	*
β_{10}	0.000	0.004	0.002	0.000	0.000	*
β_{11}	0.000	-0.003	0.003	0.000	0.000	*
β_{12}	0.000	-0.014	0.002	-0.012	0.000	*
β_{13}	0.000	0.003	0.002	0.000	0.000	*
β_{14}	0.000	-0.001	0.003	0.000	0.000	*
β_{15}	0.000	-0.001	0.002	0.000	0.000	*
α_1	0.500	0.423	0.062	0.422	0.006	-0.077
α_2	0.500	0.495	0.064	0.494	0.000	- 0.012
α_3	0.500	0.559	0.064	0.558	0.003	0.016
α_4	0.500	0.531	0.001	0.062	0.001	0.031
α_5	0.500	0.464	0.001	-0.072	0.001	0.084
α_6	0.000	0.000	0.006	0.000	0.000	*
α_7	0.000	0.000	0.006	0.000	0.000	*
α_8	0.000	0.002	0.005	0.000	0.000	*
α_9	0.000	0.001	0.005	0.000	0.000	*
α_{10}	0.000	0.001	0.007	0.000	0.000	*

Table C.6: Mean (SD) of True/False Positive Rate (TPR/FPR) and Matthews correlation coefficient of BF and LBFDR for ZINB and ZIP random effects models of Scenario 2 for DS with M = 100 simulated data with sample sizes 500 and 1000.

		ZIP			ZINB		
n		BF	LBFDR	median	BF	LBFDR	median
	TPR	0.979(0.053)	0.978(0.055)	1.000(0.000)	0.978(0.055)	1.000(0.000)	1.000(0.000)
500	FPR	0.033(0.045)	0.031(0.032)	0.080(0.024)	0.031(0.120)	0.090(0.012)	0.085(0.017)
	MCC	0.948(0.079)	0.949(0.069)	0.931(0.021)	0.949(0.069)	0.971(0.032)	0.928(0.071)
	TPR	1.000(0.000)	1.000(0.000)	0.021(0.035)	1.000(0.000)	1.000(0.000)	1.000(0.000)
1000	FPR	0.017(0.033)	0.000(0.000)	1.000(0.000)	0.000(0.000)	0.000(0.000)	0.000(0.000)
	MCC	0.980(0.039)	1.000(0.000)	0.981(0.039)	1.000(0.000)	1.000(0.000)	1.000(0.000)

Table C.7: Results of the simulation study of CS for generated data under the ZINB random effects model for scenario 1. The posterior mean, the standard deviation of estimators, the posterior median, the root of the mean squared error (RMSE), and relative bias (Rbias) for each of the parameter estimates for M= 100 simulated data with sample sizes of 500 and 1000. The generated data are analyzed with the ZINB model (*: the relative bias cannot be calculated since the real value of the related parameter is zero).

Parameter	True	mean	sd	median	RMSE	Rbias
			n=500			
β_1	1.000	0.904	0.044	0.904	0.003	-0.018
β_2	1.000	0.979	0.046	0.983	0.002	0.116
β_3	1.000	0.992	0.044	0.993	0.002	-0.074
β_4	1.000	0.912	0.043	0.912	0.002	0.024
β_5	1.000	0.960	0.044	0.961	0.002	-0.078
β_6	0.000	-0.004	0.002	-0.002	0.000	*
β_7	0.000	0.000	0.002	0.000	0.000	*
β_8	0.000	0.002	0.002	0.000	0.000	*
β_9	0.000	-0.001	0.003	-0.001	0.000	*
β_{10}	0.000	0.001	0.003	0.000	0.000	*
β_{11}	0.000	0.001	0.003	0.001	0.000	*
β_{12}	0.000	0.007	0.002	0.002	0.000	*
β_{13}	0.000	0.003	0.003	0.001	0.000	*
β_{14}	0.000	-0.029	0.004	-0.006	0.000	*
β_{15}	0.000	-0.003	0.002	-0.001	0.000	*
α_1	1.000	0.950	0.063	0.949	0.003	-0.102
α_2	1.000	0.959	0.065	0.957	0.003	0.014
α_3	1.000	0.965	0.065	0.961	0.004	0.022
α_4	1.000	0.934	0.065	0.934	0.004	-0.132
α_5	1.000	0.957	0.065	0.958	0.002	-0.084
α_6	0.000	-0.029	0.007	-0.004	0.000	*
α_7	0.000	-0.001	0.005	0.000	0.000	*
α_8	0.000	0.006	0.006	0.002	0.000	*
α_9	0.000	0.006	0.005	0.001	0.000	*
α_{10}	0.000	-0.002	0.007	0.000	0.000	*
ϕ	2.000	1.949	0.107	2.301	0.001	0.013
			n=1000			
β_1	1.000	0.988	0.043	0.987	0.001	-0.026
β_2	1.000	0.928	0.045	0.928	0.001	0.056
β_3	1.000	0.915	0.043	0.915	0.001	-0.170
β_4	1.000	0.985	0.042	0.986	0.002	0.000
β_5	1.000	0.967	0.043	0.966	0.001	-0.079
β_6	0.000	0.006	0.002	0.004	0.000	*
β_7	0.000	-0.002	0.002	-0.001	0.000	*
β_8	0.000	0.003	0.002	0.001	0.000	*
β_9	0.000	0.000	0.003	0.000	0.000	*
β_{10}	0.000	0.001	0.003	0.001	0.000	*
β_{11}	0.000	0.002	0.002	0.001	0.000	*
β_{12}	0.000	-0.001	0.001	-0.001	0.000	*
β_{13}	0.000	0.017	0.003	0.006	0.000	*
β_{14}	0.000	0.001	0.002	0.000	0.000	*
β_{15}	0.000	-0.008	0.002	-0.003	0.000	*
α_1	1.000	0.957	0.062	0.953	0.003	-0.106
α_2	1.000	0.960	0.064	0.956	0.002	-0.088
α_3	1.000	0.997	0.063	0.996	0.054	0.000
α_4	1.000	0.969	0.064	0.969	0.001	-0.136
α_5	1.000	0.975	0.064	0.974	0.001	-0.092
α_6	0.000	0.000	0.005	0.000	0.000	*
α_7	0.000	0.004	0.005	0.001	0.000	*
α_8	0.000	0.005	0.005	0.002	0.000	*
α_9	0.000	-0.001	0.004	-0.001	0.000	*
α_{10}	0.000	-0.001	0.007	-0.001	0.000	*
ϕ	2.000	2.045	0.111	2.011	0.005	0.011

Table C.8: Results of the simulation study of CS for generated data under the ZINB random effects model for scenario 2. The posterior mean, the standard deviation of estimators, the posterior median, the root of the mean squared error (RMSE), and relative bias (Rbias) for each of the parameter estimates for M= 100 simulated data with sample sizes of 500 and 1000. The generated data are analyzed with the ZINB model (*: the relative bias cannot be calculated since the real value of the related parameter is zero).

Parameter	True	mean	sd	median	RMSE	Rbias
			n=500			
β_1	0.500	0.504	0.044	0.504	0.002	0.048
β_2	0.500	0.579	0.046	0.583	0.001	0.023
β_3	0.500	0.492	0.044	0.493	0.001	-0.038
β_4	0.500	0.512	0.043	0.512	0.001	0.018
β_5	0.500	0.460	0.044	0.461	0.002	-0.043
β_6	0.000	-0.004	0.002	-0.002	0.000	*
β_7	0.000	0.000	0.002	0.000	0.000	*
β_8	0.000	0.002	0.002	0.000	0.000	*
β_9	0.000	-0.001	0.003	-0.001	0.000	*
β_{10}	0.000	0.001	0.003	0.000	0.000	*
β_{11}	0.000	0.001	0.003	0.001	0.000	*
β_{12}	0.000	0.007	0.002	0.002	0.000	*
β_{13}	0.000	0.003	0.003	0.001	0.000	*
β_{14}	0.000	-0.029	0.004	-0.006	0.000	*
β_{15}	0.000	-0.003	0.002	-0.001	0.000	*
α_1	0.500	0.450	0.063	0.449	0.003	-0.056
α_2	0.500	0.559	0.065	0.557	0.007	0.083
α_3	0.500	0.565	0.065	0.561	0.000	0.015
α_4	0.500	0.434	0.065	0.434	0.008	-0.092
α_5	0.500	0.457	0.065	0.458	0.000	-0.020
α_6	0.000	-0.029	0.007	-0.004	0.000	*
α_7	0.000	-0.001	0.005	0.000	0.000	*
α_8	0.000	0.006	0.006	0.002	0.000	*
α_9	0.000	0.006	0.005	0.001	0.000	*
α_{10}	0.000	-0.002	0.007	0.000	0.000	*
ϕ	2.000	1.949	0.107	2.301	0.301	0.050
			n=1000			
β_1	0.500	0.488	0.043	0.487	0.001	-0.055
β_2	0.500	0.528	0.045	0.528	0.001	0.056
β_3	0.500	0.415	0.043	0.415	0.001	-0.058
β_4	0.500	0.485	0.042	0.486	0.001	-0.026
β_5	0.500	0.467	0.043	0.466	0.000	-0.049
β_6	0.000	0.006	0.002	0.004	0.000	*
β_7	0.000	-0.002	0.002	-0.001	0.000	*
β_8	0.000	0.003	0.002	0.001	0.000	*
β_9	0.000	0.000	0.003	0.000	0.000	*
β_{10}	0.000	0.001	0.003	0.001	0.000	*
β_{11}	0.000	0.002	0.002	0.001	0.000	*
β_{12}	0.000	-0.001	0.001	-0.001	0.000	*
β_{13}	0.000	0.017	0.003	0.006	0.000	*
β_{14}	0.000	0.001	0.002	0.000	0.000	*
β_{15}	0.000	-0.008	0.002	-0.003	0.000	*
α_1	0.500	0.557	0.062	0.553	0.001	0.064
α_2	0.500	0.460	0.064	0.456	0.000	-0.087
α_3	0.500	0.497	0.063	0.496	0.000	-0.011
α_4	0.500	0.469	0.064	0.469	0.001	-0.095
α_5	0.500	0.475	0.064	0.474	0.005	-0.072
α_6	0.000	0.000	0.005	0.000	0.000	*
α_7	0.000	0.004	0.005	0.001	0.000	*
α_8	0.000	0.005	0.005	0.002	0.000	*
α_9	0.000	-0.001	0.004	-0.001	0.000	*
α_{10}	0.000	-0.001	0.007	-0.001	0.000	*
ϕ	2.000	2.045	0.045	2.011	0.011	0.005

Table C.9: Results of the simulation study of DS for generated data under the ZINB random effects model for scenario 1. The posterior mean, the standard deviation of estimators, the posterior median, the root of the mean squared error (RMSE), and relative bias (Rbias) for each of the parameter estimates for M= 100 simulated data with sample sizes of 500 and 1000. The generated data are analyzed with the ZINB model (*: the relative bias cannot be calculated since the real value of the related parameter is zero).

Parameter	True	mean	sd	median	RMSE	Rbias
			n=500			
β_1	1.000	1.013	0.043	1.014	0.000	0.014
β_2	1.000	1.059	0.044	1.061	0.004	0.061
β_3	1.000	0.976	0.043	0.976	0.001	-0.024
β_4	1.000	1.016	0.042	1.015	0.000	0.015
β_5	1.000	0.955	0.042	0.956	0.002	-0.044
β_6	0.000	0.004	0.001	0.000	0.000	*
β_7	0.000	-0.005	0.002	0.000	0.000	*
β_8	0.000	0.000	0.002	0.000	0.000	*
β_9	0.000	0.007	0.003	0.000	0.000	*
β_{10}	0.000	0.002	0.002	0.000	0.000	*
β_{11}	0.000	-0.001	0.003	0.000	0.000	*
β_{12}	0.000	-0.001	0.002	0.000	0.000	*
β_{13}	0.000	-0.004	0.002	0.000	0.000	*
β_{14}	0.000	-0.008	0.003	0.000	0.000	*
β_{15}	0.000	0.000	0.002	0.000	0.000	*
α_1	1.000	0.910	0.062	0.908	0.008	-0.092
α_2	1.000	1.012	0.064	1.008	0.000	0.008
α_3	1.000	1.049	0.064	1.046	0.002	0.040
α_4	1.000	0.997	0.064	0.993	0.000	-0.007
α_5	1.000	0.977	0.000	-0.022	0.001	-0.023
α_6	0.000	-0.022	0.006	0.000	0.000	*
α_7	0.000	-0.023	0.005	0.000	0.000	*
α_8	0.000	-0.004	0.005	0.000	0.000	*
α_9	0.000	0.001	0.005	0.000	0.000	*
α_{10}	0.000	-0.004	0.007	0.000	0.000	*
ϕ	2.000	2.100	0.021	2.001	0.001	0.001
			n=1000			
β_1	1.000	0.921	0.043	0.996	0.000	-0.004
β_2	1.000	1.010	0.044	1.010	0.000	0.010
β_3	1.000	0.945	0.043	0.944	0.003	-0.054
β_4	1.000	1.033	0.042	1.033	0.000	0.015
β_5	1.000	1.013	0.042	1.013	0.000	0.013
β_6	0.000	-0.003	0.001	0.000	0.000	*
β_7	0.000	0.001	0.002	0.000	0.001	*
β_8	0.000	-0.001	0.002	0.000	0.001	*
β_9	0.000	0.024	0.003	0.025	0.001	*
β_{10}	0.000	-0.001	0.002	0.000	0.001	*
β_{11}	0.000	-0.001	0.003	0.000	0.000	*
β_{12}	0.000	-0.003	0.002	0.000	0.000	*
β_{13}	0.000	0.001	0.002	0.000	0.000	*
β_{14}	0.000	0.003	0.003	0.000	0.000	*
β_{15}	0.000	-0.001	0.002	0.000	0.000	*
α_1	1.000	0.909	0.062	0.907	0.009	-0.093
α_2	1.000	0.947	0.000	0.946	-0.003	-0.064
α_3	1.000	1.016	0.064	1.015	0.000	0.015
α_4	1.000	1.022	0.064	1.021	0.000	0.021
α_5	1.000	0.960	0.064	0.958	0.002	-0.042
α_6	0.000	0.001	0.006	0.000	0.000	*
α_7	0.000	0.000	0.005	0.000	0.000	*
α_8	0.000	0.005	0.005	0.000	0.000	*
α_9	0.000	-0.002	0.005	0.000	0.000	*
α_{10}	0.000	0.000	0.007	0.000	0.000	*
ϕ	2.000	2.213	0.158	2.001	0.001	0.001

Table C.10 Results of the simulation study of DS for generated data under the ZINB random effects model for scenario 2. The posterior mean, the standard deviation of estimators, the posterior median, the root of the mean squared error (RMSE), and relative bias (Rbias) for each of the parameter estimates for M= 100 simulated data with sample sizes of 500 and 1000. The generated data are analyzed with the ZINB model (*: the relative bias cannot be calculated since the real value of the related parameter is zero).

Parameter	True	mean	sd	median	RMSE	Rbias
			n=500			
β_1	0.500	0.499	0.043	0.498	0.000	-0.004
β_2	0.500	0.535	0.044	0.534	0.001	0.068
β_3	0.500	0.438	0.043	0.438	0.004	-0.052
β_4	0.500	0.506	0.042	0.504	0.000	0.004
β_5	0.500	0.430	0.042	0.429	0.005	-0.142
β_6	0.000	0.000	0.001	0.000	0.000	*
β_7	0.000	-0.001	0.002	0.000	0.000	*
β_8	0.000	-0.032	0.002	-0.035	0.001	*
β_9	0.000	0.006	0.003	0.000	0.000	*
β_{10}	0.000	0.019	0.002	0.000	0.000	*
β_{11}	0.000	-0.003	0.003	0.000	0.000	*
β_{12}	0.000	0.000	0.002	-0.006	0.000	*
β_{13}	0.000	-0.001	0.002	0.000	0.000	*
β_{14}	0.000	-0.001	0.002	0.000	0.000	*
β_{15}	0.000	-0.003	0.003	0.000	0.000	*
α_1	0.500	0.420	0.002	0.457	0.001	-0.043
α_2	0.500	0.352	0.062	0.353	0.022	-0.147
α_3	0.500	0.542	0.064	0.540	0.002	0.080
α_4	0.500	0.546	0.064	0.544	0.002	0.088
α_5	0.500	0.488	0.064	0.487	0.000	-0.036
α_6	0.500	0.375	0.000	-0.026	0.000	*
α_7	0.000	0.001	0.006	0.000	0.000	*
α_8	0.000	0.000	0.005	0.000	0.000	*
α_9	0.000	0.000	0.005	0.000	0.000	*
α_{10}	0.000	0.002	0.007	0.000	0.000	*
ϕ	2.000	1.943	0.133	1.949	0.051	-0.031
			n=1000			
β_1	0.500	0.483	0.043	0.483	0.000	-0.034
β_2	0.500	0.502	0.044	0.502	0.000	0.004
β_3	0.500	0.448	0.043	0.449	0.003	0.102
β_4	0.500	0.522	0.042	0.522	0.000	0.022
β_5	0.500	0.499	0.042	0.499	0.000	-0.002
β_6	0.000	-0.003	0.001	0.000	0.000	*
β_7	0.000	0.001	0.002	0.000	0.000	*
β_8	0.000	0.001	0.002	0.000	0.000	*
β_9	0.000	-0.002	0.003	0.000	0.000	*
β_{10}	0.000	0.004	0.002	0.000	0.000	*
β_{11}	0.000	-0.003	0.003	0.000	0.000	*
β_{12}	0.000	-0.014	0.002	-0.012	0.000	*
β_{13}	0.000	0.003	0.002	0.000	0.000	*
β_{14}	0.000	-0.001	0.003	0.000	0.000	*
β_{15}	0.000	-0.001	0.002	0.000	0.000	*
α_1	0.500	0.423	0.062	0.422	0.006	-0.077
α_2	0.500	0.495	0.064	0.494	0.000	- 0.012
α_3	0.500	0.559	0.064	0.558	0.003	0.016
α_4	0.500	0.531	0.001	0.062	0.001	0.031
α_5	0.500	0.464	0.001	-0.072	0.001	0.084
α_6	0.000	0.000	0.006	0.000	0.000	*
α_7	0.000	0.000	0.006	0.000	0.000	*
α_8	0.000	0.002	0.005	0.000	0.000	*
α_9	0.000	0.001	0.005	0.000	0.000	*
α_{10}	0.000	0.001	0.007	0.000	0.000	*
ϕ	2.000	1.943	0.133	2.011	0.001	0.057

SUPPLEMENTARY MATERIAL D: Results of the simulation study of DS and CS for generated data under the ZIP random effects model for Scenarios 3 and 4 and results of the simulation study of DS and CS for generated data under the ZINB random effects model for Scenarios 3 and 4 with $\phi = 2$ and a sample size of 500.

Table D.1: Results of the simulation study of CS for generated data under the ZIP random effects model for Scenario 3. Posterior mean, the standard deviation of estimators, the posterior median, the root of the mean squared error (RMSE), relative bias (Rbias), BF, and LBFDR (sd) for each of the parameter estimates for M= 100 simulated data with a sample size of 500. The generated data are analyzed with the ZIP model.

Parameter	True	mean	sd	median	RMSE	Rbias	BF	LBFDR(sd)
				n=500				
β_1	0.500	0.385	0.113	0.354	0.033	-0.230	$> 10^3$	0.000(0.000)
β_2	0.500	0.483	0.093	0.469	0.059	-0.035	$> 10^3$	0.000(0.000)
β_3	0.500	0.440	0.058	0.437	0.007	-0.119	$> 10^3$	0.000(0.000)
β_4	0.500	0.538	0.056	0.540	0.003	0.076	$> 10^3$	0.000(0.000)
β_5	0.500	0.486	0.062	0.485	0.004	-0.029	$> 10^3$	0.000(0.000)
β_6	0.500	0.517	0.059	0.521	0.008	0.034	$> 10^3$	0.000(0.000)
β_7	0.500	0.402	0.054	0.400	0.061	-0.197	$> 10^3$	0.000(0.000)
β_8	0.500	0.345	0.046	0.342	0.068	-0.311	$> 10^3$	0.000(0.000)
β_9	0.500	0.351	0.051	0.349	0.062	-0.298	$> 10^3$	0.000(0.000)
β_{10}	0.500	0.557	0.070	0.561	0.009	0.114	$> 10^3$	0.000(0.000)
β_{11}	0.500	0.504	0.056	0.502	0.002	0.008	$> 10^3$	0.000(0.000)
β_{12}	0.500	0.500	0.051	0.502	0.008	0.000	$> 10^3$	0.000(0.000)
β_{13}	0.500	0.488	0.059	0.489	0.003	-0.024	$> 10^3$	0.000(0.000)
β_{14}	0.500	0.488	0.081	0.498	0.011	-0.024	$> 10^3$	0.000(0.000)
β_{15}	0.500	0.442	0.070	0.453	0.006	-0.116	$> 10^3$	0.000(0.000)
α_1	0.500	0.414	0.090	0.413	0.012	-0.172	$> 10^3$	0.000(0.000)
α_2	0.500	0.514	0.097	0.512	0.003	0.028	$> 10^3$	0.000(0.000)
α_3	0.500	0.469	0.095	0.467	0.003	-0.062	$> 10^3$	0.000(0.000)
α_4	0.500	0.415	0.091	0.414	0.021	-0.170	$> 10^3$	0.000(0.000)
α_5	0.500	0.476	0.090	0.475	0.003	-0.048	$> 10^3$	0.000(0.000)
α_6	0.500	0.457	0.094	0.455	0.002	-0.085	$> 10^3$	0.000(0.000)
α_7	0.500	0.440	0.093	0.440	0.009	-0.121	$> 10^3$	0.000(0.000)
α_8	0.500	0.476	0.098	0.476	0.004	-0.047	$> 10^3$	0.000(0.000)
α_9	0.500	0.422	0.092	0.422	0.009	-0.156	$> 10^3$	0.000(0.000)
α_{10}	0.500	0.391	0.091	0.390	0.015	-0.218	$> 10^3$	0.000(0.000)

Table D.2: Results of the simulation study of CS for generated data under the ZIP random effects model for Scenario 4. Posterior mean, the standard deviation of estimators, the posterior median, the root of the mean of squared error (RMSE), relative bias (Rbias), BF (sd), and LBFDR (sd) for each of the parameter estimates for M= 100 simulated data with a sample size of 500. The generated data are analyzed with the ZIP model.

Parameter	True	mean	sd	median	RMSE	BF (sd)	LBFDR (sd)
				n=500			
β_1	0.000	0.001	0.009	0.001	0.004	0.000(0.000)	1.000(0.000)
β_2	0.000	0.016	0.009	0.000	0.009	0.000(0.000)	1.000(0.000)
β_3	0.000	-0.003	0.010	-0.001	0.003	0.000(0.000)	1.000(0.000)
β_4	0.000	-0.004	0.008	-0.001	0.004	0.000(0.000)	1.000(0.000)
β_5	0.000	0.000	0.009	0.000	0.000	0.000(0.000)	1.000(0.000)
β_6	0.000	0.000	0.009	0.000	0.009	0.000(0.000)	1.000(0.000)
β_7	0.000	0.003	0.008	0.001	0.011	0.000(0.000)	1.000(0.000)
β_8	0.000	-0.001	0.007	-0.001	0.007	0.000(0.000)	1.000(0.000)
β_9	0.000	0.002	0.007	0.000	0.005	0.000(0.000)	1.000(0.000)
β_{10}	0.000	0.008	0.006	0.000	0.012	0.000(0.000)	1.000(0.000)
β_{11}	0.000	-0.020	0.008	-0.005	0.001	0.000(0.000)	1.000(0.000)
β_{12}	0.000	-0.006	0.010	-0.001	0.010	0.000(0.000)	1.000(0.000)
β_{13}	0.000	0.001	0.009	0.000	0.005	0.000(0.000)	1.000(0.000)
β_{14}	0.000	-0.005	0.008	-0.002	0.002	0.000(0.000)	1.000(0.000)
β_{15}	0.000	-0.006	0.009	-0.001	0.003	0.000(0.000)	1.000(0.000)
α_1	0.000	0.000	0.020	0.000	0.015	0.000(0.000)	1.000(0.000)
α_2	0.000	0.011	0.036	0.002	0.016	0.000(0.000)	1.000(0.000)
α_3	0.000	0.003	0.015	0.000	0.010	0.000(0.000)	1.000(0.000)
α_4	0.000	0.000	0.029	0.000	0.048	0.000(0.000)	1.000(0.000)
α_5	0.000	-0.004	0.030	-0.001	0.008	0.000(0.000)	1.000(0.000)
α_6	0.000	-0.024	0.014	-0.004	0.014	0.000(0.000)	1.000(0.000)
α_7	0.000	-0.003	0.023	-0.001	0.006	0.000(0.000)	1.000(0.000)
α_8	0.000	-0.001	0.025	-0.001	0.026	0.000(0.000)	1.000(0.000)
α_9	0.000	0.000	0.023	0.000	0.005	0.000(0.000)	1.000(0.000)
α_{10}	0.000	-0.039	0.059	-0.008	0.011	0.000(0.000)	1.000(0.000)

Table D.3: Results of the simulation study of DS for generated data under the ZIP random effects model for Scenario 3. Posterior mean, the standard deviation of estimators, the posterior median, the root of the mean squared error (RMSE), relative bias (Rbias), BF, and LBFDR (sd) for each of the parameter estimates for M= 100 simulated data with a sample size of 500. The generated data are analyzed with the ZIP model.

Parameter	True	mean	sd	median	RMSE	Rbias	BF	LBFDR(sd)
				n=500				
β_1	0.500	0.514	0.000	0.518	0.006	0.028	$> 10^3$	0.000(0.000)
β_2	0.500	0.583	0.000	0.581	0.017	0.166	$> 10^3$	0.000(0.000)
β_3	0.500	0.465	0.000	0.465	0.002	-0.070	$> 10^3$	0.000(0.000)
β_4	0.500	0.524	0.000	0.527	0.001	0.048	$> 10^3$	0.000(0.000)
β_5	0.500	0.442	0.000	0.444	0.005	-0.115	$> 10^3$	0.000(0.000)
β_6	0.500	0.488	0.000	0.487	0.009	-0.024	$> 10^3$	0.000(0.000)
β_7	0.500	0.558	0.000	0.557	0.005	0.115	$> 10^3$	0.000(0.000)
β_8	0.500	0.468	0.000	0.468	0.003	-0.064	$> 10^3$	0.000(0.000)
β_9	0.500	0.485	0.000	0.488	0.002	-0.029	$> 10^3$	0.000(0.000)
β_{10}	0.500	0.579	0.000	0.578	0.008	0.158	$> 10^3$	0.000(0.000)
β_{11}	0.500	0.518	0.000	0.518	0.003	0.036	$> 10^3$	0.000(0.000)
β_{12}	0.500	0.492	0.000	0.488	0.007	-0.016	$> 10^3$	0.000(0.000)
β_{13}	0.500	0.489	0.000	0.493	0.005	-0.021	$> 10^3$	0.000(0.000)
β_{14}	0.500	0.502	0.000	0.499	0.012	0.004	$> 10^3$	0.000(0.000)
β_{15}	0.500	0.452	0.000	0.463	0.003	-0.096	$> 10^3$	0.000(0.000)
α_1	0.500	0.410	0.103	0.416	0.014	-0.179	$> 10^3$	0.000(0.000)
α_2	0.500	0.530	0.098	0.530	0.004	0.061	$> 10^3$	0.000(0.000)
α_3	0.500	0.469	0.097	0.469	0.005	-0.061	$> 10^3$	0.000(0.000)
α_4	0.500	0.420	0.097	0.424	0.021	-0.159	$> 10^3$	0.000(0.000)
α_5	0.500	0.487	0.092	0.484	0.003	-0.027	$> 10^3$	0.000(0.000)
α_6	0.500	0.452	0.093	0.452	0.003	-0.096	$> 10^3$	0.000(0.000)
α_7	0.500	0.447	0.096	0.448	0.006	-0.106	$> 10^3$	0.000(0.000)
α_8	0.500	0.472	0.095	0.470	0.005	-0.055	$> 10^3$	0.000(0.000)
α_9	0.500	0.432	0.093	0.432	0.007	-0.136	$> 10^3$	0.000(0.000)
α_{10}	0.500	0.402	0.108	0.410	0.012	-0.196	$> 10^3$	0.000(0.000)

Table D.4: Results of the simulation study of DS for generated data under the ZIP random effects model for Scenario 4. Posterior mean, the standard deviation of estimators, the posterior median, the root of the mean of squared error (RMSE), relative bias (Rbias), BF (sd), and LBFDR (sd) for each of the parameter estimates for M= 100 simulated data with a sample size of 500. The generated data are analyzed with the ZIP model.

Parameter	True	mean	sd	median	RMSE	BF(sd)	LBFDR(sd)
				n=500			
β_1	0.000	0.000	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_2	0.000	0.004	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_3	0.000	0.000	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_4	0.000	0.000	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_5	0.000	0.000	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_6	0.000	-0.011	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_7	0.000	-0.003	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_8	0.000	-0.003	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_9	0.000	0.004	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_{10}	0.000	0.001	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_{11}	0.000	-0.010	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_{12}	0.000	-0.002	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_{13}	0.000	0.001	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_{14}	0.000	-0.018	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_{15}	0.000	0.000	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
α_1	0.000	-0.007	0.002	0.000	0.000	0.000(0.000)	1.000(0.000)
α_2	0.000	0.004	0.005	0.000	0.000	0.000(0.000)	1.000(0.000)
α_3	0.000	0.005	0.008	0.000	0.001	0.000(0.000)	1.000(0.000)
α_4	0.000	-0.003	0.004	0.000	0.000	0.000(0.000)	1.000(0.000)
α_5	0.000	-0.019	0.009	0.000	0.001	0.000(0.000)	1.000(0.000)
α_6	0.000	-0.005	0.003	0.000	0.000	0.000(0.000)	1.000(0.000)
α_7	0.000	0.000	0.002	0.000	0.000	0.000(0.000)	1.000(0.000)
α_8	0.000	0.000	0.006	0.000	0.000	0.000(0.000)	1.000(0.000)
α_9	0.000	-0.003	0.009	0.000	0.000	0.000(0.000)	1.000(0.000)
α_{10}	0.000	-0.001	0.008	0.000	0.000	0.000(0.000)	1.000(0.000)

Table D.5: Results of the simulation study of CS for generated data under the ZINB random effects model for Scenario 3. Posterior mean, the standard deviation of estimators, the posterior median, the root of the mean of squared error (RMSE), relative bias (Rbias), BF, and LBFDR (sd)for each of the parameter estimates for M= 100 simulated data with a sample size of 500. The generated data are analyzed with the ZINB model.

Parameter	True	mean	sd	median	RMSE	Rbias	BF	LBFDR(sd)
				n=500				
β_1	0.500	0.549	0.012	0.547	0.003	0.097	$> 10^3$	0.000(0.000)
β_2	0.500	0.566	0.021	0.567	0.005	0.131	$> 10^3$	0.000(0.000)
β_3	0.500	0.503	0.032	0.503	0.001	0.006	$> 10^3$	0.000(0.000)
β_4	0.500	0.485	0.0311	0.481	0.002	-0.030	$> 10^3$	0.000(0.000)
β_5	0.500	0.419	0.012	0.418	0.009	-0.162	$> 10^3$	0.000(0.000)
β_6	0.500	0.542	0.031	0.543	0.002	0.084	$> 10^3$	0.000(0.000)
β_7	0.500	0.427	0.011	0.427	0.006	-0.145	$> 10^3$	0.000(0.000)
β_8	0.500	0.501	0.44	0.501	0.002	0.001	$> 10^3$	0.000(0.000)
β_9	0.500	0.530	0.042	0.527	0.004	0.061	$> 10^3$	0.000(0.000)
β_{10}	0.500	0.488	0.012	0.489	0.001	-0.024	$> 10^3$	0.000(0.000)
β_{11}	0.500	0.527	0.031	0.527	0.001	0.054	$> 10^3$	0.000(0.000)
β_{12}	0.500	0.464	0.081	0.462	0.002	-0.072	$> 10^3$	0.000(0.000)
β_{13}	0.500	0.468	0.055	0.468	0.002	-0.063	$> 10^3$	0.000(0.000)
β_{14}	0.500	0.402	0.054	0.399	0.013	-0.196	$> 10^3$	0.000(0.000)
β_{15}	0.500	0.458	0.062	0.458	0.003	-0.084	$> 10^3$	0.000(0.000)
α_1	0.500	0.383	0.084	0.383	0.014	-0.234	$> 10^3$	0.000(0.000)
α_2	0.500	0.489	0.105	0.487	0.000	-0.023	$> 10^3$	0.000(0.000)
α_3	0.500	0.507	0.096	0.506	0.003	0.014	$> 10^3$	0.000(0.000)
α_4	0.500	0.457	0.102	0.452	0.014	-0.085	$> 10^3$	0.000(0.000)
α_5	0.500	0.561	0.095	0.560	0.008	0.121	$> 10^3$	0.000(0.000)
α_6	0.500	0.481	0.096	0.478	0.001	-0.038	$> 10^3$	0.000(0.000)
α_7	0.500	0.427	0.109	0.434	0.016	-0.145	$> 10^3$	0.000(0.000)
α_8	0.500	0.532	0.109	0.531	0.001	0.064	$> 10^3$	0.000(0.000)
α_9	0.500	0.508	0.095	0.508	0.001	0.016	$> 10^3$	0.000(0.000)
α_{10}	0.500	0.392	0.095	0.388	0.014	-0.216	$> 10^3$	0.000(0.000)
ϕ	2.000	2.189	0.055	2.175	0.037	0.087		

Table D.6: Results of the simulation study of CS for generated data under the ZINB random effects model for Scenario 4. Posterior mean, the standard deviation of estimators, the posterior median, the root of the mean of squared error (RMSE), relative bias (Rbias), BF (sd), and LBFDR (sd) for each of the parameter estimates for M= 100 simulated data with a sample size of 500. The generated data are analyzed with the ZINB model.

Parameter	True	mean	sd	median	RMSE	BF (sd)	LBFDR (sd)
				n=500			
β_1	0.000	0.006	0.009	0.000	0.000	0.000(0.000)	1.000(0.000)
β_2	0.000	0.004	0.009	0.002	0.000	0.000(0.000)	1.000(0.000)
β_3	0.000	-0.002	0.008	-0.001	0.000	0.000(0.000)	1.000(0.000)
β_4	0.000	-0.003	0.009	-0.002	0.000	0.000(0.000)	1.000(0.000)
β_5	0.000	-0.001	0.008	0.000	0.000	0.000(0.000)	1.000(0.000)
β_6	0.000	-0.003	0.004	-0.001	0.000	0.000(0.000)	1.000(0.000)
β_7	0.000	-0.004	0.007	-0.002	0.000	0.000(0.000)	1.000(0.000)
β_8	0.000	-0.014	0.009	-0.004	0.001	0.000(0.000)	1.000(0.000)
β_9	0.000	0.005	0.008	0.002	0.000	0.000(0.000)	1.000(0.000)
β_{10}	0.000	0.001	0.007	0.000	0.000	0.000(0.000)	1.000(0.000)
β_{11}	0.000	-0.005	0.008	-0.002	0.000	0.000(0.000)	1.000(0.000)
β_{12}	0.000	-0.016	0.009	-0.003	0.001	0.000(0.000)	1.000(0.000)
β_{13}	0.000	-0.002	0.008	-0.001	0.000	0.000(0.000)	1.000(0.000)
β_{14}	0.000	0.039	0.007	0.001	0.008	0.000(0.000)	1.000(0.000)
β_{15}	0.000	0.000	0.009	0.000	0.000	0.000(0.000)	1.000(0.000)
α_1	0.000	-0.003	0.004	-0.001	0.000	0.000(0.000)	1.000(0.000)
α_2	0.000	0.010	0.008	0.001	0.000	0.000(0.000)	1.000(0.000)
α_3	0.000	0.020	0.008	0.000	0.001	0.000(0.000)	1.000(0.000)
α_4	0.000	0.001	0.009	0.000	0.000	0.000(0.000)	1.000(0.000)
α_5	0.000	-0.012	0.015	-0.002	0.001	0.000(0.000)	1.000(0.000)
α_6	0.000	-0.016	0.016	-0.002	0.001	0.000(0.000)	1.000(0.000)
α_7	0.000	0.001	0.004	0.000	0.000	0.000(0.000)	1.000(0.000)
α_8	0.000	0.002	0.012	0.000	0.000	0.000(0.000)	1.000(0.000)
α_9	0.000	0.000	0.023	0.000	0.000	0.000(0.000)	1.000(0.000)
α_{10}	0.000	0.001	0.026	0.000	0.000	0.000(0.000)	1.000(0.000)
ϕ	2.000	1.796	0.558	1.895	0.021		

Table D.7: Results of the simulation study of DS for generated data under the ZINB random effects model for Scenario 3. Posterior mean, the standard deviation of estimators, the posterior median, the root of the mean squared error (RMSE), relative bias (Rbias), BF, and LBFDR (sd) for each of the parameter estimates for M= 100 simulated data with a sample size of 500. The generated data are analyzed with the ZINB model.

Parameter	True	mean	sd	median	RMSE	Rbias	BF	LBFDR (sd)
				n=500				
β_1	0.5	0.481	0.070	0.482	0.003	-0.037	$> 10^3$	0.000(0.000)
β_2	0.500	0.550	0.070	0.549	0.008	0.100	$> 10^3$	0.000(0.000)
β_3	0.500	0.440	0.065	0.441	0.005	-0.120	$> 10^3$	0.000(0.000)
β_4	0.500	0.505	0.071	0.504	0.003	0.009	$> 10^3$	0.000(0.000)
β_5	0.500	0.421	0.067	0.422	0.008	-0.159	$> 10^3$	0.000(0.000)
β_6	0.500	0.520	0.075	0.524	0.001	0.101	$> 10^3$	0.000(0.000)
β_7	0.500	0.478	0.083	0.483	0.005	-0.111	$> 10^3$	0.000(0.000)
β_8	0.500	0.471	0.058	0.470	0.011	-0.143	$> 10^3$	0.000(0.000)
β_9	0.500	0.477	0.074	0.470	0.009	-0.113	$> 10^3$	0.000(0.000)
β_{10}	0.500	0.567	0.072	0.569	0.006	0.337	$> 10^3$	0.000(0.000)
β_{11}	0.500	0.422	0.065	0.411	0.016	-0.390	$> 10^3$	0.000(0.000)
β_{12}	0.500	0.439	0.055	0.435	0.030	-0.307	$> 10^3$	0.000(0.000)
β_{13}	0.500	0.487	0.072	0.457	0.020	-0.564	$> 10^3$	0.000(0.000)
β_{14}	0.500	0.433	0.055	0.430	0.017	-0.333	$> 10^3$	0.000(0.000)
β_{15}	0.500	0.450	0.074	0.447	0.009	-0.549	$> 10^3$	0.000(0.000)
α_1	0.500	0.412	0.095	0.418	0.010	-0.177	$> 10^3$	0.000(0.000)
α_2	0.500	0.521	0.102	0.519	0.005	0.042	$> 10^3$	0.000(0.000)
α_3	0.500	0.536	0.101	0.537	0.012	0.073	$> 10^3$	0.000(0.000)
α_4	0.500	0.454	0.096	0.453	0.003	-0.092	$> 10^3$	0.000(0.000)
α_5	0.500	0.485	0.095	0.486	0.011	-0.030	$> 10^3$	0.000(0.000)
α_6	0.500	0.471	0.065	0.504	0.027	-0.646	$> 10^3$	0.000(0.000)
α_7	0.500	0.466	0.066	0.475	0.027	-0.670	$> 10^3$	0.000(0.000)
α_8	0.500	0.421	0.044	0.460	0.033	-0.893	$> 10^3$	0.000(0.000)
α_9	0.500	0.421	0.087	0.446	0.017	-0.397	$> 10^3$	0.000(0.000)
α_{10}	0.500	0.419	0.067	0.409	0.022	-0.405	$> 10^3$	0.000(0.000)
ϕ	2.000	1.808	0.133	1.928	0.050	-0.095		

Table D.8: Results of the simulation study of DS for generated data under the ZINB random effects model for Scenario 4. Posterior mean, the standard deviation of estimators, the posterior median, the root of the mean of squared error (RMSE), relative bias (Rbias), BF (sd), and LBFDR (sd) for each of the parameter estimates for M= 100 simulated data with a sample size of 500. The generated data are analyzed with the ZINB model.

Parameter	True	mean	sd	median	RMSE	BF (sd)	LBFDR (sd)
				n=500			
β_1	0.000	0.013	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_2	0.000	0.000	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_3	0.000	0.001	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_4	0.000	-0.001	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_5	0.000	0.000	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_6	0.000	0.000	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_7	0.000	0.001	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_8	0.000	0.009	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_9	0.000	0.000	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_{10}	0.000	0.000	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_{11}	0.000	-0.001	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_{12}	0.000	0.001	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_{13}	0.000	0.000	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_{14}	0.000	0.000	0.000	0.000	0.000	0.000(0.000)	1.000(0.000)
β_{15}	0.000	0.033	0.000	0.000	0.002	0.000(0.000)	1.000(0.000)
α_1	0.000	-0.002	0.020	0.000	0.000	0.000(0.000)	1.000(0.000)
α_2	0.000	0.004	0.030	0.000	0.000	0.000(0.000)	1.000(0.000)
α_3	0.000	0.000	0.016	0.000	0.000	0.000(0.000)	1.000(0.000)
α_4	0.000	-0.006	0.028	0.000	0.000	0.000(0.000)	1.000(0.000)
α_5	0.000	-0.014	0.041	0.000	0.000	0.000(0.000)	1.000(0.000)
α_6	0.000	-0.001	0.014	0.000	0.000	0.000(0.000)	1.000(0.000)
α_7	0.000	0.007	0.030	0.000	0.000	0.000(0.000)	1.000(0.000)
α_8	0.000	-0.003	0.026	0.000	0.000	0.000(0.000)	1.000(0.000)
α_9	0.000	0.006	0.030	0.000	0.000	0.000(0.000)	1.000(0.000)
α_{10}	0.000	-0.001	0.015	0.000	0.000	0.000(0.000)	1.000(0.000)
ϕ	2.000	1.908	0.113	1.933	0.020		

SUPPLEMENTARY MATERIAL E: Results of a simulation study of DS and CS for generated data under ZINB random effects model with $\phi = 0.25$ for Scenarios 1 and 2

Table E.1: Results of the simulation study of DS for generated data under the ZINB random effects model for scenario 1. The posterior mean, the standard deviation of estimators, the posterior median, the root of the mean squared error (RMSE), and relative bias (Rbias) for each of the parameter estimates for M= 100 simulated data with sample sizes of 500 and 1000. The generated data are analyzed with the ZINB model (*: the relative bias cannot be calculated since the real value of the related parameter is zero).

Parameter	True	mean	sd	median	RMSE	Rbias
			n=500			
β_1	1.000	0.960	0.094	0.960	0.0015	-0.040
β_2	1.000	1.110	0.098	1.109	0.012	0.109
β_3	1.000	1.003	0.092	1.003	0.0014	0.003
β_4	1.000	1.016	0.101	1.016	0.0013	0.016
β_5	1.000	0.969	0.093	0.969	0.0015	-0.031
β_6	0.000	-0.028	0.063	0.000	0.000	*
β_7	0.000	0.003	0.029	0.000	0.000	*
β_8	0.000	-0.007	0.041	0.000	0.000	*
β_9	0.000	0.000	0.034	0.000	0.000	*
β_{10}	0.000	0.030	0.061	0.000	0.000	*
β_{11}	0.000	-0.008	0.035	0.000	0.000	*
β_{12}	0.000	0.000	0.015	0.000	0.000	*
β_{13}	0.000	-0.007	0.041	0.000	0.000	*
β_{14}	0.000	-0.002	0.036	0.000	0.000	*
β_{15}	0.000	-0.007	0.048	0.000	0.000	*
α_1	1.000	1.018	0.162	1.012	0.007	0.012
α_2	1.000	0.941	0.165	0.930	0.009	-0.070
α_3	1.000	1.126	0.174	1.113	0.017	0.113
α_4	1.000	0.936	0.160	0.924	0.011	-0.036
α_5	1.000	0.971	0.156	0.964	0.012	-0.036
α_6	0.000	-0.011	0.053	0.000	0.000	*
α_7	0.000	-0.005	0.036	0.000	0.000	*
α_8	0.000	-0.001	0.022	0.000	0.000	*
α_9	0.000	0.001	0.039	0.000	0.000	*
α_{10}	0.000	0.000	0.025	0.000	0.000	*
ϕ	0.25	0.253	0.017	0.255	0.005	0.020
			n=1000			
β_1	1.000	0.951	0.069	0.952	0.002	-0.048
β_2	1.000	1.002	0.067	1.001	0.001	0.001
β_3	1.000	0.958	0.070	0.958	0.002	-0.042
β_4	1.000	0.970	0.069	0.970	0.001	-0.030
β_5	1.000	0.941	0.070	0.942	0.003	-0.058
β_6	0.000	0.001	0.018	0.000	0.000	*
β_7	0.000	-0.003	0.029	0.000	0.000	*
β_8	0.000	0.001	0.012	0.000	0.000	*
β_9	0.000	0.004	0.056	0.000	0.000	*
β_{10}	0.000	0.000	0.014	0.000	0.000	*
β_{11}	0.000	0.001	0.021	0.000	0.000	*
β_{12}	0.000	0.004	0.047	0.000	0.000	*
β_{13}	0.000	0.001	0.019	0.000	0.000	*
β_{14}	0.000	-0.001	0.017	0.000	0.000	*
β_{15}	0.000	-0.001	0.019	0.000	0.000	*
α_1	1.000	0.816	0.104	0.813	0.005	-0.187
α_2	1.000	0.882	0.102	0.880	0.004	-0.120
α_3	1.000	0.923	0.105	0.921	0.006	-0.079
α_4	1.000	0.986	0.104	0.982	0.005	-0.038
α_5	1.000	0.900	0.102	0.899	0.010	-0.101
α_6	0.000	-0.006	0.036	0.000	0.000	*
α_7	0.000	-0.001	0.017	0.000	0.000	*
α_8	0.000	0.001	0.017	0.000	0.000	*
α_9	0.000	-0.004	0.024	0.000	0.000	*
α_{10}	0.000	-0.003	0.030	0.000	0.000	*
ϕ	0.25	0.264	0.026	0.255	0.005	0.020

Table E.2: Mean (SD) of true/false positive rate (TPR/FPR) and Matthews correlation coefficient of BF and LBFDR for ZINB random effects model of Scenario 1 for DS with M = 100 simulated data with sample sizes 500 and 1000.

		ZINB		
n		BF	LBFDR	median
500	TPR	1.000(0.000)	1.000(0.000)	1.000(0.000)
	FPR	0.000(0.000)	0.000(0.000)	0.000(0.000)
	MCC	1.000(0.000)	1.000(0.000)	1.000(0.000)
1000	TPR	1.000(0.000)	1.000(0.000)	1.000(0.000)
	FPR	0.000(0.000)	0.000(0.000)	0.000(0.000)
	MCC	1.000(0.000)	1.000(0.000)	1.000(0.000)

Table E.3: Results of the simulation study of DS for generated data under the ZINB random effects model for scenario 2. The posterior mean, the standard deviation of estimators, the posterior median, the root of the mean squared error (RMSE), and relative bias (Rbias) for each of the parameter estimates for M= 100 simulated data with sample sizes of 500 and 1000. The generated data are analyzed with the ZINB model (*: the relative bias cannot be calculated since the real value of the related parameter is zero).

Parameter	True	mean	sd	median	RMSE	Rbias
			n=500			
β_1	0.500	0.532	0.089	0.532	0.002	0.048
β_2	0.500	0.483	0.089	0.482	0.000	-0.001
β_3	0.500	0.423	0.086	0.423	0.002	-0.042
β_4	0.500	0.412	0.102	0.419	0.001	-0.030
β_5	0.500	0.420	0.086	0.420	0.003	-0.058
β_6	0.000	-0.002	0.033	0.000	0.000	*
β_7	0.000	0.000	0.030	0.000	0.000	*
β_8	0.000	-0.001	0.026	0.000	0.000	*
β_9	0.000	-0.002	0.023	0.000	0.000	*
β_{10}	0.000	0.001	0.010	0.000	0.000	*
β_{11}	0.000	0.005	0.024	0.000	0.000	*
β_{12}	0.000	-0.007	0.031	0.000	0.000	*
β_{13}	0.000	0.002	0.024	0.000	0.000	*
β_{14}	0.000	0.002	0.049	0.000	0.000	*
β_{15}	0.000	-0.004	0.039	0.000	0.000	*
α_1	0.500	0.467	0.118	0.464	0.035	-0.187
α_2	0.500	0.483	0.129	0.482	0.014	-0.120
α_3	0.500	0.632	0.124	0.630	0.006	0.079
α_4	0.500	0.646	0.127	0.642	0.000	0.018
α_5	0.500	0.472	0.120	0.471	0.010	-0.101
α_6	0.000	0.001	0.026	0.000	0.000	*
α_7	0.000	0.003	0.025	0.000	0.000	*
α_8	0.000	0.001	0.030	0.000	0.000	*
α_9	0.000	-0.002	0.026	0.000	0.000	*
α_{10}	0.000	0.000	0.026	0.000	0.000	*
ϕ	0.25	0.264	0.012	0.275	0.025	0.025
			n=1000			
β_1	0.500	0.449	0.067	0.449	0.003	-0.102
β_2	0.5	0.456	0.067	0.456	0.002	-0.088
β_3	0.500	0.445	0.067	0.444	0.003	-0.112
β_4	0.500	0.578	0.066	0.578	0.006	0.056
β_5	0.500	0.430	0.070	0.431	0.005	-0.138
β_6	0.000	-0.001	0.013	0.000	0.000	*
β_7	0.000	0.003	0.018	0.000	0.000	*
β_8	0.000	0.000	0.016	0.000	0.000	*
β_9	0.000	0.001	0.023	0.000	0.000	*
β_{10}	0.000	0.002	0.014	0.000	0.000	*
β_{11}	0.000	0.002	0.013	0.000	0.000	*
β_{12}	0.000	-0.057	0.030	0.000	0.000	*
β_{13}	0.000	0.000	0.021	0.000	0.000	*
β_{14}	0.000	-0.001	0.014	0.000	0.000	*
β_{15}	0.000	-0.001	0.015	0.000	0.000	*
α_1	0.500	0.440	0.094	0.438	0.004	-0.124
α_2	0.500	0.425	0.093	0.423	0.006	-0.154
α_3	0.500	0.515	0.093	0.514	0.000	0.028
α_4	0.500	0.543	0.094	0.543	0.002	0.086
α_5	0.500	0.452	0.094	0.451	0.002	-0.098
α_6	0.000	-0.001	0.012	0.000	0.000	*
α_7	0.000	0.001	0.012	0.000	0.000	*
α_8	0.000	0.000	0.007	0.000	0.000	*
α_9	0.000	-0.001	0.015	0.000	0.000	*
α_{10}	0.000	-0.002	0.017	0.000	0.000	*
ϕ	0.25	0.259	0.013	0.255	0.005	0.020

Table E.4: Mean (SD) of true/false positive rate (TPR/FPR) and Matthews correlation coefficient (MCC) of BF and LBFDR for ZINB random effects model of Scenario 2 for DS with M = 100 simulated data with sample sizes 500 and 1000.

		ZINB		
n		BF	LBFDR	median
	TPR	1.000(0.000)	0.975(0.05)	1.000(0.000)
500	FPR	0.250(0.001)	0.243(0.012)	0.407(0.273)
	MCC	0.612(0.002)	0.599(0.024)	0.472(0.360)
	TPR	1.000(0.000)	1.000(0.000)	1.000(0.000)
1000	FPR	0.262(0.025)	0.262(0.025)	0.022(0.045)
	MCC	0.600(0.024)	0.600(0.024)	0.980(0.039)

Table E.5: Results of the simulation study of CS for generated data under the ZINB random effects model for scenario 1. The posterior mean, the standard deviation of estimators, the posterior median, the root of the mean squared error (RMSE), and relative bias (Rbias) for each of the parameter estimates for M= 100 simulated data with sample sizes of 500 and 1000. The generated data are analyzed with the ZINB model (*: the relative bias cannot be calculated since the real value of the related parameter is zero).

Parameter	True	mean	sd	median	RMSE	Rbias
			n=500			
β_1	1.000	0.996	0.098	0.997	0.003	-0.003
β_2	1.000	1.015	0.100	1.016	0.004	0.016
β_3	1.000	1.001	0.095	1.004	0.005	0.004
β_4	1.000	0.980	0.102	0.980	0.006	-0.020
β_5	1.000	0.976	0.095	0.977	0.004	-0.023
β_6	0.000	-0.003	0.028	-0.001	0.000	*
β_7	0.000	0.008	0.035	0.002	0.000	*
β_8	0.000	0.000	0.016	0.000	0.000	*
β_9	0.000	-0.001	0.026	0.000	0.000	*
β_{10}	0.000	0.000	0.029	0.000	0.000	*
β_{11}	0.000	-0.004	0.027	-0.001	0.000	*
β_{12}	0.000	0.001	0.023	0.000	0.000	*
β_{13}	0.000	-0.006	0.037	0.000	0.000	*
β_{14}	0.000	0.010	0.044	0.001	0.000	*
β_{15}	0.000	-0.004	0.033	-0.001	0.000	*
α_1	1.000	1.014	0.155	1.009	0.002	0.009
α_2	1.000	1.036	0.166	1.030	0.002	0.030
α_3	1.000	1.086	0.158	1.081	0.007	0.081
α_4	1.000	0.919	0.157	0.916	0.007	-0.084
α_5	1.000	1.015	0.162	1.012	0.002	0.012
α_6	0.000	0.025	0.079	0.002	0.000	*
α_7	0.000	0.007	0.034	0.001	0.000	*
α_8	0.000	0.016	0.070	0.002	0.000	*
α_9	0.000	0.011	0.045	0.002	0.000	*
α_{10}	0.000	0.002	0.039	0.000	0.000	*
ϕ	0.25	0.231	0.029	0.215	0.035	-0.004
			n=1000			
β_1	1.000	0.960	0.094	0.998	0.002	-0.040
β_2	1.000	1.111	0.096	1.112	0.003	0.012
β_3	1.000	1.001	0.094	1.001	0.002	0.001
β_4	1.000	1.019	0.098	1.019	0.003	-0.029
β_5	1.000	0.968	0.092	0.968	0.004	-0.032
β_6	0.000	-0.024	0.044	0.000	0.000	*
β_7	0.000	0.005	0.027	0.000	0.000	*
β_8	0.000	-0.008	0.035	0.000	0.000	*
β_9	0.000	0.001	0.032	0.000	0.000	*
β_{10}	0.000	0.024	0.057	0.000	0.000	*
β_{11}	0.000	-0.003	0.044	0.000	0.000	*
β_{12}	0.000	0.000	0.026	0.000	0.000	*
β_{13}	0.000	0.001	0.026	0.000	0.000	*
β_{14}	0.000	-0.005	0.028	0.000	0.000	*
β_{15}	0.000	0.005	0.044	0.000	0.000	*
α_1	1.000	1.024	0.158	1.016	0.001	0.006
α_2	1.000	0.946	0.162	0.938	0.001	-0.062
α_3	1.000	1.143	0.167	1.132	0.01	0.032
α_4	1.000	0.954	0.161	0.947	0.001	-0.093
α_5	1.000	0.981	0.155	0.975	0.001	-0.025
α_6	0.000	-0.008	0.043	0.000	0.000	*
α_7	0.000	-0.005	0.044	0.000	0.000	*
α_8	0.000	0.000	0.029	0.000	0.000	*
α_9	0.000	0.002	0.031	0.000	0.000	*
α_{10}	0.000	0.002	0.032	0.000	0.000	*
ϕ	0.25	0.253	0.015	0.250	0.000	0.001

Table E.6: Mean (SD) of true/false positive rate (TPR/FPR) and Matthews correlation coefficient (MCC) of BF and LBFDR for ZINB random effects model of Scenario 1 for CS with M = 100 simulated data with sample sizes 500 and 1000.

		ZINB		
n		BF	LBFDR	Median
500	TPR	1.000(0.000)	1.000(0.000)	1.000(0.000)
	FPR	0.000(0.000)	0.000(0.000)	0.000(0.000)
	MCC	1.000(0.000)	1.000(0.000)	1.000(0.000)
1000	TPR	1.000(0.000)	1.000(0.000)	1.000(0.000)
	FPR	0.000(0.000)	0.000(0.000)	0.000(0.000)
	MCC	1.000(0.000)	1.000(0.000)	1.000(0.000)

Table E.7: Results of the simulation study of CS for generated data under the ZINB random effects model for scenario 2. The posterior mean, the standard deviation of estimators, the posterior median, the root of the mean squared error (RMSE), and relative bias (Rbias) for each of the parameter estimates for M= 100 simulated data with sample sizes of 500 and 1000. The generated data are analyzed with the ZINB model (*: the relative bias cannot be calculated since the real value of the related parameter is zero).

Parameter	True	mean	sd	median	RMSE	Rbias
			n=500			
β_1	0.500	0.395	0.097	0.396	0.011	-0.080
β_2	0.500	0.542	0.093	0.542	0.002	0.084
β_3	0.500	0.480	0.091	0.480	0.003	-0.040
β_4	0.500	0.555	0.097	0.556	0.003	0.012
β_5	0.500	0.463	0.091	0.462	0.003	-0.076
β_6	0.000	-0.004	0.029	-0.001	0.000	*
β_7	0.000	0.000	0.026	0.000	0.000	*
β_8	0.000	0.003	0.025	0.001	0.000	*
β_9	0.000	-0.004	0.023	-0.001	0.000	*
β_{10}	0.000	0.006	0.031	0.001	0.000	*
β_{11}	0.000	-0.004	0.028	-0.001	0.000	*
β_{12}	0.000	0.003	0.023	0.001	0.000	*
β_{13}	0.000	-0.008	0.038	-0.002	0.000	*
β_{14}	0.000	0.000	0.033	0.000	0.000	*
β_{15}	0.000	-0.002	0.030	0.000	0.000	*
α_1	0.500	0.443	0.125	0.440	0.004	-0.020
α_2	0.500	0.320	0.130	0.290	0.044	-0.020
α_3	0.500	0.559	0.131	0.553	0.003	0.106
α_4	0.500	0.413	0.115	0.408	0.008	-0.184
α_5	0.500	0.409	0.140	0.369	0.017	-0.062
α_6	0.000	-0.007	0.042	-0.001	0.000	*
α_7	0.000	0.011	0.044	0.001	0.000	*
α_8	0.000	-0.013	0.050	-0.002	0.000	*
α_9	0.000	0.004	0.036	0.000	0.000	*
α_{10}	0.000	-0.001	0.034	0.000	0.000	*
ϕ	0.25	0.255	0.043	0.250	0.000	0.002
			n=1000			
β_1	0.500	0.460	0.066	0.459	0.002	-0.082
β_2	0.500	0.547	0.068	0.548	0.002	0.076
β_3	0.500	0.397	0.071	0.397	0.001	-0.056
β_4	0.500	0.496	0.067	0.495	0.002	-0.010
β_5	0.500	0.495	0.069	0.495	0.002	-0.090
β_6	0.000	0.002	0.020	0.001	0.000	*
β_7	0.000	0.001	0.015	0.000	0.000	*
β_8	0.000	0.001	0.017	0.001	0.000	*
β_9	0.000	0.004	0.031	0.001	0.000	*
β_{10}	0.000	-0.013	0.042	-0.002	0.000	*
β_{11}	0.000	0.001	0.024	0.000	0.000	*
β_{12}	0.000	-0.045	0.037	-0.048	0.000	*
β_{13}	0.000	-0.003	0.025	-0.001	0.000	*
β_{14}	0.000	0.002	0.028	0.000	0.000	*
β_{15}	0.000	-0.001	0.020	0.000	0.000	*
α_1	0.500	0.480	0.120	0.466	0.001	-0.068
α_2	0.500	0.554	0.114	0.521	0.000	0.012
α_3	0.500	0.427	0.110	0.424	0.006	-0.152
α_4	0.500	0.439	0.116	0.536	0.001	0.192
α_5	0.500	0.460	0.112	0.458	0.002	-0.084
α_6	0.000	0.004	0.031	0.000	0.000	*
α_7	0.000	-0.005	0.033	-0.001	0.000	*
α_8	0.000	0.004	0.022	0.001	0.000	*
α_9	0.000	-0.009	0.036	-0.001	0.000	*
α_{10}	0.000	0.000	0.019	0.000	0.000	*
ϕ	0.25	0.246	0.026	0.269	0.019	0.076

Table E.8: Mean (SD) of true/false positive rate (TPR/FPR) and Matthews correlation coefficient (MCC) of BF and LBFDR for ZINB random effects model of Scenario 2 for CS with M = 100 simulated data with sample sizes 500 and 1000.

		ZINB	
n		BF	LBFDR
500	TPR	0.903(0.081)	0.904(0.081)
	FPR	0.225(0.020)	0.215(0.020)
	MCC	0.562(0.039)	0.512(0.039)
1000	TPR	1.000(0.000)	1.000(0.000)
	FPR	0.000(0.000)	0.000(0.000)
	MCC	1.000(0.000)	1.000(0.000)

SUPPLEMENTARY MATERIAL F: Results of Application

Table F.1: Parameter estimates (Est.), standard deviation (SD.), 2.5%: lower bound of 95% credible interval, 97.5%: upper bound of 95% credible interval, local Bayesian false discovery rate (LBFDR), and Gelman-Rubin statistics ($\hat{R}$) for analyzing RAND data using the ZIP model.

	Est.	SD.	2.5%	97.5%	LBFDR	$\hat{R}$
	Model of the rate (μ_{ij})					
Intercept (β_0)	0.887	0.038	0.809	0.956	0.000	1.068
time (β_1)	0.000	0.000	0.000	0.000	1.000	1.000
time^2 (β_2)	0.000	0.000	0.000	0.000	0.999	1.000
IDP (β_3)	-0.136	0.031	-0.195	-0.076	0.000	1.007
LPI (β_4)	0.000	0.000	0.000	0.000	0.998	1.015
FMDE (β_5)	-0.125	0.013	-0.150	-0.100	0.000	1.023
LINC (β_6)	0.083	0.016	0.052	0.114	0.000	1.009
FEMALE (β_7)	0.152	0.027	0.100	0.207	0.000	1.017
PHYSLM (β_8)	0.244	0.042	0.164	0.326	0.000	1.012
BLACK (β_9)	-0.377	0.046	-0.468	-0.284	0.000	1.002
EDUCDEC (β_{10})	-0.001	0.013	-0.003	0.000	0.964	1.015
NDISEASE (β_{11})	0.168	0.014	0.141	0.194	0.000	1.002
HLTHF (β_{12})	0.202	0.054	0.100	0.312	0.000	1.004
HLTHG (β_{13})	0.001	0.006	0.000	0.000	0.968	1.098
HLTHP (β_{14})	0.421	0.114	0.167	0.617	0.014	1.037
AGE (β_{15})	0.000	0.000	0.000	0.000	1.000	1.000
CHILD (β_{16})	0.001	0.007	0.000	0.007	0.972	1.182
LFAM (β_{17})	-0.103	0.024	-0.149	-0.057	0.000	1.023
LC (β_{18})	0.000	0.009	0.000	0.000	0.975	1.011
FEMCHILD (β_{19})	-0.001	0.009	-0.014	0.000	0.966	1.139
	Model of the probability (π_{ij})					
Intercept (α_0)	2.779	0.159	2.481	3.096	0.000	1.014
time (α_1)	0.000	0.005	0.000	0.000	0.973	1.014
IDP (α_2)	-0.802	0.138	-1.072	-0.533	0.000	1.003
LPI (α_3)	0.310	0.072	0.170	0.455	0.000	1.005
FMDE (α_4)	-0.536	0.075	-0.686	-0.386	0.000	1.039
LINC (α_5)	0.214	0.060	0.091	0.321	0.016	1.020
FEMALE (α_6)	1.555	0.181	1.209	1.905	0.000	1.061
PHYSLM (α_7)	0.021	0.085	0.000	0.324	0.889	1.020
BLACK (α_8)	-2.803	0.172	-3.158	-2.485	0.000	1.061
EDUCDEC (α_9)	-0.010	0.072	-0.253	0.049	0.891	1.020
NDISEASE (α_{10})	0.379	0.067	0.247	0.512	0.000	1.011
HLTHF (α_{11})	0.007	0.069	-0.096	0.211	0.886	1.003
HLTHG (α_{12})	-0.003	0.037	-0.095	0.018	0.921	1.001
HLTHP (α_{13})	0.226	0.410	-0.015	1.358	0.633	1.156
AGE (α_{14})	0.000	0.000	0.000	0.000	0.992	1.018
CHILD (α_{15})	1.168	0.188	0.806	1.542	0.000	1.039
LFAM (α_{16})	-0.002	0.029	-0.065	0.000	0.938	1.010
LC (α_{17})	-0.122	0.214	-0.682	0.000	0.646	1.053
FEMCHILD (α_{18})	-1.980	0.272	-2.512	-1.447	0.000	1.023
D_{11}	1.043	0.012	0.172	2.807	-	1.110
$D_{12}(D_{21})$	0.302	0.012	0.251	0.990	-	1.101
D_{22}	1.421	0.097	0.261	2.907	-	1.102
DIC	183523.7					

Table F.2: Parameter estimates (Est.), standard deviation (SD.), 2.5%: lower bound of 95% credible interval, 97.5%: upper bound of 95% credible interval, local Bayesian false discovery rate (LBFDR), and Gelman-Rubin statistics ($\hat{R}$) for analyzing RAND data using the NB model.

	Est.	SD.	2.5%	97.5%	LBFDR	$\hat{R}$
Intercept (β_0)	1.022	0.029	0.964	1.080	0.000	1.001
time (β_1)	0.000	0.001	0.000	0.000	0.989	1.091
time^2 (β_2)	0.000	0.000	0.000	0.000	0.998	1.000
IDP (β_3)	-0.204	0.022	-0.247	-0.162	0.000	1.001
LPI (β_4)	0.056	0.011	0.035	0.078	0.000	1.003
FMDE (β_5)	-0.167	0.011	-0.188	-0.147	0.000	1.002
LINC (β_6)	0.120	0.010	0.100	0.141	0.000	1.001
FEMALE (β_7)	0.357	0.024	0.310	0.404	0.000	1.001
PHYSLM (β_8)	0.265	0.030	0.208	0.323	0.000	1.001
BLACK (β_9)	-0.725	0.027	-0.779	-0.671	0.000	1.001
EDUCDEC (β_{10})	0.001	0.007	0.000	0.000	0.979	1.086
NDISEASE (β_{11})	0.185	0.010	0.166	0.204	0.000	1.001
HLTHF (β_{12})	0.217	0.036	0.149	0.287	0.000	1.001
HLTHG (β_{13})	0.000	0.003	0.000	0.000	0.983	1.090
HLTHP (β_{14})	0.376	0.073	0.235	0.520	0.000	1.001
AGE (β_{15})	0.000	0.000	0.000	0.000	1.000	1.000
CHILD (β_{16})	0.216	0.028	0.161	0.269	0.000	1.001
LFAM (β_{17})	-0.133	0.020	-0.172	-0.096	0.000	1.003
LC (β_{18})	-0.020	0.041	-0.134	0.000	0.760	1.001
FEMCHILD (β_{19})	-0.374	0.038	-0.446	-0.300	0.000	1.002
σ_b^2	0.143	0.012	0.121	1.807	-	1.100
ϕ	0.837	0.013	0.812	0.862	-	1.001
DIC	283127.5					

Table F.3: Parameter estimates (Est.), standard deviation (SD.), 2.5%: lower bound of 95% credible interval, 97.5%: upper bound of 95% credible interval, local Bayesian false discovery rate (LBFDR), and Gelman-Rubin statistics ($\hat{R}$) for analyzing RAND data using the Poisson model.

	Est.	SD.	2.5%	97.5%	LBFDR	$\hat{R}$
Intercept (β_0)	1.255	0.098	1.130	1.382	0.000	1.026
time (β_1)	-0.001	0.004	-0.011	0.000	0.928	1.015
time^2 (β_2)	0.000	0.000	0.000	0.000	0.983	1.007
IDP (β_3)	-0.211	0.011	-0.230	-0.191	0.000	1.004
LPI (β_4)	0.050	0.006	0.040	0.060	0.006	1.006
FMDE (β_5)	-0.134	0.006	-0.144	-0.125	0.000	1.002
LINC (β_6)	0.115	0.014	0.100	0.133	0.010	1.157
FEMALE (β_7)	0.180	0.159	0.000	0.355	0.422	1.022
PHYSLM (β_8)	0.323	0.022	0.292	0.350	0.003	1.142
BLACK (β_9)	-0.643	0.021	-0.680	-0.609	0.000	1.024
EDUCDEC (β_{10})	-0.043	0.031	-0.102	0.000	0.172	1.012
NDISEASE (β_{11})	0.186	0.014	0.167	0.207	0.000	1.016
HLTHF (β_{12})	0.006	0.032	0.000	0.139	0.964	1.008
HLTHG (β_{13})	-0.027	0.017	-0.056	0.000	0.158	1.003
HLTHP (β_{14})	0.336	0.031	0.280	0.401	0.000	1.005
AGE (β_{15})	0.000	0.000	0.000	0.000	0.975	1.004
CHILD (β_{16})	0.122	0.139	-0.005	0.300	0.512	1.006
LFAM (β_{17})	-0.134	0.018	-0.159	-0.093	0.002	1.024
LC (β_{18})	-0.134	0.026	-0.172	-0.093	0.019	1.004
FEMCHILD (β_{19})	-0.201	0.183	-0.424	0.000	0.236	1.013
σ_b^2	1.013	0.012	0.276	2.101	-	1.110
DIC	311804.8					

Table F.4: Variable definitions and summary statistics for the RAND Health Insurance Experiment data set.

Variables	Definition	Mean	Sd
MD	Yearly number of outpatient visits to physicians	2.86	4.50
Year	Study year	2.42	1.21
IDP	Indicator for individual deductible plan	0.26	0.43
LPI	ln(max(1,annual participation incentive payment))	4.71	2.69
FMDE	log(max(medical deductible expenditure))	4.03	3.47
LINC	ln(annual family income) in US dollars	8.71	1.22
FEMALE	Indicator for female	0.52	0.49
PHYSLIM	Indicator for physical limitations	0.12	0.32
BLACK	1 if the race of the household head is black	0.18	0.38
EDUCDEC	Education of head of household in years	11.97	2.80
NDISEASE	Index of chronic diseases	11.24	6.74
HLTHF	1 if self-rated health is fair	0.08	0.26
HLTHG	1 if self-rated health is good	0.36	0.48
HLTHP	1 if self-rated health is poor	0.02	0.12
AGE	Age in years	25.72	16.76
CHILD	Indicator for age less than 18	0.40	0.49
LFAM	ln(family size)	1.25	0.53
LC	ln(coinsurance+1), $0 \leq$ coinsurance rate ≤ 100	2.38	2.04
FEMCHILD	Interaction between Female and child (FEMALE×CHILD)	0.19	0.39

SUPPLEMENTARY MATERIAL G: The trace, autocorrelation, and density plots of the Markov chain samples for ZINB random effects model

In this paper, the convergence of the chains is checked using Brooks-Gelman-Rubin (BGR) diagnostics. Also, in this supplementary material, the trace, autocorrelation, and density plots for one of the replications of the simulation study are presented. For this purpose, the second scenario of the simulation studies in Section 4.2 is considered. Figures G.1. and G.2. shows the plots of nonzero fixed-effect regression coefficients in the rate model (μ_{ij}) and the probability model (π_{ij}), respectively. All the plots visually confirmed the assessment of the convergence of MCMC output for parameters.

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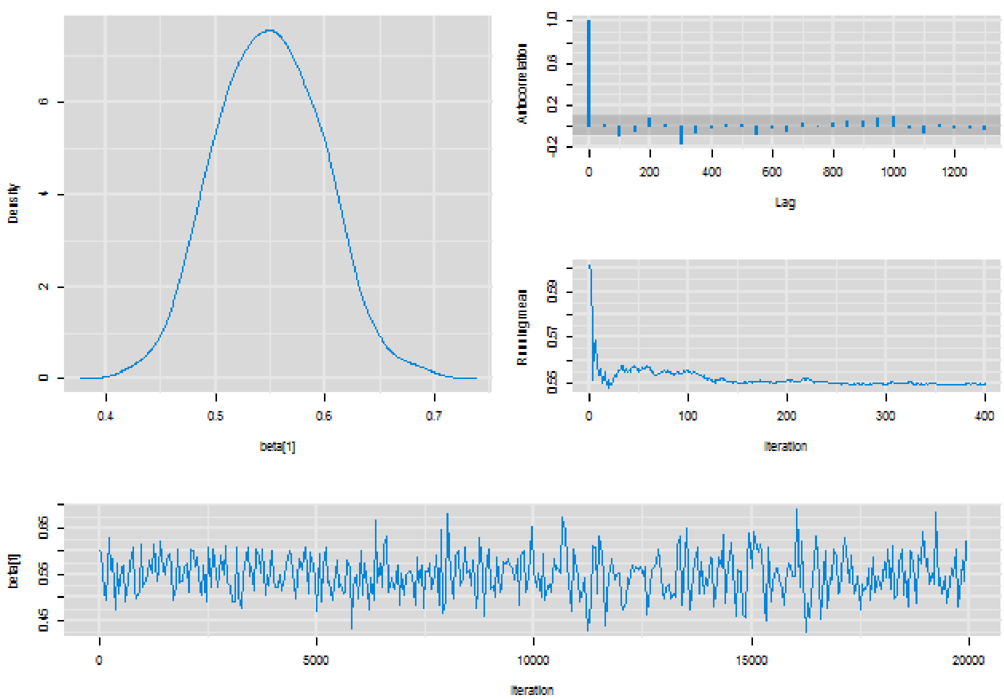

Figure G.1: Diagnostic plots for β_1

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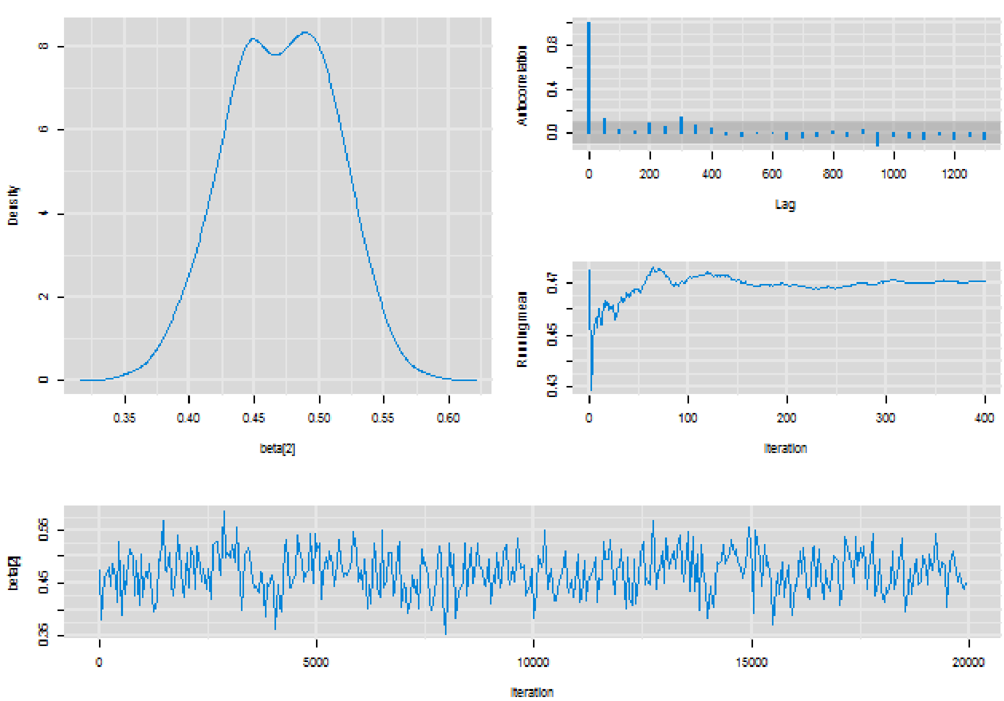

Figure G.2: Diagnostic plots for β_2

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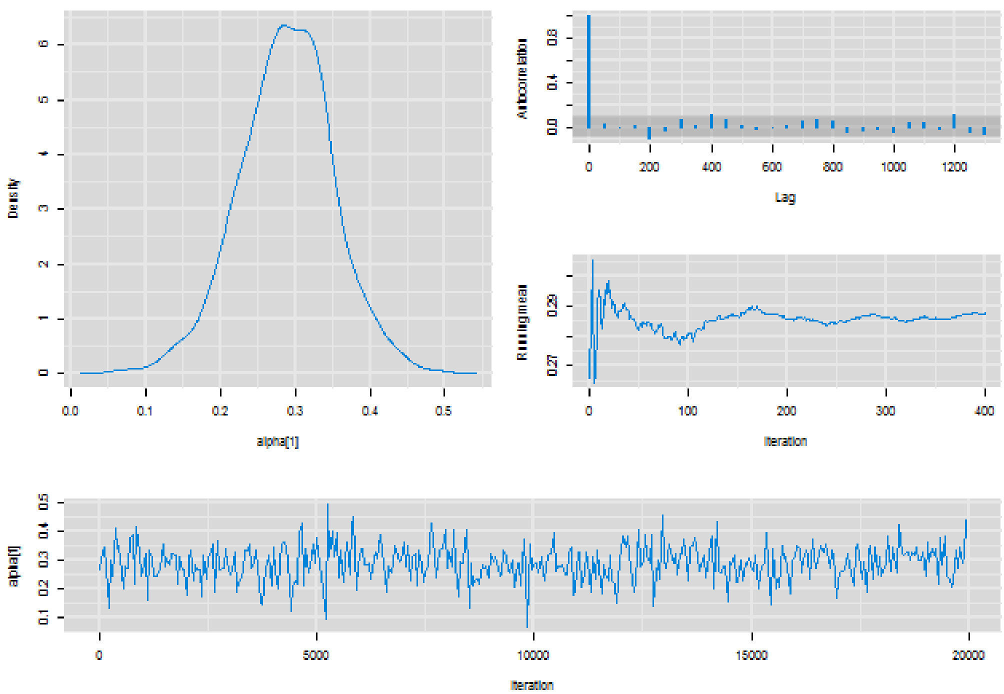

Figure G.7: Diagnostic plots for α_1

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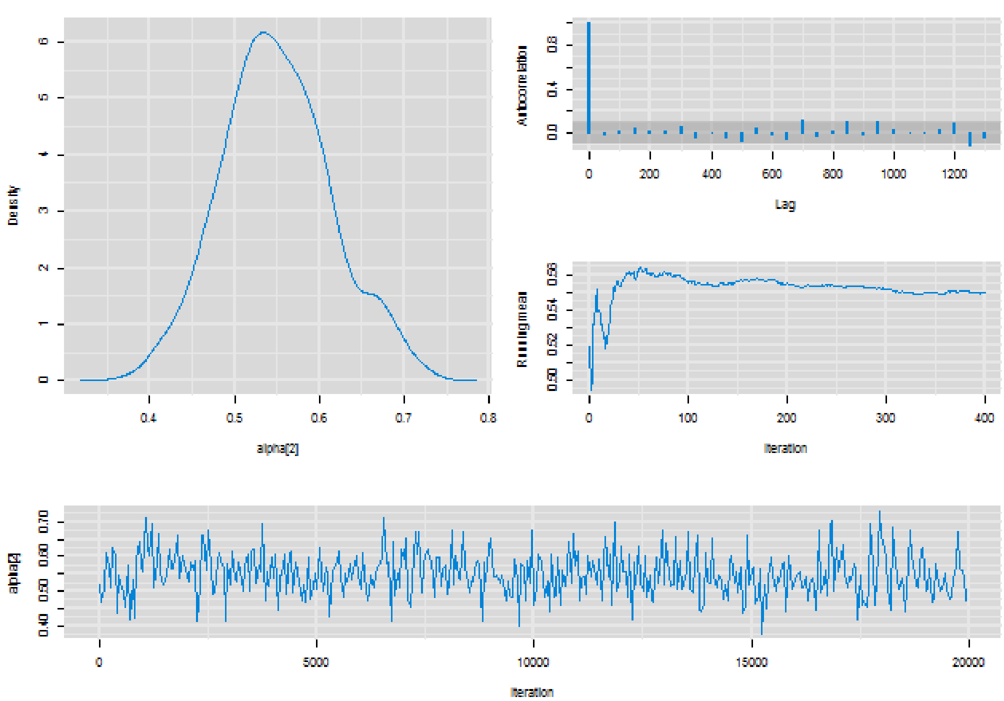

Figure G.8: Diagnostic plots for α_2

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