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ECONOMIC AND ECONOMIC-STATISTICAL DESIGNS OF MULTIVARIATE COEFFICIENT OF VARIATION CHART

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Abstract:

- From the economic perspective, cost minimization is an important part of Statistical Process Control (SPC). The conventional approach in SPC focuses on monitoring the process mean and variance for possible shifts. In some processes, such as clinical and financial investments, the process mean and variance are not independent of one another. Thus, a separate monitoring of the mean and variance using two different control charts is not meaningful. Therefore, the coefficient of variation chart that measures the ratio of the process variance to the mean needs to be employed. In multivariate SPC, the quality characteristics that jointly control the process quality are correlated. Thus, the multivariate coefficient of variation (MCV) chart is used in process monitoring to monitor the process MCV. This work studies the economic and economic-statistical designs of the MCV chart. Optimal parameters that minimize the cost function of the MCV chart are computed. Furthermore, it is shown that adding statistical constraints to the economic design of the MCV chart improves the chart's statistical performance with only a minimal increase in cost.

Keywords:

- *multivariate coefficient of variation (MCV); economic design; economic-statistical design; cost model.*

AMS Subject Classification:

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1. INTRODUCTION

The coefficient of variation (CV) chart is commonly used in SPC for processes which require the reproducibility of measuring tools or methods [3, 20]. Operators usually demand a lower CV profile for better equipment and/or method precision while maintaining the accuracy of the process with an in-control state [8, 17]. Examples of the use of CV are laboratory assay techniques in medicine and biology [19, 36], monitoring the associated stand-alone risk in actuarial finance [24], factory processes in mechanical industries [4], to name a few.

Kang *et al.* [9] proposed the first Shewhart-type univariate CV chart. Since then, the univariate CV charts continue to receive attention among researchers (see [4] and [28], to name a few) but not the multivariate CV (MCV) chart. Yeong *et al.* [32] was the first to propose a control chart for the MCV. More recent studies on MCV charts include studies by Giner-Bosch *et al.* [6] on the EWMA MCV chart and Nguyen *et al.* [16] on one-sided synthetic MCV charts. Some crucial applications of MCV in laboratories and industries are in the correlation of phenotypic variation [25], affymetrix gene expression [7], comparison of serum protein electrophoresis techniques [35], multivariate gage repeatability and reproducibility studies [18, 27], and several others.

The advancement in hardware technologies enabled more automation techniques to be easily applied in various aspects of living. Newly developed equipment and methods can produce large pool of useful data and results with high efficiency. The generalization of CV to the multivariate setting is required to accommodate the part-to-part variability measurements and the correlations of higher dimensional variables. However, the definition of MCV is not as straight forward as that of the univariate CV, i.e. lacking in the generality. Currently, the available definitions of MCV were those by Reyment [21], Van Valen [29], Voinov and Nikulin [30], and Albert and Zhang [2]. Similar to existing MCV type control charts (see for example, Yeong *et al.* [32], Abbasi and Adegoke [1], Khaw *et al.* [11] and Khatun *et al.* [10]), this work adopts the Voinov and Nikulin's [30] definition of MCV.

A pure statistical design of a control chart may not be cost effective in industrial practices. An optimal economic design of a control chart will enhance the competency of the chart from the cost perspective [26]. The idea of an economic model was first presented by Duncan [5], and later improved by Lorenzen and Vance [13]. Saniga [23] expanded the model by incorporating statistical constraints into the cost function, resulting in an economic-statistical model. The unified cost model by Lorenzen and Vance [13] is widely accepted and used in many types of control charts. Some published works which are closely related to this study include Linderman and Love [12] and Molnau *et al.* [14] on economic and economic-statistical designs of multivariate EWMA control chart.

Despite being over three decades old, the Lorenzen and Vance's [13] model is one of the most inclusive cost models in the literature, where it considers all possible sources of cost assumptions, phases of a process and evaluations of expenses. As the Lorenzen and Vance's [13] model is easy to be implemented, it continues to be adopted by researchers until now. Some of the recent works that adopted the Lorenzen and Vance's [13] model are Safe *et al.* [22] and Wan and Zhu [31] who used the model on variable sampling interval type control charts; and Ng *et al.* [15] who employed the model on auxiliary information based $\bar{X}$, synthetic and EWMA charts. Note that the numerical example presented in Lorenzen and Vance [13] and adopted by the above-mentioned researchers, to name a few, is based on a real casting operation process from the General Motors Company.

This study proposes the economic and economic-statistical designs of MCV chart as they are currently not available in the literature. In each of the designs, optimal parameters will be computed to minimize the cost. A comparison between purely economic design and economic-statistical design will also be presented.

This paper is organized in the following order: The properties of MCV and the MCV chart will be explained in Section 2. Following that is a brief review on Lorenzen and Vance [13] cost model in Section 3. Subsequently, a set of numerical examples along with comparisons of different parameter settings and designs are given in Section 4. A sum up of the paper with some general remarks and findings are given in Section 5.

2. PROPERTIES OF MCV AND MCV CHART

Section 2.1 discusses the cumulative distribution function (cdf) and inverse cdf of the sample MCV derived by Yeong *et al.* [32] while the MCV chart is discussed in Section 2.2.

2.1. Distribution of the sample MCV

Suppose that a random vector, $\mathbf{X_i}$, in a sample of size n with mean vector, $\boldsymbol{\mu}$ and covariance matrix, $\boldsymbol{\Sigma}$ follows a p-variate normal distribution, i.e. $\mathbf{X}_i \sim N_p(\boldsymbol{\mu}, \boldsymbol{\Sigma})$, where $\mathbf{X}_i^\mathsf{T} = (X_{i1}, X_{i2}, ..., X_{ip})$, for $1 \le i \le n$. A general definition of the population MCV by Voinov and Nikulin [30] is

$$\gamma = \left(\boldsymbol{\mu}^\mathsf{T}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}\right)^{-\frac{1}{2}}. \tag{2.1}$$

Yeong *et al.* [32] derived an estimator of the process MCV, $\hat{\gamma}$ based on Equation (2.1), where $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ are estimated using the sample mean vector, $\bar{\mathbf{X}}$ and the sample covariance matrix, $\mathbf{S}$, respectively. Here,

$$\bar{\mathbf{X}}^\mathsf{T} = \left(\frac{1}{n}\sum_{i=1}^{n} X_{i1}, \frac{1}{n}\sum_{i=1}^{n} X_{i2}, ..., \frac{1}{n}\sum_{i=1}^{n} X_{ip}\right), \tag{2.2}$$

and

$$\mathbf{S} = \frac{1}{n-1}\sum_{i=1}^{n}(\mathbf{X}_i - \bar{\mathbf{X}})(\mathbf{X}_i - \bar{\mathbf{X}})^\mathsf{T}. \tag{2.3}$$

Then, $\hat{\gamma}$ takes the form

$$\hat{\gamma} = (\bar{\mathbf{X}}^\mathsf{T}\mathbf{S}^{-1}\bar{\mathbf{X}})^{-\frac{1}{2}}. \tag{2.4}$$

The cdf of $\hat{\gamma}$ was derived by Yeong *et al.* [32] to be

$$F_{\hat{\gamma}}(x|n,p,\delta) = 1 - F_F\left(\frac{n(n-p)}{(n-1)px^2}|p, n-p, \delta\right), \tag{2.5}$$

where $F_F(\cdot|p, n-p, \delta)$ is the non-central F distribution with p and $n-p$ degrees of freedom and non-centrality parameter $\delta = n\boldsymbol{\mu}^{\mathsf{T}}\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}$ (which can be written as $\delta = \frac{n}{\gamma^2}$). Yeong *et al.* [32] also derived the inverse cdf of $\hat{\gamma}$ (or the α quantile of $F_{\hat{\gamma}}$) as follows:

$$F_{\hat{\gamma}}^{-1}(\alpha|n,p,\delta) = \sqrt{\frac{n(n-p)}{(n-1)p}\left[\frac{1}{F_F^{-1}(1-\alpha|p,n-p,\delta)}\right]}. \tag{2.6}$$

Note that $F_F^{-1}(\cdot|p, n-p, \delta)$ is the inverse cdf of the non-central F distribution with p and $n-p$ degrees of freedom and non-centrality parameter δ.

2.2. MCV chart

The MCV chart is a Shewhart type chart where the statistic plotted on the chart is the sample MCV, $\hat{\gamma}$. To justify the use of the MCV chart, a check for the constant MCV assumption needs to be conducted. This check is conducted by plotting the rational group MCV, $\hat{\gamma}_t^2$ versus $\bar{\mathbf{X}}_t^{\mathsf{T}}\bar{\mathbf{X}}_t$, followed by a formal test of the regression slope [32].

Yeong *et al.* [32] suggested estimating the in-control sample MCV, $\hat{\gamma}_0$ using the root mean square method as this method has high statistical efficiency and the estimate can be easily computed. Consequently, $\hat{\gamma}_0$ is computed as

$$\hat{\gamma}_0 = \sqrt{\frac{1}{m}\sum_{t=1}^{m}\hat{\gamma}_t^2}\,, \tag{2.7}$$

where m is the number of Phase-I sample MCVs. As the distribution of $\hat{\gamma}$ is not symmetric, the use of two-sided limits will result in an average run length (ARL) biased chart. Therefore, Yeong *et al.* [32] suggested adopting two separate one-sided (an upward and a downward) charts to overcome this drawback. Using two separate one-sided charts allow the upper and lower limits of the respective charts to be determined independently based on the desired in-control ARL value.

For the downward MCV chart in detecting decreasing shifts in the process MCV, its lower control limit (LCL) is computed as

$$\mathrm{LCL} = F_{\hat{\gamma}}^{-1}(\alpha|n,p,\delta_0), \tag{2.8}$$

where α is the Type-I error probability and $\delta_0 = \frac{n}{\gamma_0^2}$ with γ_0 representing the in-control process MCV. The statistical performance of MCV chart can be measured using the ARL criterion. The corresponding value of the in-control average run length (ARL_0) computed using the LCL in Equation (2.8) is

$$\mathrm{ARL}_0 = \frac{1}{\alpha}. \tag{2.9}$$

In like manner, for the upward MCV chart in detecting increasing shifts in the process MCV, its upper control limit (UCL) is obtained as

$$\text{UCL} = F_{\hat{\gamma}}^{-1}(1-\alpha|n,p,\delta_0) \tag{2.10}$$

which gives the ARL_0 value in Equation (2.9). The process MCV is considered as out-of-control when $\hat{\gamma} < \text{LCL}$ (for the downward chart) or $\hat{\gamma} > \text{UCL}$ (for the upward chart).

The out-of-control process MCV is represented by $\gamma_1 = \tau\gamma_0$. Here, τ is the shift size in the process MCV, where $\tau < 1$ ($\gamma_1 < \gamma_0$) indicates process improvement, while $\tau > 1$ ($\gamma_1 > \gamma_0$) implies process deterioration. The probability of detecting a shift by the downward and upward MCV charts are

$$P = \Pr(\hat{\gamma} < \text{LCL}) = F_{\hat{\gamma}}(\text{LCL}|n,p,\delta_1) \tag{2.11}$$

and

$$P = \Pr(\hat{\gamma} > \text{UCL}) = 1 - F_{\hat{\gamma}}(\text{UCL}|n,p,\delta_1), \tag{2.12}$$

respectively, where $\delta_1 = \dfrac{n}{\gamma_1^2}$. The out-of-control average run length (ARL_1) is computed as

$$\text{ARL}_1 = \frac{1}{P}. \tag{2.13}$$

3. LORENZEN AND VANCE COST MODEL

The unified cost model proposed by Lorenzen and Vance [13] is adopted for the economic and economic-statistical designs of the MCV chart. The functional form of this model only requires the computation of ARL, sample size and control limit of the chart at hand. Thus, Lorenzen and Vance [13] cost model can be used on any type of control chart, regardless of the quality characteristics. Table 1 provides the list of notations for this cost model.

The total cost per hour as defined by this model includes the costs during the in-control and out-of-control states, cost of false alarms, cost of repair and cost of sampling. In Lorenzen and Vance [13] cost model, the assignable cause is assumed to occur randomly once in every λ hours. Another assumption is that the shift in the process MCV is due to only a single assignable cause. Lorenzen and Vance [13] cost function is defined as

$$C = \frac{\dfrac{C_0}{\lambda} + C_1 B + \dfrac{b+cn}{h}\left(\dfrac{1}{\lambda} + B\right) + \dfrac{sY}{\text{ARL}_0} + W}{\dfrac{1}{\lambda} + \dfrac{(1-\varphi_1)sT_0}{\text{ARL}_0} + EH}, \tag{3.1}$$

where

$$B = (\text{ARL}_1 - 0.5)h + F,$$
$$F = ne + \varphi_1 T_1 + \varphi_2 T_2,$$
$$EH = (\text{ARL}_1 - 0.5)h + G,$$
$$G = ne + T_1 + T_2,$$

and

$$s = \frac{1}{\lambda h} - \frac{1}{2}.$$

Table 1: List of notations for Lorenzen and Vance (1986) cost model.

b	Fixed cost per sample
c	Variable cost per unit sampled
C	Cost per hour
C_0	Quality cost per hour while in-control
C_1	Quality cost per hour while out-of-control
e	Time to sample and interpret one unit
h	Sampling interval
n	Sample size
s	Expected number of samples taken while in-control
T_0	Expected search time during false alarm
T_1	Expected time to find the assignable cause
T_2	Expected time to repair the process
W	Cost to locate and remove the assignable cause
Y	Cost of false alarms
φ_1	$= 1$ if process continues during search $= 0$ if process stops during search
φ_2	$= 1$ if process continues during repair $= 0$ if process stops during repair
λ	Rate of occurrence of assignable cause

The objective of the economic design of the MCV chart is to obtain the optimal parameters n, h and α in minimizing the cost function, C in Equation (3.1), for specified values of p, τ and γ_0. Note that the parameters p, τ and γ_0 are not included in the optimization procedure because they are intrinsic properties of the process.

With the same objective, the economic-statistical design adds additional constraints on ARL_0 and ARL_1 while minimizing the cost function, C in Equation (3.1). Here, ARL_0 must be greater than a lower bound value while ARL_1 must be less than an upper bound value. The aim of these constraints is to ensure that the MCV chart gives acceptably high ARL_0 value when the process is in-control and low ARL_1 value when the process is out-of-control. In this research, the constraints $\text{ARL}_0 \geq 250$ and $\text{ARL}_1 \leq 20$, i.e. similar to those used by Yeong *et al.* [34] are adopted.

The optimal sampling interval, h can be computed as follows [33]:

$$h = \frac{-r_2 + \sqrt{r_2^2 - 4r_1r_3}}{2r_1}, \tag{3.2}$$

where

$$\begin{aligned} r_1 \;=\; & \frac{\text{ARL}_1 - 0.5}{2\lambda\text{ARL}_0} \Big\{ \lambda\big(Y + C_1T_0(-1+\varphi_1)\big) \\ & - 2\,\text{ARL}_0\Big[C_0 + \lambda\big((\text{ARL}_1 - 0.5)b + (\text{ARL}_1 - 0.5)cn + W\big) + C_1(-1 + F\lambda - G\lambda)\Big]\Big\}, \\ r_2 \;=\; & -\frac{2(\text{ARL}_1 - 0.5)\Big[Y + C_1T_0(-1+\varphi_1) + \text{ARL}_0(b+cn)(1+F\lambda)\Big]}{\lambda\text{ARL}_0}, \end{aligned}$$

and

$$\begin{aligned} r_3 = &-\frac{1}{2\lambda^2\mathrm{ARL}_0}\Big\{2Y + 2C_0T_0(-1+\varphi_1) - bT_0\lambda - 2(\mathrm{ARL}_1-0.5)bT_0\lambda - 2C_1FT_0\lambda \\ &- cnT_0\lambda - 2(\mathrm{ARL}_1-0.5)cnT_0\lambda - 2T_0W\lambda + 2GY\lambda + bT_0\varphi_1\lambda \\ &+ 2(\mathrm{ARL}_1-0.5)bT_0\varphi_1\lambda + 2C_1FT_0\varphi_1\lambda + cnT_0\varphi_1\lambda \\ &+ 2(\mathrm{ARL}_1-0.5)cnT_0\varphi_1\lambda + 2T_0W\varphi_1\lambda - bFT_0\lambda^2 - cFnT_0\lambda^2 \\ &+ bFT_0\varphi_1\lambda^2 + cFnT_0\varphi_1\lambda^2 + 2\mathrm{ARL}_0(b+cn)(1+F\lambda)(1+G\lambda)\Big\}. \end{aligned}$$

From Equations (3.1) and (3.2), it is clear that both ARL_0 and ARL_1 need to be computed first before the computation of C and h can be made. The formulae for computing ARL_0 and ARL_1 are dependent on n, α, p, τ and γ_0. As the exact values of p, τ, γ_0 and the desired values of the thirteen input parameters in Table 2, i.e. λ, C_0, C_1, Y, W, b, c, e, T_0, T_1, T_2, φ_1 and φ_2 are specified, the parameters that control the cost minimization iteration in this case are n and α. The desired values of these thirteen input parameters are adopted from Lorenzen and Vance [13], where they are taken as the control case (Case 1) in Table 2.

Table 2: Input parameters for the cost function, C and the variations of each input parameters, labelled with case numbering.

Case	Changes	λ	C_0	C_1	Y	W	b	c	e	T_0	T_1	T_2	φ_1	φ_2
1	Control	0.02	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
2	$\lambda 2$	**0.01**	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
3	$\lambda 3$	**0.04**	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
4	$C_0 2$	0.02	**57.12**	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
5	$C_0 3$	0.02	**228.48**	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
6	$C_1 2$	0.02	114.24	**474.6**	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
7	$C_1 3$	0.02	114.24	**1898.4**	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
8	$Y2$	0.02	114.24	949.2	**488.7**	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
9	$Y3$	0.02	114.24	949.2	**1954.8**	977.4	0	4.22	0.083	0.083	0.083	0.75	1	0
10	$W2$	0.02	114.24	949.2	977.4	**488.7**	0	4.22	0.083	0.083	0.083	0.75	1	0
11	$W3$	0.02	114.24	949.2	977.4	**1954.8**	0	4.22	0.083	0.083	0.083	0.75	1	0
12	$b2$	0.02	114.24	949.2	977.4	977.4	**5**	4.22	0.083	0.083	0.083	0.75	1	0
13	$b3$	0.02	114.24	949.2	977.4	977.4	**10**	4.22	0.083	0.083	0.083	0.75	1	0
14	$c2$	0.02	114.24	949.2	977.4	977.4	0	**2.11**	0.083	0.083	0.083	0.75	1	0
15	$c3$	0.02	114.24	949.2	977.4	977.4	0	**8.44**	0.083	0.083	0.083	0.75	1	0
16	$e2$	0.02	114.24	949.2	977.4	977.4	0	4.22	**0.042**	0.083	0.083	0.75	1	0
17	$e3$	0.02	114.24	949.2	977.4	977.4	0	4.22	**0.166**	0.083	0.083	0.75	1	0
18	$T_0 2$	0.02	114.24	949.2	977.4	977.4	0	4.22	0.083	**0.042**	0.083	0.75	1	0
19	$T_0 3$	0.02	114.24	949.2	977.4	977.4	0	4.22	0.083	**0.166**	0.083	0.75	1	0
20	$T_1 2$	0.02	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	**0.042**	0.75	1	0
21	$T_1 3$	0.02	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	**0.166**	0.75	1	0
22	$T_2 2$	0.02	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	**0.375**	1	0
23	$T_2 3$	0.02	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	**1.5**	1	0
24	$\varphi_1\varphi_2 2$	0.02	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	**0**	**0**
25	$\varphi_1\varphi_2 3$	0.02	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	**0**	**1**
26	$\varphi_1\varphi_2 4$	0.02	114.24	949.2	977.4	977.4	0	4.22	0.083	0.083	0.083	0.75	**1**	**1**

The computations of the control values of these thirteen input parameters will be explained in detail in Section 4.

In order to impose changes to each of the thirteen input parameters of the control case (Case 1) in Table 2, each of these input parameters (except b, φ_1 and φ_2) is either increased (i.e. doubled) or decreased (i.e. halved). For example, $\lambda 2$ $(= 0.01)$ (Case 2) is half of its control value $(\lambda = 0.02)$ in Case 1, while $\lambda 3$ $(= 0.04)$ (Case 3) is twice of its control value in Case 1. The notations $\lambda 2$ and $\lambda 3$ are used to represent the second and third variations of the control value of λ, as not every input parameter (such as b, φ_1 and φ_2) is doubled or halved. For instance, the fixed cost per sample, b is set at \$0 for the control case (Case 1), while $b2$ involves a raise to \$5 (Case 12) and $b3$ to \$10 (Case 13).

In this research, the sample sizes, $n \in \{2, 3, ..., 30\}$ are considered. The upper limit of n $(= 30)$ is chosen because from a practical perspective, $n = 30$ is considered as a large sample size. In addition, the Type-I error probabilities $\alpha \in \{0.0010, 0.0011, ..., 0.05\}$ are adopted for the economic design, while $\alpha \in \{0.0010, 0.0011, ..., 0.004\}$ are adopted for the economic-statistical design. Note that the Type-I error rate for the economic-statistical design is kept at a maximum of $\alpha = 0.004$, in order to correspond to the constraint $\text{ARL}_0 \geq 250$ specified earlier. An optimization program is written in the MATLAB software to compute the optimal parameters n, α and h that minimize the cost function, C in Equation (3.1), based on the specified values of p, τ, γ_0 and thirteen input parameters in Table 2.

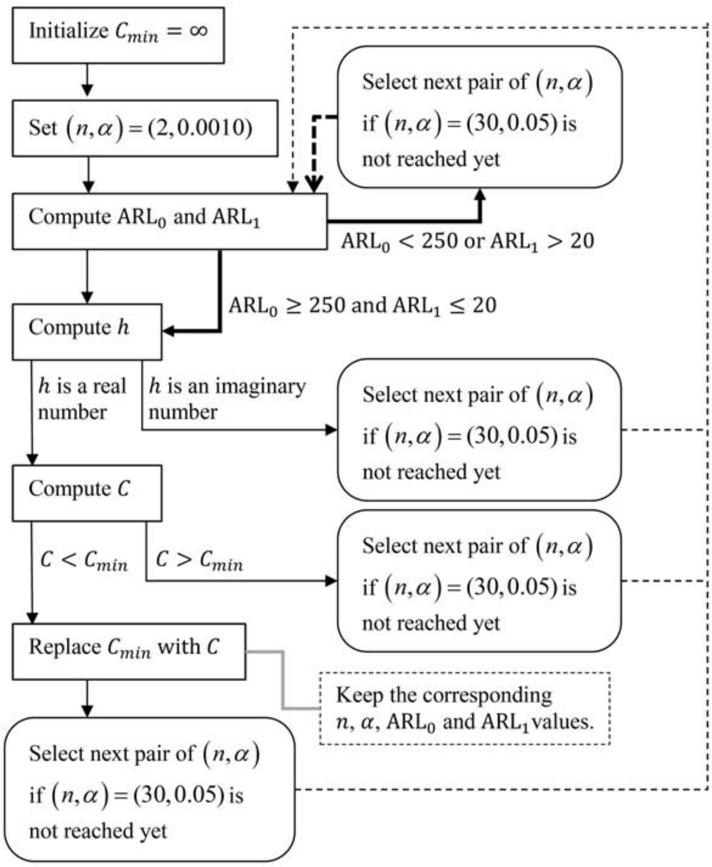

Figure 1: A flowchart explaining the minimization of the cost function, C in Equation (3.1), where thick arrows indicate additional steps for the economic-statistical design model.

The program starts with an assumingly large value of the cost per hour, C, which will be replaced by a new value of C each time a smaller one is obtained. For the controlled parameters, the first pair $(n, \alpha) = (2, 0.0010)$ is iteratively increased as $(2, 0.0011)$, $(2, 0.0012)$, ..., $(2, 0.05)$, $(3, 0.0010)$, $(3, 0.0011)$, ..., until it reaches (30, 0.05) for the economic design. However, for the economic-statistical design, the pair (n, α) is iteratively increased as $(2, 0.0010)$, $(2, 0.0011)$, ..., $(2, 0.004)$, $(3, 0.0010)$, $(3, 0.0011)$, ..., $(3, 0.004)$, ..., $(30, 0.0010)$, $(30, 0.0011)$, ..., $(30, 0.004)$. After the completion of all the iterations, the lowest cost per hour, C $(= C_{\min})$ is recorded, together with the corresponding optimal parameters n, α and h that produce the cost $C_{\min}$. The ARL_0 and ARL_1 values associated with these optimal parameter values are also recorded. Figure 1 shows a flowchart in minimizing C. In this flowchart, the statistical constraints imposed on the economic-statistical design of the MCV chart are shown as additional steps with thicker arrows.

4. NUMERICAL EXAMPLES

The thirteen input parameters and their values given in Lorenzen and Vance [13] for a real case problem of a casting operation process producing 84 castings per hour will be adopted in the numerical analyses in this section. These values are taken as the control values of the thirteen input parameters. In practice, the control values of these input parameters can be computed from historical data and prior knowledge of the process.

To demonstrate the computations of the control values of these thirteen input parameters in a real case problem, the following discussions adopted from Lorenzen and Vance [13] is provided. In this case study, the variable cost per unit sampled (c) is \$4.22 and it requires approximately 5 minutes to sample a single unit. The cost of each nonconforming unit produced is \$100. Historical data indicate that the process produces about 1.36% nonconforming units when it is in-control and about 11.3% nonconforming units when it is out-of-control, and the process stays in-control for an average of 50 hours. When an out-of-control signal is detected, a search for assignable cause is conducted. When one is found, the manufacturing system is stopped for repair, otherwise, the system is allowed to continue running. After repair is completed, the manufacturing system is restarted. The search for an assignable cause requires about 5 minutes, while repair requires 45 minutes. The repair cost is \$22.80 per hour and the downtime cost is \$21.34 per minute.

From the above paragraph, $\lambda = 1/50 = 0.02$ is the occurrence rate of assignable cause per hour. The time per unit sampled (e), expected search time during false alarm (T_0) and expected time to find the assignable cause (T_1) are $e = T_0 = T_1 = 5/60 = 0.083$ hour; while the expected time to repair the process is $T_2 = 45/60 = 0.75$ hour. During the search for the assignable cause, the process continues, thus $\varphi_1 = 1$, whereas the process is stopped during repair, hence, $\varphi_2 = 0$. The quality cost per hour while the process is in-control (C_0) is computed as follows: $C_0 = \$100$ (per nonconforming unit) $\times$ 84 (castings / units per hour) $\times$ 1.36% (nonconforming units) = \$114.24. Additionally, the quality cost per hour while the process is out-of-control (C_1) is calculated as follows: $C_1 = \$100 \times 84 \times 11.3\% = \$949.20$. Next, the cost of locating and removing the assignable cause (W) is obtained as the sum of the downtime cost and repair cost, i.e. $W = 45 \times \$21.34 + (45/60) \times \$22.80 = \$977.40$. It is assumed that the cost of false alarms (Y) is the same as the cost, W, hence, $Y = \$977.40$ is considered. Lastly, there is no fixed cost per sample, thus $b = \$0$.

Tables 3 and 4 provide the optimal parameters n, α and h of the MCV chart in minimizing the cost function, C in Equation (3.1), for the economic and economic-statistical designs of the aforementioned chart. The minimum cost, $C_{\min}$ and corresponding ARL_0 and ARL_1 values are also given in these tables. In Table 3, $p = 2$, $\gamma_0 = 0.1$ and $\tau = 0.5$ are considered for the downward MCV chart while in Table 4, $p = 2$, $\gamma_0 = 0.1$ and $\tau = 1.5$ are used for the upward MCV chart.

Table 3: Optimal parameters n, α and h in minimizing the cost function, C and the corresponding minimum cost ($C_{\min}$), ARL_0 and ARL_1 values computed for the downward MCV chart when $p = 2$, $\gamma_0 = 0.1$ and $\tau = 0.5$.

Case	Economic design						Economic-statistical design					
	n	α	h	$C_{\min}$	ARL_0	ARL_1	n	α	h	$C_{\min}$	ARL_0	ARL_1
1	13	0.0294	2.9112	206.7028	34.0136	1.1744	19	0.0039	2.8236	217.3567	256.4103	1.2426
2	14	0.0255	4.1072	**173.8845**	39.2157	1.1479	20	0.0039	4.1215	**180.1477**	256.4103	1.1865
3	12	0.0345	2.1108	*258.1688*	28.9855	1.2028	17	0.0040	1.8151	*275.8835*	250.0000	1.4009
4	13	0.0294	2.8124	**153.6580**	34.0136	1.1744	19	0.0039	2.7275	**164.9246**	256.4103	1.2426
5	13	0.0295	3.1474	*312.5772*	33.8983	1.1736	19	0.0039	3.0500	*321.9959*	256.4103	1.2426
6	14	0.0267	4.7588	**175.5130**	37.4532	1.1396	20	0.0039	4.7535	**180.7912**	256.4103	1.1865
7	12	0.0329	1.8891	*254.6568*	30.3951	1.2150	17	0.0040	1.6308	*274.8385*	250.0000	1.4009
8	11	0.0500	2.6535	**200.1941**	20.0000	1.1805	19	0.0039	2.7912	**216.7512**	256.4103	1.2426
9	15	0.0158	3.0772	*213.0151*	63.2911	1.1876	19	0.0039	2.8873	*218.5457*	256.4103	1.2426
10	13	0.0294	2.8935	**197.6308**	34.0136	1.1744	19	0.0039	2.8064	**208.3897**	256.4103	1.2426
11	13	0.0295	2.9507	*224.8406*	33.8983	1.1736	19	0.0039	2.8588	*235.2841*	256.4103	1.2426
12	13	0.0309	3.0492	*208.3568*	32.3625	1.1638	19	0.0039	2.9099	*219.0768*	256.4103	1.2426
13	13	0.0323	3.1805	*209.9396*	30.9598	1.1548	19	0.0039	2.9941	*220.7473*	256.4103	1.2426
14	14	0.0174	2.0716	**195.5321**	57.4713	1.2310	17	0.0040	1.7404	**200.6940**	250.0000	1.4009
15	11	0.0500	3.8064	*221.1789*	20.0000	1.1805	20	0.0039	4.2830	*240.5155*	256.4103	1.1865
16	14	0.0259	2.9664	**198.8200**	38.6100	1.1451	20	0.0039	2.9661	**206.0657**	256.4103	1.1865
17	11	0.0384	2.7917	*220.0642*	26.0417	1.2543	17	0.0040	2.4938	*236.8859*	250.0000	1.4009
18	13	0.0294	2.9112	206.7028	34.0136	1.1744	19	0.0039	2.8236	217.3567	256.4103	1.2426
19	13	0.0294	2.9112	206.7028	34.0136	1.1744	19	0.0039	2.8236	217.3567	256.4103	1.2426
20	13	0.0294	2.9092	**206.1229**	34.0136	1.1744	19	0.0039	2.8214	**216.7844**	256.4103	1.2426
21	13	0.0295	2.9183	*207.8738*	33.8983	1.1736	19	0.0039	2.8280	*218.5125*	256.4103	1.2426
22	13	0.0294	2.9140	*208.1516*	34.0136	1.1744	19	0.0039	2.8265	*218.8624*	256.4103	1.2426
23	13	0.0294	2.9056	**203.8647**	34.0136	1.1744	19	0.0039	2.8179	**214.4064**	256.4103	1.2426
24	13	0.0297	2.9074	**205.0555**	33.6700	1.1722	19	0.0039	2.8173	**215.8458**	256.4103	1.2426
25	13	0.0300	2.9575	*218.5185*	33.3333	1.1700	19	0.0039	2.8632	*229.2907*	256.4103	1.2426
26	13	0.0297	2.9620	*220.1732*	33.6700	1.1722	19	0.0039	2.8696	*230.8008*	256.4103	1.2426

In Tables 3 and 4, the italicized $C_{\min}$ values represent poorer performance (an increase in cost) while the boldfaced ones represent better performance (a decrease in cost) when the values of the input parameters are varied from the control values in case 1. The following discussions are based on the observations in Tables 3 and 4. It is found that the effects of changes in the input parameters on $C_{\min}$, ARL_0, ARL_1, n, α, and h for the economic design are almost similar to that for the economic-statistical design. In this section, the case number hereafter refers to the cases in Tables 3 and 4, unless stated otherwise.

Table 4: Optimal parameters n, α and h in minimizing the cost function, C and the corresponding minimum cost ($C_{\min}$), ARL_0 and ARL_1 values computed for the upward MCV chart when $p = 2$, $\gamma_0 = 0.1$ and $\tau = 1.5$.

Case	Economic design						Economic-statistical design					
	n	α	h	$C_{\min}$	ARL_0	ARL_1	n	α	h	$C_{\min}$	ARL_0	ARL_1
1	11	0.0286	1.8598	226.8698	34.9650	2.0070	13	0.0040	1.3199	240.2701	250.0000	2.9300
2	13	0.0287	2.9321	**188.9809**	34.8432	1.7783	15	0.0040	2.1137	**198.1568**	250.0000	2.5281
3	10	0.0294	1.2885	*283.8155*	34.0136	2.1403	10	0.0040	0.7368	*302.8548*	250.0000	3.8952
4	11	0.0285	1.7935	**174.4656**	35.0877	2.0088	13	0.0040	1.2741	**188.4030**	250.0000	2.9308
5	12	0.0295	2.1823	*331.3248*	33.8983	1.8689	13	0.0040	1.4282	*343.6606*	250.0000	2.9308
6	14	0.0316	3.7171	**188.3456**	31.6456	1.6559	17	0.0040	2.8399	**196.3893**	250.0000	2.2335
7	9	0.0260	1.0184	*283.2810*	38.4615	2.4111	10	0.0040	0.6580	*303.6778*	250.0000	3.8952
8	9	0.0500	1.7034	**217.5766**	20.0000	1.9849	12	0.0040	1.1850	**238.8530**	250.0000	3.1912
9	14	0.0154	2.0966	*235.8959*	64.9351	1.9456	15	0.0040	1.5939	*242.7479*	250.0000	2.5281
10	11	0.0286	1.8483	**217.9094**	34.9650	2.0070	13	0.0040	1.3117	**231.4024**	250.0000	2.9308
11	11	0.0286	1.8833	*244.7820*	34.9650	2.0070	13	0.0040	1.3367	*257.9956*	250.0000	2.9308
12	12	0.0322	2.1482	*229.2706*	31.0559	1.8291	15	0.0040	1.6083	*243.6569*	250.0000	2.5281
13	13	0.0353	2.4240	*231.4296*	28.3286	1.6959	17	0.0040	1.9016	*246.5326*	250.0000	2.2335
14	12	0.0144	1.2457	**211.4058**	69.4444	2.2442	12	0.0040	0.8688	**216.2331**	250.0000	3.1912
15	10	0.0500	2.6657	245.8762	20.0000	1.8487	15	0.0040	2.2106	*273.8320*	250.0000	2.5281
16	15	0.0300	2.4123	**219.5414**	33.3333	1.6013	20	0.0040	2.1005	**231.4886**	250.0000	1.9190
17	8	0.0258	1.3946	*237.4666*	38.7597	2.6647	9	0.0040	0.8917	*251.5718*	250.0000	4.3840
18	11	0.0286	1.8598	226.8698	34.9650	2.0070	13	0.0040	1.3199	240.2701	250.0000	2.9308
19	11	0.0286	1.8598	226.8698	34.9650	2.0070	13	0.0040	1.3199	240.2701	250.0000	2.9308
20	11	0.0286	1.8586	**226.3077**	34.9650	2.0070	13	0.0040	1.3189	**239.7114**	250.0000	2.9308
21	11	0.0286	1.8622	*228.0051*	34.9650	2.0070	13	0.0040	1.3220	*241.3983*	250.0000	2.9308
22	11	0.0286	1.8618	*228.4402*	34.9650	2.0070	19	0.0039	1.3214	**218.8624**	256.4103	2.9308
23	11	0.0286	1.8558	**223.7926**	34.9650	2.0070	19	0.0039	1.3169	**214.4064**	256.4103	2.9308
24	11	0.0291	1.8616	**225.1268**	34.3643	1.9978	19	0.0039	1.3166	**215.8458**	256.4103	2.9308
25	11	0.0296	1.8989	*238.5002*	33.7838	1.9889	19	0.0039	1.3383	**229.2907**	256.4103	2.9308
26	11	0.0290	1.8954	*240.2556*	34.4828	1.9996	19	0.0039	1.3417	**230.8008**	256.4103	2.9308

The thirteen input parameters of Lorenzen and Vance [13] cost model can be classified as expenses related parameters (C_0, C_1, Y, W, b, c), time related parameters (e, T_0, T_1, T_2) and process related parameters (λ, φ_1, φ_2). For a more effective and systematic way of discussing the effects of each input parameters on the minimum cost, ARLs and optimal parameters, this section is organized as follows: Firstly, the effects of expenses related parameters are discussed in Section 4.1, then those of time related parameters are enumerated in Section 4.2 and finally that of process related parameters are explained in Section 4.3. Additionally, the effects of the shift size τ in the process MCV is included in Section 4.3. Lastly, a comparison between economic and economic-statistical designs of the MCV chart is presented in Section 4.4.

4.1. Effects of expenses related parameters on $C_{\min}$, ARLs and optimal parameters

An increase in the quality cost (due to nonconformities produced) per hour while in-control, C_0 or out-of-control, C_1 results in an increase in the minimum cost, $C_{\min}$; and vice-versa (see cases 4–7). Although C_1 is larger than C_0 (Table 2, cases 4–7), C_0 has a more noticeable effect on the minimum cost ($C_{\min}$) as it results in a larger change in $C_{\min}$. It is also seen that an increase in C_0 (see case 5) or a decrease in C_1 (see case 6) leads to an increase in h, as compared to the control case (case 1). Note that a larger sampling interval, h is adopted when C_0 increases so that less frequent sampling is made when the process is in-control in order to offset the increase in quality cost per hour while the process is in-control. On a similar note, a decrease in C_1 indicates a lower quality cost per hour while the process is out-of-control, implying that sampling can be made less frequently (with an increase in h) so that the model remains economically viable. The same explanation applies for a decrease in h when C_0 decreases or C_1 increases.

Another cost parameter worthy of discussion is the cost of false alarm, Y. It is found that increasing (decreasing) Y only results in a slight increase (decrease) in the minimum cost, $C_{\min}$ but it substantially increases (decreases) the ARL_0 value for the economic design of the chart (see cases 8 and 9). An increased (decreased) ARL_0 value translates into a lower (higher) false alarm rate, hence a smaller (larger) α value (see case 9 for the economic design). A larger cost of false alarm (see case 9 in Table 2, where $Y = \$1954.8$ instead of the control value of \$977.4) will reduce the sampling frequency (larger h of 3.0772 instead of the control value of 2.9112 — see Table 3) for the economic design model. To compensate for the less frequent sampling, a larger sample size (larger n, increasing from 13 to 15) is adopted (see cases 1 and 9 for the economic design in Table 3). Note that the effect of changing Y on the optimal parameters, minimum cost and ARLs under the economic-statistical design model is less pronounced.

Comparing to Y, varying the cost to locate and remove the assignable cause, W poses no significant changes to the optimal parameters n, α and h. However, W has a greater influence on the minimum cost $C_{\min}$ than Y. As an example, increasing W from \$977.4 to \$1954.8 (see case 11 in Table 2) causes $C_{\min}$ to increase from \$226.8698 to \$244.7820 (see case 11 for economic design in Table 4) while the same amount of increment in Y (see case 9 in Table 2) results in a smaller increase in $C_{\min}$, i.e. from \$226.8698 to \$235.8959 (see case 9 for the economic design in Table 4). Likewise, $C_{\min}$ decreases at a quicker rate when W decreases compared to that for the same amount of a decrease in Y. Using another example based on the economic-statistical design in Table 3, decreasing W and Y to half of their original values causes $C_{\min}$ to decrease by \$8.9670 (i.e. \$217.3567 − \$208.3897 or the difference between $C_{\min}$ of cases 1 and 10) versus \$0.6055 (i.e. \$217.3567 − \$216.7512 or the difference between $C_{\min}$ of cases 1 and 8), respectively.

The sampling cost is affected by two different parameters, namely the fixed cost per sample, b and the variable cost per unit sampled, c. The control value of b is \$0. When b increases to \$5 and \$10, it is found that the minimum cost, $C_{\min}$ for case 13 is larger than that for case 12 but the $C_{\min}$ values for these two cases are larger than the control cost in case 1. In fact, increasing any cost parameter, including the variable cost per unit sampled, c will always result in an increase in $C_{\min}$, as expected. Increasing the cost b and (or) c (see cases 12, 13 and 15) results in a larger optimal sampling interval (larger h) for both economic and economic-statistical design models and a smaller ARL_0 value for the economic design model. The exact opposite results are observed by decreasing c (case 14 in Table 2), which results in smaller h, lower $C_{\min}$ and larger ARLs (see the economic design for both downward and upward charts in Tables 3 and 4). Note that the ARL_0 values in Tables 3 and 4 do not vary much in the economic-statistical design model in satisfying the constraint $\text{ARL}_0 \geq 250$.

4.2. Effects of time related parameters on $C_{\min}$, ARLs and optimal parameters

Besides the expenses related parameters, Lorenzen and Vance's [13] cost model also includes the time related parameters, namely e, T_0, T_1 and T_2. Other than the time to sample and interpret one unit (e), the remaining time related parameters have minimal effect on the optimal parameters, $C_{\min}$, ARL_0 and ARL_1 values (see cases 18–23). An increase (decrease) in e causes the minimum cost, $C_{\min}$ to increase (decrease) (see cases 16 and 17). As e increases (from 0.083 hours to 0.166 hours), smaller sample sizes (for example, see case 17, where $n = 11$ in Table 3 and $n = 8$ in Table 4 for the economic design) are adopted to offset the increase in $C_{\min}$. Consequently, shorter sampling intervals (see case 17, where $h = 2.7917$ hours in Table 3 and $h = 1.3946$ hours in Table 4 for the economic design) are adopted as more frequent samplings are needed to compensate for the smaller sample sizes used. In addition, increasing (decreasing) the value of e leads to a larger (smaller) ARL_1 value (see cases 16 and 17). Using an example from the economic-statistical design, increasing e causes ARL_1 to increase from 1.2426 to 1.4009 for the downward MCV chart (see case 17 in Table 3) and from 2.9300 to 4.3840 for the upward MCV chart (see case 17 in Table 4). In addition, decreasing e causes ARL_1 to decrease from 1.2426 to 1.1865 for the downward MCV chart (see case 16 in Table 3) and from 2.9300 to 1.9190 for the upward MCV chart (see case 16 in Table 4).

4.3. Effects of process related parameters on $C_{\min}$, ARLs and optimal parameters

The rate of occurrence of assignable cause, λ has a significant effect on the optimal sample size, n, optimal sampling interval, h and minimum cost, $C_{\min}$ (see cases 2 and 3). For example, when λ decreases from 0.02 to 0.01 (see cases 1 and 2), $C_{\min}$ decreases from \$206.7028 to \$173.8845 (see case 2 for the economic design in Table 3) because the process failure rate decreases. In contrast, when λ increases from 0.02 to 0.04 (see cases 1 and 3), $C_{\min}$ increases from \$206.7028 to \$258.1688 (see case 3 for the economic design in Table 3). To enable this undesirable condition (an increase in λ) to be detected quickly by the MCV chart, more frequent samplings (decreasing h) are needed while smaller sample sizes (decreasing n) are adopted in order to remain economically favourable (see cases 1 and 3 in Tables 3 and 4, for both economic and economic statistical designs).

The parameters φ_1 and φ_2 determine whether the process continues or stops during search and repair, respectively. As shown in Table 1, φ_1 (φ_2) has:

(i) the value 1 if the process continues while *searching for the assignable cause (repairing following the occurrence of an assignable cause)*;

(ii) the value 0 if the process stops during *search (repair)*.

By comparing cases 1, 24, 25 and 26, it is observed that case 24 (where $(\varphi_1, \varphi_2 = (0, 0))$ has the lowest minimum cost, $C_{\min}$ (see Tables 3 and 4). This is expected because when the process stops during both search and repair, the cost will be minimized. For example, for the economic design in Table 3, $C_{\min} \in \{\$205.0555, 206.7028, 218.5185, 220.1732\}$ for $(\varphi_1, \varphi_2) \in \{(0, 0), (1, 0), (0, 1), (1, 1)\}$, where the lowest $C_{\min}$ $(= \$205.0555)$ occurs at $(\varphi_1, \varphi_2) = (0, 0)$, i.e. when the process stops during both search and repair. On the contrary, case 26, i.e. the process continues during both search and repair ($(\varphi_1, \varphi_2) = (1,1)$) undoubtedly results in the highest minimum cost, $C_{\min}$. Note that the effect of the same pair of (φ_1, φ_2) values on $C_{\min}$ is similar for both economic and economic-statistical designs of the downward and upward MCV charts.

Another interesting observation obtained is the influence of the shift, τ $(= \gamma_1/\gamma_0)$ on the minimum cost, $C_{\min}$. Table 3 deals with a 50% decreasing shift in the process MCV while Table 4 involves an increasing shift of 50%, hence, the size of shifts in both tables is the same. It is found that for the same size of shift in the process MCV, generally, the upward MCV chart incurs a higher $C_{\min}$ than that of the downward MCV chart. As an example, for the economic-statistical design in Table 3, $C_{\min} \in \{\$217.3567, 180.1477, 275.8835, 164.9246, 321.9959\}$ while in Table 4, $C_{\min} \in \{\$240.2701, 198.1568, 302.8548, 188.4030, 343.6606\}$ for cases 1, 2, 3, 4 and 5, respectively. This example clearly shows that $C_{\min}$ for the upward MCV chart is higher than the corresponding one for the downward MCV chart. It is noteworthy that a larger $C_{\min}$ for the upward MCV chart corresponds to detecting an increasing shift ($\tau = 1.5$) in the process MCV, which simply means process deterioration. In contrast, a smaller $C_{\min}$ incurred by the downward MCV chart is associated with the detection of a decreasing MCV shift ($\tau = 0.5$) or simply process improvement. As $C_{\min}$ incurred by the upward MCV chart is higher, smaller sample sizes, n must be adopted by this chart to offset the increase in cost. This is evident as n in Table 4 is generally lower than the corresponding one in Table 3. For example, based on the economic-statistical design in cases 1, 2, 3, 4 and 5, it is noticed that $n \in \{19, 20, 17, 19, 19\}$ and $n \in \{13, 15, 10, 13, 13\}$ in Tables 3 and 4, respectively, where it is obvious that the sample sizes in Table 4 are lower than the corresponding ones in Table 3. Consequently, to compensate for the smaller sample sizes adopted by the upward MCV chart in Table 4, samples must be taken more frequently, hence a smaller sampling interval, h is adopted. For the same example, $h \in \{1.3199, 2.1137, 0.7368, 1.2741, 1.4282\}$ are adopted for cases 1–5 in Table 4 while $h \in \{2.8236, 4.1215, 1.8151, 2.7275, 3.0500\}$ are employed for the same cases in Table 3. Evidently, the h values in Table 4 are smaller than that in Table 3.

4.4. Comparisons between economic and economic-statistical designs

It is shown in Tables 3 and 4 that imposing statistical constraints in the economic design of the MCV chart significantly improves the statistical performance of the chart as it results in larger ARL_0 values at the expense of slight increases in the minimum cost ($C_{\min}$) and ARL_1 values. For a better analysis, Table 5 shows the percentage of increase in the ARL_0 value for each of the 26 cases in Table 2 when the economic-statistical design is used in place of the economic design. Additionally, Table 5 shows the percentage of the slight increase in the minimum cost ($C_{\min}$) and ARL_1 values as a result of adding the statistical constraints (in the economic-statistical design). In Table 5, $p = 2$ and $\gamma_0 = 0.1$ are considered for the downward ($\tau = 0.5$) and upward ($\tau = 1.5$) MCV charts. It is found in Table 5 that by employing the economic-statistical design model, the ARL_0 value increases by at least 305.13% (case 9) and 260% (case 14), for the downward and upward MCV charts, respectively. In contrast, the chart's performances in terms of $C_{\min}$ and ARL_1 criteria only deteriorate slightly. For example, $C_{\min}$ increases by at most 8.79% (case 15) for the downward MCV chart and 11.42% (case 15) for the upward MCV chart. On similar lines, the ARL_1 increases by at most 16.47% (case 3) and 81.99% (case 3) for the downward and upward MCV charts, respectively.

Table 5: Percentages of increase in the minimum cost ($C_{\min}$), ARL_0 and ARL_1 values by using economic-statistical design in place of economic design for the downward and upward MCV charts when $p = 2$ and $\gamma_0 = 0.1$.

Case	Downward MCV chart			Upward MCV chart		
	% increase in $C_{\min}$	% increase in ARL_0	% increase in ARL_1	% increase in $C_{\min}$	% increase in ARL_0	% increase in ARL_1
1	5.15	653.85	5.81	5.91	615.00	45.99
2	3.60	553.85	3.36	4.86	617.50	42.16
3	6.86	762.50	16.47	6.71	635.00	81.99
4	7.33	653.85	5.81	7.99	612.50	45.90
5	3.01	656.41	5.88	3.72	637.50	56.82
6	3.01	584.62	4.12	4.27	690.00	34.88
7	7.93	722.50	15.30	7.20	550.00	61.55
8	8.31	1510.25	4.02	9.79	1255.00	64.56
9	2.60	305.13	4.63	2.90	285.00	29.94
10	5.44	653.85	5.81	6.19	615.00	46.03
11	4.64	656.41	5.88	5.40	615.00	46.03
12	5.15	692.31	6.77	6.27	705.00	38.22
13	5.15	728.20	7.60	6.53	782.50	31.70
14	2.64	335.00	13.80	2.28	260.00	42.20
15	8.79	1325.64	2.67	11.42	1342.50	42.08
16	3.64	564.10	3.62	5.44	650.00	19.84
17	7.64	860.00	11.69	5.94	545.00	64.52
18	5.15	653.85	5.81	5.91	615.00	46.03
19	5.15	653.85	5.81	5.91	615.00	46.03
20	5.17	653.85	5.81	5.92	615.00	46.03
21	5.12	656.41	5.88	5.87	615.00	46.03
22	5.15	653.85	5.81	5.90	615.00	46.03
23	5.17	653.85	5.81	5.92	615.00	46.03
24	5.26	661.54	6.01	6.04	627.50	46.70
25	4.93	669.23	6.21	5.74	640.00	47.36
26	4.83	661.54	6.01	5.61	625.00	46.57
Average	5.26	697.55	6.78	5.99	653.65	46.59

The last row in Table 5 shows the average percentages of increase in $C_{\min}$, ARL_0 and ARL_1 values when the economic-statistical design is used instead of the economic design. For the downward MCV chart, it is found that there is a huge average increase in the ARL_0 value, i.e. 697.55% as compared to significantly smaller average increase in $C_{\min}$ and ARL_1 values, i.e. at only 5.26% and 6.78%, respectively. Similarly, for the upward MCV chart, a large average increase in ARL_0, i.e. 653.65% is obtained at the expense of enormously smaller average increases in $C_{\min}$ (5.99%) and ARL_1 (46.59%) values. It is obviously seen in Table 5 that when the economic-statistical design is adopted in lieu of the economic design, the downward MCV chart (average increase of 6.78%) results in a smaller increase in the value compared to the upward MCV chart (average increase of 46.59%).

Additional analyses are conducted for the number of correlated variables, $p \geq 3$, where the same trends as that for $p = 2$ are observed. Thus, the results for $p \geq 3$ are not given here so as not to increase the length of this manuscript unnecessarily.

5. CONCLUSIONS

The MCV chart is used in the monitoring of the process MCV. The use of the MCV chart in process monitoring requires not only the statistical consideration in assessing its performance but also from a cost point of view. In line with this requirement, this research studies the economic and economic-statistical designs of the MCV chart. The economic design takes into account of minimizing the cost, but it ignores the statistical evaluation of the chart. Therefore, the economic design exposes the MCV chart to a poor statistical performance, resulting in an undesirable Type-I error rate. To circumvent this setback, statistical constraints, in terms of the ARL_0 and ARL_1 considerations, are imposed on the cost minimization model, resulting in the economic-statistical design of the chart. The effects of changes in the input parameters on the minimum cost and the corresponding optimal parameters of the MCV chart, as well as the char's ARL_0 and ARL_1 values are enumerated. Additionally, this work also compares the impact of adding statistical constraints on the performance of the MCV chart. It is found that the economic-statistical design significantly improves the ARL_0 performance of the MCV chart at the expense of slight increases in minimum cost ($C_{\min}$) and ARL_1 values.

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REFERENCES

[1] Abbasi, S.A. and Adegoke, N.A. (2018). Multivariate coefficient of variation control charts in phase I of SPC, *The International Journal of Advanced Manufacturing Technology*, **99**(5–8), 1903–1916.

[2] Albert, A. and Zhang, L. (2010). A novel definition of the multivariate coefficient of variation, *Biometrical Journal*, **52**(5), 667–675.

[3] Carstensen, B. (2010). *Comparing Clinical Measurement Methods: A Practical Guide*, John Wiley & Sons, Ltd, Chapter **23**, 107–114.

[4] Castagliola, P.; Celano, G. and Psarakis, S. (2011). Monitoring the coefficient of variation using EWMA charts, *Journal of Quality Technology*, **43**(3), 249–265.

[5] Duncan, A.J. (1956). The economic design of $\bar{X}$ charts used to maintain current control of a process, *Journal of the American Statistical Association*, **51**(274), 228–242.

[6] Giner-Bosch, V.; Tran, K.P.; Castagliola, P. and Khoo, M.B.C. (2019). An EWMA control for the multivariate coefficient of variation, *Quality and Reliability Engineering International*, **35**(6), 1515–1541.

[7] Golub, T.R.; Slonim, D.K.; Tomayo, P.; Huard, C.; Gaasenbeek, M.; Mesirov, J.P.; Coller, H.; Loh, M.L.; Downing, J.R.; Caligiuri, M.A.; Bloomfield, C.D. and Lander, E.S. (1999). Molecular classification of cancer: class discovery and class prediction by gene expression monitoring, *Science*, **286**(5439), 531–537.

[8] Hua, Y.; Qiu, W.; Xiao, Q. and Wu, Q. (2018). Precision (repeatability and reproducibility) of ocular parameters obtained by the tomey OA-2000 biometer compared to the IOLMaster in healthy eyes, *PLoS ONE*, **13**(2), e0193023.

[9] Kang, C.W.; Lee, M.S.; Seong, Y.J. and Hawkins, D.M. (2007). A control chart for the coefficient of variation, *Journal of Quality Technology*, **39**(2), 151–158.

[10] Khatun, M.; Khoo, M.B.C.; Lee, M.H. and Castagliola, P. (2019). One-sided control charts for monitoring the multivariate coefficient of variation in short production runs, *Transactions of the Institute of Measurement and Control*, **41**(6), 1712–1728.

[11] Khaw, K.W.; Khoo, M.B.C.; Castagliola, P. and Rahim, M.A. (2018). New adaptive control charts for monitoring the multivariate coefficient of variation, *Computers & Industrial Engineering*, **126**, 595–610.

[12] Linderman, K. and Love, T.E. (2000). Economic and economic statistical designs for MEWMA control charts, *Journal of Quality Technology*, **32**(4), 410–417.

[13] Lorenzen, T.J. and Vance, L.C. (1986). The economic design of control chart: a unified approach, *Technometrics*, **28**(1), 3–10.

[14] Molnau, W.E.; Montgomery, D.C. and Runger, G.C. (2011). Statistically constrained economic design of the multivariate exponentially weighted moving average control chart, *Quality and Reliability Engineering International*, **17**(1), 39–49.

[15] Ng, P.S.; Khoo, M.B.C.; Saha, S. and Yeong, W.C. (2019). Economic and economic-statistical designs of auxiliary information based $\bar{X}$, synthetic and EWMA charts, *Communications in Statistics – Simulation and Computation*, DOI: 10.1080/03610918.2019.1664575.

[16] Nguyen, Q.T.; Tran, K.P.; Castagliola, P.; Celano, G. and Lardjane, S. (2019). One-sided synthetic control charts for monitoring the multivariate coefficient of variation, *Journal of Statistical Computation and Simulation*, **89**(10), 1841–1862.

[17] Onkelinx, S.; Cornelissen, V.; Goetschalckx, K.; Thomaes, T.; Verhamme, P. and Vanhees, L. (2012). Reproducibility of different methods to measure the endothelial function, *Vascular Medicine*, **17**(2), 79–84.

[18] Peruchi, R.S.; Balestrassi, P.P.; de Paiva, A.P.; Ferreira, J.R. and de Santana Carmelossi, M. (2013). A new multivariate gage R&R method for correlated characteristics, *International Journal of Production Economics*, **144**(1), 301–315.

[19] Reed, G.F.; Lynn, F. and Meade, B.D. (2002). Use of coefficient of variation in assessing variability of quantitative assays, *Clinical and Diagnostic Laboratory Immunology*, **9**(6), 1235–1239.

[20] Reinstein, D.Z.; Archer, T.J.; Silverman, R.H. and Coleman, D.J. (2006). Accuracy, repeatability, and reproducibility of artemis very high-frequency digital ultrasound arc-scan lateral dimension measurements, *Journal of Cataract & Refractive Surgery*, **32**(11), 1799–1802.

[21] Reyment, R.A. (1960). Studies on nigerian upper cretaceous and lower tertiary ostracoda: part 1, senonian and maastrichtian ostracoda, *Stockholm Contributions in Geology*, **7**, 1–238.

[22] Safe, H.; Kazemzadeh, R.B. and Gholipour Kanami, Y. (2018). A Markov chain approach for double-objective economic statistical design of the variable sampling interval control chart, *Communications in Statistics – Theory and Methods*, **47**(2), 277–288.

[23] Saniga, E.M. (1989). Economic statistical control-chart designs with an application to $\bar{X}$ and R charts, *Technometrics*, **31**(3), 313–320.

[24] Sharpe, W.F. (1994). The sharpe ratio, *The Journal of Portfolio Management*, **21**(1), 49–58.

[25] Soulé, M.E. and Zegers, G.P. (1996). Phenetics of natural populations. V. Genetic correlates of phenotypic variation in the pocket gopher (Thomomys bottae) in California, *The Journal of Heredity*, **87**(5), 341–350.

[26] Surtihadi, J. and Raghavachar, M. (1994). Exact economic design of $\bar{X}$ charts for general time in-control distributions, *International Journal of Production Research*, **32**(7), 2287–2302.

[27] Sweeney, S. (2007). Analysis of two-dimensional gage repeatability and reproducibility, *Quality Engineering*, **19**(1), 29–37.

[28] Teoh, W.L.; Khoo, M.B.C.; Castagliola, P.; Yeong, W.C. and Teh, S.Y. (2017). Run-sum control charts for monitoring the coefficient of variation, *European Journal of Operational Research*, **257**(1), 144–158.

[29] Van Valen, L. (1974). Multivariate structural statistics in natural history, *Journal of Theoretical Biology*, **45**(1), 235–247.

[30] Voinov, V.G. and Nikulin, M.S. (1996). *Unbiased Estimators and Their Applications, Multivariate Case*, Vol. 2, Kluwer, Dordrecht.

[31] Wan, Q. and Zhu, M. (2021). Detecting a shift in variance using economically designed VSI control chart with combined attribute-variable inspection, *Communications in Statistics – Simulation and Computation*, **50**(11), 3483–3497.

[32] Yeong, W.C.; Khoo, M.B.C.; Teoh, W.L. and Castagliola, P. (2016). A control chart for the multivariate coefficient of variation, *Quality and Reliability Engineering International*, **32**(3), 1213–1225.

[33] Yeong, W.C.; Khoo, M.B.C.; Wu, Z. and Castagliola, P. (2012). Economically optimum design of a synthetic $\bar{X}$ chart, *Quality and Reliability Engineering International*, **28**(7), 725–741.

[34] Yeong, W.C.; Lim, S.L.; Khoo, M.B.C.; Chuah, M.H. and Lim, A.J.X. (2018). The economic and economic-statistical designs of the synthetic chart for the coefficient of variation, *Journal of Testing and Evaluation*, **46**(3), 1175–1195.

[35] Zhang, L.; Albarède, S.; Dumont, G.; Campenhout, C.V.; Libeer, J.C. and Albert, A. (2010). The multivariate coefficient of variation for comparing serum protein electrophoresis techniques in external quality assessment schemes, *Accreditation and Quality Assurance*, **15**(6), 351–357.

[36] Zhang, L.; Campenhout, C.V.; Devleeschouwer, N.; Libeer, J.C. and Albert, A. (2008). Statistical analysis of serum protein electrophoresis results in external quality assessment schemes, *Accreditation and Quality Assurance*, **13**(3), 149–155.