THE EXTREMAL INDEX OF SUB-SAMPLED PERIODIC SEQUENCES WITH STRONG LOCAL DEPENDENCE

Authors: H. Ferreira
– Department of Mathematics, University of Beira Interior, Portugal
(ferreira@fenix2.ubi.pt)

A.P. Martins
– Department of Mathematics, University of Beira Interior, Portugal
(amartins@noe.ubi.pt)

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Abstract:

- Let $\mathbf{X} = \{X_n\}_{n \geq 1}$ be a T-periodic sequence. We define a family of local dependence conditions $D_T^{(k)}(\mathbf{u})$, $k \geq 1$, and calculate the extremal index $\theta_{\mathbf{X}}$ from the distributions of k consecutive variables of $\mathbf{X}$. For a periodic sub-sampled sequence $\mathbf{Y} = \{X_{g(n)}\}_{n \geq 1}$, where g generates blocks of I_1 observations separated by J observations, we present results on local and long range dependence conditions and compute the extremal index $\theta_{\mathbf{Y}}$.

Key-Words:

- *sub-sampling; periodic sequences; extremal index; extreme values.*

1. INTRODUCTION

In this paper we consider that $\mathbf{X} = \{X_n\}_{n\geq 1}$ is a T-periodic sequence of random variables, i.e., there exists an integer $T \geq 1$ such that, for each choice of integers $1 \leq i_1 < ... < i_n$, $(X_{i_1}, ..., X_{i_n})$ and $(X_{i_1+T}, ..., X_{i_n+T})$ have the same distribution. The period T will be considered the smallest integer satisfying the above definition.

We say that a T-periodic sequence $\mathbf{X}$ has extremal index $\theta_{\mathbf{X}}$ when, $\forall\, \tau > 0$, $\exists\, \mathbf{u}^{(\tau)} = \{u_n^{(\tau)}\}_{n\geq 1}$ such that

$$\lim_{n\to\infty} n\, \frac{1}{T} \sum_{i=1}^{T} P\Big(X_i > u_n^{(\tau)}\Big) \;=\; \tau$$

and

$$\lim_{n\to\infty} P\Big(\max\{X_1, ..., X_n) \leq u_n^{(\tau)}\Big) \;=\; e^{-\theta_{\mathbf{X}}\tau} \;.$$

The elements of $\mathbf{u}^{(\tau)}$ are called normalized levels for $\mathbf{X}$.

Such as happens for stationary sequences, the extremal index of a periodic sequence (Alpuim ([1]), Ferreira ([4])) enables us to infer the limiting behaviour of M_n from the limiting behaviour of $\hat{M}_n = \max\{\hat{X}_1, ..., \hat{X}_n\}$, $n \geq 1$, where $\hat{\mathbf{X}} = \{\hat{X}_n\}_{n\geq 1}$ is a periodic sequence of independent variables such that $F_{X_i} = F_{\hat{X}_i}$, $\forall\, i \geq 1$. Specifically,

$$\lim_{n\to\infty} P\Big(\max\{X_1, ..., X_n) \leq u_n^{(\tau)}\Big) \;=\; \left(\lim_{n\to\infty} P\Big(\max\{\hat{X}_1, ..., \hat{X}_n) \leq u_n^{(\tau)}\Big)\right)^{\theta_{\mathbf{X}}}$$

holds true.

By evaluating its extremal index $\theta_{\mathbf{X}}$, we describe in section 2 the asymptotic behaviour of the partial maximum $M_n = \max\{X_1, ..., X_n\}$, $n \geq 1$, under the condition $D(\mathbf{u})$ of Leadbetter ([5]) and a local dependence condition that generalizes the $D^{(k)}(\mathbf{u})$ of Chernick et al. ([2]).

In section 3 we give sufficient conditions for the analogous dependence conditions to hold for a sub-sampled sequence $\mathbf{Y} = \{X_{g(n)}\}_{n\geq 1}$ and we relate the extremal indexes $\theta_{\mathbf{X}}$ and $\theta_{\mathbf{Y}}$.

There are important situations in finance, for instance, where it seems reasonable to sub-sample the process by blocks matching them with bussiness periods (Dacorogna et al. ([3])). For a complete description of the extremal behavior of sub-sampled sequences $\mathbf{Y}$ from moving averages $\mathbf{X}$ with regularly varying tails see Scotto and Ferreira ([10]) and references therein.

Robinson and Tawn ([9]) pointed out the importance of the sampling frequency on the extremal properties and they have showed that if the sequence

$\mathbf{X} = \{X_n\}_{n\geq 1}$ and the sub-sampled sequence $\mathbf{Y} = \{X_{Tn}\}_{n\geq 1}$ have extremal indexes $\theta_{\mathbf{X}}$ and $\theta_{\mathbf{Y}}$, respectively, then

$$\theta_{\mathbf{X}} \;\leq\; \theta_{\mathbf{Y}} \;\leq\; T\,\theta_{\mathbf{X}} \left(1 - \sum_{j=1}^{T-1} \left(1 - \frac{j}{T}\right) \Pi(j)\right) ,$$

where $\Pi(j),\ j \geq 1$, are the asymptotic cluster size distributions for $\mathbf{X}$. Moreover, the upper bound is obtained under the condition $D''(u_n)$ from Leadbetter and Nandagopalan ([6]).

Our results in section 3 enable the computation of the extremal index of periodic sub-sampled sequences $\mathbf{Y}{=}\{X_{g(n)}\}_{n\geq 1}$ for g such that $\lim_{n\to\infty} \frac{g(n)}{n} = G$, under a family of local dependence conditions for T-periodic sequences. They generalize the main result in Martins and Ferreira ([7]) concerning stationary sequences satisfying the condition $D''(u_n)$ and g defined as $g(n) = (n-1) \bmod I + T\left[\frac{(n-1)}{I}\right],\ n \geq 1$.

2. COMPUTING THE EXTREMAL INDEX UNDER $D_T^{(k)}(\mathbf{u})$

We introduce a family of local dependence conditions for T-periodic sequences satisfying the long range dependence condition $D(\mathbf{u})$ from Leadbetter ([5]). The sequence of dependence coefficients in this condition will be referred as $\alpha^{(\mathbf{X},\mathbf{u})} = \{\alpha_{n,l}^{(\mathbf{X},\mathbf{u})}\}_{n\geq 1}$ and it is such that $\alpha_{n,l_n}^{(\mathbf{X},\mathbf{u})} = o(1)$ for some $l_n = o(n)$. For simplicity we omit the sequences $\mathbf{X}$ and $\mathbf{u}$ in these notations whenever no doubt is created.

Definition 2.1. Let $k \geq 1$ be a fixed integer and $\mathbf{X}$ a T-periodic sequence satisfying $D(\mathbf{u})$. The condition $D_T^{(k)}(\mathbf{u})$ holds for $\mathbf{X}$ when there exists a sequence of integers $\mathbf{k} = \{k_n\}_{n\geq 1}$ such that

$$\lim_{n\to\infty} k_n = +\infty \ , \qquad \lim_{n\to\infty} k_n \frac{l_n}{n} = 0 \ , \qquad \lim_{n\to\infty} k_n \alpha_{n,l_n} = 0 \ , \tag{2.1}$$

and

$$\lim_{n\to\infty} S^{(k)}_{[\frac{n}{k_n T}]} = 0 \ ,$$

where

$$S^{(1)}_{[\frac{n}{k_n T}]} \;=\; n\,\frac{1}{T} \sum_{i=1}^{T} \sum_{j=i+1}^{[\frac{n}{k_n T}]T} P\Big(X_i > u_n,\ X_j > u_n\Big)$$

and, for $k \geq 2$,

$$S^{(k)}_{[\frac{n}{k_n T}]} \;=\; n\,\frac{1}{T} \sum_{i=1}^{T} \sum_{j=i+k}^{[\frac{n}{k_n T}]T} P\Big(X_i > u_n,\ X_{j-1} \leq u_n < X_j\Big) \ .$$

The extremal behaviour of $\mathbf{X}$ has already been considered in Ferreira ([4]) under the conditions $D_T^{(k)}(\mathbf{u})$, for $k=1,2$.

If $\max\{X_i, X_{i+1}, ..., X_j\}$ is denoted by $M_{i,j}^{(\mathbf{X})}$ and we put $M_{i,j}^{(\mathbf{X})} = -\infty$ for $i>j$, then $\lim_{n\to\infty} S^{(k)}_{[\frac{n}{k_n T}]} = 0$ implies

$$\lim_{n\to\infty} n\,\frac{1}{T} \sum_{i=1}^{T} \sum_{j=i+k}^{[\frac{n}{k_n T}]T} P\Big(X_i > u_n \ge M_{i+1,i+k-1},\, X_j > u_n\Big) \;=\; 0 \;,$$

which leads to

$$\lim_{n\to\infty} n\,\frac{1}{T} \sum_{i=1}^{T} P\Big(X_i > u_n \ge M_{i+1,i+k-1},\, M_{i+k,[\frac{n}{k_n T}]T} > u_n\Big) \;=\; 0 \;.$$

This last restriction, when $T=1$, is the one considered in $D^{(k)}(\mathbf{u})$ by Chernick et al. ([2]) for stationary sequences. Under $D^{(k)}(\mathbf{u})$ they compute $\theta_{\mathbf{X}}$ from the distribution of the first k variables of $\mathbf{X}$ and apply the result to several autoregressive sequences. In the following we will extend their results for periodic sequences.

Proposition 2.1. *If the T-periodic sequence $\mathbf{X}$ satisfies $D(\mathbf{u})$ and $D_T^{(k)}(\mathbf{u})$ then*

$$P\Big(M_n \le u_n\Big) - \exp\left(\frac{n}{T} \sum_{i=1}^{T} P\Big(X_i > u_n \ge M_{i+1,i+k-1}\Big)\right) \;=\; o(1) \;.$$

Proof: Under $D(\mathbf{u})$ we have, for $\mathbf{k}$ as in (2.1),

$$P\Big(M_n \le u_n\Big) - P^{k_n}\Big(M_{[\frac{n}{k_n T}]T} \le u_n\Big) \;=\; o(1) \;,$$

and therefore it is enough to proof that

$$P\Big(M_{[\frac{n}{k_n T}]T} > u_n\Big) - \frac{\dfrac{n}{T} \displaystyle\sum_{i=1}^{T} P\Big(X_i > u_n \ge M_{i+1,i+k-1}\Big)}{k_n} \;=\; o(1) \;. \tag{2.2}$$

Since, by applying $D_T^{(k)}(\mathbf{u})$,

$$\begin{aligned} P\Big(M_{[\frac{n}{k_n T}]T} > u_n\Big) \;&=\; P\left(\bigcup_{i=1}^{[\frac{n}{k_n T}]T} \Big\{X_i > u_n \ge M_{i+1,[\frac{n}{k_n T}]T}\Big\}\right) \\ &=\; \left[\frac{n}{k_n T}\right] \sum_{i=1}^{T} P\Big(X_i > u_n \ge M_{i+1,i+k-1}\Big) - A_n \;, \end{aligned}$$

holds with $k_n A_n \le S^{(k)}_{[\frac{n}{k_n T}]} = o(1)$, we conclude (2.2). □

As a consequence of this result we compute the extremal index as follows.

Corollary 2.1. *If the T-periodic sequence $\mathbf{X}$ satisfies $D(\mathbf{u})$ for all $\mathbf{u}=\mathbf{u}^{(\tau)}$ and $D_T^{(k)}(\mathbf{v})$ for some $\mathbf{v}=\mathbf{v}^{(\tau_0)}$ then there exists $\theta_{\mathbf{X}}$ if and only if there exists*

$$\nu_{\mathbf{X}} \;=\; \lim_{n\to\infty} n\,\frac{1}{T}\,\sum_{i=1}^{T} P\Big(X_i > v_n \geq M_{i+1,i+k-1}\Big)\ ,$$

and in this case it holds

$$\theta_{\mathbf{X}} = \frac{\nu_{\mathbf{X}}}{\tau_0}\ . \qquad \square$$

We can apply this result to calculate the extremal index of a T-periodic moving average, following the approach of Chernick et al. ([2]) for the stationary case.

Let $\mathbf{Z} = \{Z_n\}_{n\geq 1}$ be a T-periodic sequence of independent variables with regularly varying equivalent tails with exponent $-\alpha$ satisfying

$$\lim_{x\to\infty} \frac{P(Z_i > x)}{P(Z_j > x)} = \gamma_{i,j}^{(+)} > 0, \quad \lim_{x\to\infty} \frac{P(Z_i < -x)}{P(Z_j < -x)} = \gamma_{i,j}^{(-)} > 0\ , \qquad i,j = 1,...,T\ ,$$

and

$$\lim_{x\to\infty} \frac{P(Z_i > x)}{P(|Z_i| > x)} \;=\; p_i \,\in\, [0,1]\ , \qquad i = 1,...,T\ .$$

For $\tau_i > 0$, $i=1,...,T$, and $\tau = \frac{1}{T}\sum_{i=1}^{T}\tau_i$, let $\mathbf{u}^{(\tau)}$ be defined by

$$\lim_{n\to\infty} nP(|Z_i| > u_n) = \tau_i \Bigg/ \left\{ p_i \sum_{s=0}^{T-1} \gamma_{i-s,i}^{(+)} \sum_{j=-\infty}^{\infty} [c_{jT+s}^{+}]^{\alpha} + q_i \sum_{s=0}^{T-1} \gamma_{i-s,i}^{(-)} \sum_{j=-\infty}^{\infty} [c_{jT+s}^{-}]^{\alpha} \right\},$$

where $q_i = 1 - p_i$, $c_j^{+} = \max\{c_j, 0\}$, $c_j^{-} = \max\{-c_j, 0\}$ and $\mathbf{c} = \{c_j\}$ is a sequence of constants such that $\sum_{j=-\infty}^{+\infty} |c_j|^{\delta} < +\infty$ for some $\delta < \min\{\alpha, 1\}$.

For the T-periodic moving average $X_n = \sum_{j=-\infty}^{+\infty} c_j Z_{n-j}$, $n \geq 1$, by applying our result to the $2m$-dependent T-periodic sequence $X_n^{(m)} = \sum_{j=-m}^{m} c_j Z_{n-j}$ and following in a straighforward way the reasoning of Chernick et al. ([2]), we find

$$\theta \;=\; \frac{\displaystyle\sum_{i=1}^{T} \gamma_{i,1} \left\{ p_i \sum_{s=0}^{T-1} \gamma_{i-s,i}^{(+)}\ c_s^{+}(\alpha) \;+\; q_i \sum_{s=0}^{T-1} \gamma_{i-s,i}^{(-)}\ c_s^{-}(\alpha) \right\}}{\displaystyle\sum_{i=1}^{T} \gamma_{i,1} \left\{ p_i \sum_{s=0}^{T-1} \gamma_{i-s,i}^{(+)} \sum_{j=-\infty}^{\infty} [c_{jT+s}^{+}]^{\alpha} \;+\; q_i \sum_{s=0}^{T-1} \gamma_{i-s,i}^{(-)} \sum_{j=-\infty}^{\infty} [c_{jT+s}^{-}]^{\alpha} \right\}}\ ,$$

where

$$c_s^{+}(\alpha) = \sum_{j=-\infty}^{\infty} \Big([c_{jT+s}^{+}]^{\alpha} - \max_{r > jT+s} \{c_r^{+}\}^{\alpha} \Big)^{+}\ , \quad c_s^{-}(\alpha) = \sum_{j=-\infty}^{\infty} \Big([c_{jT+s}^{-}]^{\alpha} - \max_{r > jT+s} \{c_r^{-}\}^{\alpha} \Big)^{+}\ .$$

For details on the proofs of this example see Martins and Ferreira ([8]).

3. PERIODIC SUB-SAMPLED SEQUENCE

We first set sufficient conditions for the previous results to hold for $\mathbf{Y} = \{X_{g(n)}\}_{n\geq 1}$. Let $g\colon \mathbb{N}\to\mathbb{N}$ be a strictly increasing function for which there exists positive integers I_1 and I_2 such that, $\forall\, n,k \in \mathbb{N}$, it holds $g(n+kI_1) = g(n)+kI_2$. We will refer such g as an I_1, I_2-periodic function and suppose that I_1 and I_2 are the smallest integers satisfying the definition.

Therefore $\mathbf{Y}=\{X_{g(n)}\}_{n\geq 1}$ is obtained from $\mathbf{X}$ by sub-sampling blocks of I_1 variables separated by $J = I_2 - (g(I_1)-g(1)) - 1 \geq 1$ variables.

In a particular case considered in Scotto and Ferreira ([10]), $\mathbf{X}$ is a stationary moving average with heavy-tailed innovations and g generates blocks of I_1 consecutive observations separated by $J \geq 1$ observations.

Proposition 3.1. *If $\mathbf{X}$ is a T-periodic sequence and g is an I_1, I_2-periodic function with I_2 a multiple of T, then $\mathbf{Y} = \{X_{g(n)}\}$ is an I_1-periodic sequence.*

Proof: For each choice of integers $1 \leq i_1 < ... < i_n,\ p \geq 1$, we have

$$(Y_{i_1+I_1}, ..., Y_{i_n+I_1}) \;=\; (X_{g(i_1+I_1)}, ..., X_{g(i_n+I_1)}) \;=$$
$$= (X_{g(i_1)+I_2}, ..., X_{g(i_n)+I_2}) \overset{d}{=} (X_{g(i_1)}, ..., X_{g(i_n)}) \;=\; (Y_{i_1}, ..., Y_{i_n}) \;. \quad \square$$

In the next result, we denote a sequence $\mathbf{u}$ such that $\lim_{n\to\infty} nP(X_i > u_n^{(\tau_i)}) = \tau_i$ by $\mathbf{u} = \mathbf{u}^{(\tau_i, X_i)}$. From the definition of normalized levels and $\mathbf{Y} \subset \mathbf{X}$ we give a simple procedure to get $\mathbf{v}=\mathbf{v}^{(\tau,\mathbf{Y})}$ with $\tau = \frac{1}{I_1}\sum_{i=1}^{I_1} G^{-1}\tau_{g(i)}$ and $G = \lim_{n\to\infty}\dfrac{g(n)}{n}$.

Proposition 3.2. *Let $\mathbf{X}$ be a T-periodic sequence and g an I_1, I_2-periodic function with I_2 a multiple of T. If $\lim_{n\to\infty}\dfrac{g(n)}{n} = G$ and $\mathbf{u} = \mathbf{u}^{(\tau_i, X_i)}$, $i = 1, ..., T$, then $\mathbf{v} = \{u_{g(n)}\}$ satisfies:*

(i) $\mathbf{v} = \mathbf{v}^{(G^{-1}\tau_i, X_i)},\ i = 1, ..., T.$

(ii) $\mathbf{v} = \mathbf{v}^{(G^{-1}\tau_{g(i)}, Y_i)},\ i = 1, ..., I_1,$ *and* $\{\tau_{g(1)}, ..., \tau_{g(I_1)}\} \subset \{\tau_1, ..., \tau_T\}.$ $\square$

For $\mathbf{u} = \mathbf{u}^{(\tau_i', X_i)}$, with $\tau_i' = G\tau_i$, $i=1, ..., T$, we have $\mathbf{v} = \{u_{g(n)}\} = \mathbf{v}^{(\tau_i, Y_i)}$ and we can easily get $\alpha^{(\mathbf{Y},\mathbf{v})}_{n, l^{(\mathbf{X})}_{g(n)}} \leq \alpha^{(\mathbf{X},\mathbf{u})}_{g(n), l^{(\mathbf{X})}_{g(n)}}$ with $l^{(\mathbf{X})}_{g(n)} = o(n)$.

Moreover, if $\mathbf{v} = \mathbf{v}^{(\tau_{0,i}, X_i)}$, $i=1, ..., T$, then $\mathbf{w} = \{v_{[nI_2/I_1]}\}$ satisfies

$$\mathbf{w} = \mathbf{w}^{(\tau_{0,i} I_1/I_2, X_i)}\;, \qquad i = 1, ..., T\;,$$
$$\mathbf{w} = \mathbf{w}^{(\tau_{0,g(i)} I_1/I_2, Y_i)}\;, \qquad i = 1, ..., I_1$$

and
$$S^{(k,\mathbf{Y},\mathbf{w})}_{[\frac{n}{k_n I_1}]} \leq A\, S^{(k,\mathbf{X},\mathbf{w})}_{[\frac{n}{k'_n T}]}\ ,$$
where A is a constant and $k'_n = k_{[nI_1/I_2]}$.

These are the main arguments to obtain the following result.

Proposition 3.3. *Let* $\mathbf{X}$ *be a* T*-periodic sequence* $\mathbf{X}$ *satisfying* $D(\mathbf{u})$ *for all* $\mathbf{u} = \mathbf{u}^{(\tau_i, X_i)}$ *for some* $i \in \{1,...,T\}$ *and* $D^{(k)}_T(\mathbf{v})$ *for some* $\mathbf{v} = \mathbf{v}^{(\tau_{0,i}, X_i)}$, $i = 1,...,T$, *with* $\mathbf{k}' = \{k_{[nI_1/I_2]}\}$ *and* $\mathbf{k} = \{k_n\}$ *as in (2.1). Then, for* g *as in the above proposition,* $\mathbf{Y} = \{X_{g(n)}\}$ *satisfies:*

(i) $D(\mathbf{u})$ *for all* $\mathbf{u} = \mathbf{u}^{(\tau_i, Y_i)}$, $i = 1,...,I_1$,

(ii) $D^{(k)}_{I_1}(\mathbf{w})$ *for* $\mathbf{w} = \{v_{[nI_2/I_1]}\} = \mathbf{w}^{(\tau_{0,g(i)I_1/I_2}, Y_i)}$, $i = 1,...,I_1$, *with* $\mathbf{k} = \{k_n\}$. □

We will assume that $\mathbf{X}$ is in the conditions of Proposition 3.3 and calculate the extremal index of the periodic sub-sampled sequence $\mathbf{Y} = \{X_{g(n)}\}$ as a consequence of this proposition and Corollary 2.1.

Proposition 3.4. *Let* $\mathbf{X}$ *be a* T*-periodic sequence* $\mathbf{X}$ *satisfying* $D(\mathbf{u})$ *for all* $\mathbf{u} = \mathbf{u}^{(\tau_i, X_i)}$ *for some* $i \in \{1,...,T\}$ *and* $D^{(k)}_T(\mathbf{v})$ *for some* $\mathbf{v} = \mathbf{v}^{(\tau_{0,i}, X_i)}$, $i = 1,...,T$, *with* $\mathbf{k}' = \{k_{[nI_1/I_2]}\}$ *and* $\mathbf{k} = \{k_n\}$ *as in (2.1). Then, for* g *as in the above proposition,* $\mathbf{Y} = \{X_{g(n)}\}$ *has extremal index* $\theta_{\mathbf{Y}}$ *if and only if there exists*

$$\nu_{\mathbf{Y}} = \lim_{n\to\infty} n\, \frac{1}{I_1} \sum_{i=1}^{I_1} P\Big(X_{g(i)} > v_{[nI_2/I_1]} \geq \max\Big\{X_{g(i+1)}, X_{g(i+2)}, ..., X_{g(i+k-1)}\Big\}\Big)\ .$$

In this case
$$\theta_{\mathbf{Y}} = \frac{I_1\, \nu_{\mathbf{Y}}}{\sum\limits_{i=1}^{I_1} \tau_{0,g(i)}}\ .$$
□

Let
$$\nu_{\mathbf{X}} = \lim_{n\to\infty} n\, \frac{1}{T} \sum_{i=1}^{T} P\Big(X_i > v_n \geq M^{(\mathbf{X})}_{i+1,i+k-1}\Big)\ ,$$
and $\theta_{\mathbf{X}} = \frac{\nu_{\mathbf{X}}}{\tau_0}$, with $\tau_0 = \frac{1}{T}\sum_{i=1}^{T} \tau_{0,i}$.

For the particular case of $I_1 = T$ and $g(i{+}1) = g(i)$, for $i = 1,...,I_1$, we find $\theta_{\mathbf{Y}} = \theta_{\mathbf{X}} + \frac{\rho}{T\tau_0}$ where

$$\rho = \lim_{n\to\infty} n\, P\Big(X_{g(I_1)} > v_{[nI_2/I_1]} \geq \max\Big\{X_{g(1)+I_2}, X_{g(2)+I_2}, ..., X_{g(k-1)+I_2}\Big\}\Big) - \lim_{n\to\infty} n\, P\Big(X_{g(I_1)} > v_{[nI_2/I_1]} \geq M^{(\mathbf{X})}_{g(I_1)+1, g(I_1)+k-1}\Big)\ .$$

If $k=1$ then $\rho=0$, as expected, and for the particular cases where $1=T=I_1$ and $k=2$ we have very simple expressions for ρ (Martins and Ferreira ([7])). They can be applied, for instance, to calculate the extremal index of the sub-sampled ARMAX(α) process considered in Robinson and Tawn ([9]). For that example we find

$$\theta_{\mathbf{Y}} \;=\; \theta_{\mathbf{X}} + \frac{\rho}{\tau_0} \;=\; 1-\alpha + \frac{\alpha\,(1-\alpha^{I_2-1})\,\tau_0}{\tau_0} \;=\; 1-\alpha^{I_2}\;,$$

equal to the value of Robinson and Tawn ([9]) for the sampling case $\mathbf{Y}=\{X_{nI_2}\}$.

4. CONCLUDING REMARKS

Under the local dependence condition $D_T^{(k)}(\mathbf{u}^{(\tau)})$ we compute the extremal index of the T-periodic sequence $\mathbf{X}$ from the T distributions of k consecutive variables as well as the extremal index of some sub-sampled I_1-periodic sequences $\mathbf{Y}=\{X_{g(n)}\}$.

It would be interesting to apply these results to functions g used in applications and moving averages or Markov sequences $\mathbf{X}$ where $D''(u_n)$ fails. This remains as topic of future research.

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