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# GENERALIZED ESTIMATORS OF STATIONARY RANDOM-COEFFICIENTS PANEL DATA MODELS: ASYMPTOTIC AND SMALL SAMPLE PROPERTIES

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Abstract:

- This article provides generalized estimators for the random-coefficients panel data (RCPD) model where the errors are cross-sectional heteroskedastic and contemporaneously correlated as well as with the first-order autocorrelation of the time series errors. Of course, under the new assumptions of the error, the conventional estimators are not suitable for RCPD model. Therefore, the suitable estimator for this model and other alternative estimators have been provided and examined in this article. Furthermore, the efficiency comparisons for these estimators have been carried out in small samples and also we examine the asymptotic distributions of them. The Monte Carlo simulation study indicates that the new estimators are more efficient than the conventional estimators, especially in small samples.

Key-Words:

- *classical pooling estimation; contemporaneous covariance; first-order autocorrelation; heteroskedasticity; mean group estimation; random coefficient regression.*

AMS Subject Classification:

- 91G70, 97K80.

## 1. INTRODUCTION

The econometrics literature reveals a type of data called "panel data", which refers to the pooling of observations on a cross-section of households, countries, and firms over several time periods. Pooling this data achieves a deep analysis of the data and gives a richer source of variation which allows for more efficient estimation of the parameters. With additional, more informative data, one can get more reliable estimates and test more sophisticated behavioral models with less restrictive assumptions. Also, panel data sets are more effective in identifying and estimating effects that are simply not detectable in pure cross-sectional or pure time series data. In particular, panel data sets are more effective in studying complex issues of dynamic behavior. Some of the benefits and limitations of using panel data sets are listed in Baltagi (2013) and Hsiao (2014).

The pooled least squares (classical pooling) estimator for pooled cross-sectional and time series data (panel data) models is the best linear unbiased estimator (BLUE) under the classical assumptions as in the general linear regression model.[1] An important assumption for panel data models is that the individuals in our database are drawn from a population with a common regression coefficient vector. In other words, the coefficients of a panel data model must be fixed. In fact, this assumption is not satisfied in most economic models, see, e.g., Livingston *et al.* (2010) and Alcacer *et al.* (2013). In this article, the panel data models are studied when this assumption is relaxed. In this case, the model is called "random-coefficients panel data (RCPD) model". The RCPD model has been examined by Swamy in several publications (Swamy 1970, 1973, and 1974), Rao (1982), Dielman (1992a, b), Beck and Katz (2007), Youssef and Abonazel (2009), and Mousa *et al.* (2011). Some statistical and econometric publications refer to this model as Swamy's model or as the random coefficient regression (RCR) model, see, e.g., Poi (2003), Abonazel (2009), and Elhorst (2014, ch.3). In RCR model, Swamy assumes that the individuals in our panel data are drawn from a population with a common regression parameter, which is a fixed component, and a random component, that will allow the coefficients to differ from unit to unit. This model has been developed by many researchers, see, e.g., Beran and Millar (1994), Chelliah (1998), Anh and Chelliah (1999), Murtazashvili and Wooldridge (2008), Cheng *et al.* (2013), Fu and Fu (2015), Elster and Wübbeler (2017), and Horváth and Trapani (2016).

The random-coefficients models have been applied in different fields and they constitute a unifying setup for many statistical problems. Moreover, several applications of Swamy's model have appeared in the literature of finance and

[1]Dielman (1983, 1989) discussed these assumptions. In the next section in this article, we will discuss different types of classical pooling estimators under different assumptions.

economics.[2] Boot and Frankfurter (1972) used the RCR model to examine the optimal mix of short and long-term debt for firms. Feige and Swamy (1974) applied this model to estimate demand equations for liquid assets, while Boness and Frankfurter (1977) used it to examine the concept of risk-classes in finance. Recently, Westerlund and Narayan (2015) used the random-coefficients approach to predictăthe stock returns at the New York Stock Exchange. Swamy *et al.* (2015) applied a random-coefficient framework to deal with two problems frequently encountered in applied work; these problems are correcting for misspecifications in a small area level model and resolving Simpson's paradox.

Dziechciarz (1989) and Hsiao and Pesaran (2008) classified the random-coefficients models into two categories (stationary and non-stationary models), depending on the type of assumption about the coefficient variation. Stationary random-coefficients models regard the coefficients as having constant means and variance-covariances, like Swamy's (1970) model. On the other hand, the coefficients in non-stationary random-coefficients models do not have a constant mean and/or variance and can vary systematically; these models are relevant mainly for modeling the systematic structural variation in time, like the Cooley–Prescott (1973) model.[3]

The main objective of this article is to provide the researchers with general and more efficient estimators for the stationary RCPD models. To achieve this objective, we propose and examine alternative estimators of these models under an assumption that the errors are cross-sectional heteroskedastic and contemporaneously correlated as well as with the first-order autocorrelation of the time series errors.

The rest of the article is organized as follows. Section 2 presents the classical pooling (CP) estimators of fixed-coefficients models. Section 3 provides generalized least squares (GLS) estimators of the different random-coefficients models. In section 4, we examine the efficiency of these estimators, theoretically. In section 5, we discuss alternative estimators for these models. The Monte Carlo comparisons between various estimators have been carried out in section 6. Finally, section 7 offers the concluding remarks.

[2]The RCR model has been applied also in different sciences fields, see, e.g., Bodhlyera *et al.* (2014).

[3]Cooley and Prescott (1973) suggested a model where coefficients vary from one time period to another on the basis of a non-stationary process. Similar models have been considered by Sant (1977) and Rausser *et al.* (1982).

## 2. FIXED-COEFFICIENTS MODELS

Suppose the variable $y$ for the $i$th cross-sectional unit at time period $t$ is specified as a linear function of $K$ strictly exogenous variables, $x_{kit}$, in the following form:

$$y_{it} = \sum_{k=1}^{K} \alpha_{ki} x_{kit} + u_{it} = \mathrm{x}_{it}\alpha_i + u_{it}\,, \qquad i = 1, 2, ..., N; \quad t = 1, 2, ..., T\,, \tag{2.1}$$

where $u_{it}$ denotes the random error term, $x_{it}$ is a $1 \times K$ vector of exogenous variables, and $\alpha_i$ is the $K \times 1$ vector of coefficients. Stacking equation (2.1) over time, we obtain:

$$y_i = X_i \alpha_i + u_i\,, \tag{2.2}$$

where $y_i = (y_{i1},\ ...,\ y_{iT})'$, $X_i = (\mathrm{x}'_{i1}, ...,\ \mathrm{x}'_{iT})'$, $\alpha_i = (\alpha_{i1},\ ..., \alpha_{iK})'$, and $u_i = (u_{i1},\ ..., u_{iT})'$.

When the performance of one individual from the database is of interest, separate equation regressions can be estimated for each individual unit using the ordinary least squares (OLS) method. The OLS estimator of $\alpha_i$, is given by:

$$\hat{\alpha}_i = \left(X'_i X_i\right)^{-1} X'_i y_i\,. \tag{2.3}$$

Under the following assumptions, $\hat{\alpha}_i$ is a BLUE of $\alpha_i$:

**Assumption 1**: The errors have zero mean, i.e., $E(u_i) = 0; \forall\, i = 1, 2, ..., N$.

**Assumption 2**: The errors have the same variance for each individual:

$$E\left(u_i u'_j\right) = \begin{cases} \sigma_u^2 I_T & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \qquad i, j = 1, 2, ..., N\,.$$

**Assumption 3**: The exogenous variables are non-stochastic, i.e., fixed in repeated samples, and hence, not correlated with the errors. Also, $rank\,(X_i) = K < T;\ \forall\ i = 1, 2, ..., N$.

These conditions are sufficient but not necessary for the optimality of the OLS estimator.[4] When OLS is not optimal, estimation can still proceed equation by equation in many cases. For example, if variance of $u_i$ is not constant, the errors are either heteroskedastic and/or serially correlated, and the GLS method will provide relatively more efficient estimates than OLS, even if GLS was applied to each equation separately as in OLS.

[4] For more information about the optimality of the OLS estimators, see, e.g., Rao and Mitra (1971, ch. 8) and Srivastava and Giles (1987, pp. 17–21).

Another case, If the covariances between $u_i$ and $u_j$ $(i, j = 1, 2, ..., N)$ do not equal to zero as in assumption (2) above, then contemporaneous correlation is present, and we have what Zellner (1962) termed as seemingly unrelated regression (SUR) equations, where the equations are related through cross-equation correlation of errors. If the $X_i\,(i = 1, 2, ..., N)$ matrices do not span the same column space and contemporaneous correlation exists, a relatively more efficient estimator of $\alpha_i$ than equation by equation OLS is the GLS estimator applied to the entire equation system, as shown in Zellner (1962).

With either separate equation estimation or the SUR methodology, we obtain parameter estimates for each individual unit in the database. Now suppose it is necessary to summarize individual relationships and to draw inferences about certain population parameters. Alternatively, the process may be viewed as building a single model to describe the entire group of individuals rather than building a separate model for each. Again, assume that assumptions 1–3 are satisfied and add the following assumption:

**Assumption 4**: The individuals in the database are drawn from a population with a common regression parameter vector $\bar{\alpha}$, i.e., $\alpha_1 = \alpha_2 = \cdots = \alpha_N = \bar{\alpha}$.

Under this assumption, the observations for each individual can be pooled, and a single regression performed to obtain an efficient estimator of $\bar{\alpha}$. Now, the equation system is written as:

$$Y = X\bar{\alpha} + u\,, \tag{2.4}$$

where $Y = (y_1', ..., y_N')'$, $X = (X_1', ..., X_N')'$, $u = (u_1', ..., u_N')'$, and $\bar{\alpha} = (\bar{\alpha}_1, ..., \bar{\alpha}_K)'$ is a vector of fixed coefficients which to be estimated. We will differentiate between two cases to estimate $\bar{\alpha}$ in (2.4) based on the variance-covariance structure of $u$. In the first case, the errors have the same variance for each individual as given in assumption 2. In this case, the efficient and unbiased estimator of $\bar{\alpha}$ under assumptions 1–4 is:

$$\widehat{\bar{\alpha}}_{CP-OLS} = \left(X'X\right)^{-1} X'Y.$$

This estimator has been termed the classical pooling-ordinary least squares (CP-OLS) estimator. In the second case, which the errors have different variances along individuals and are contemporaneously correlated as in the SUR framework:

**Assumption 5**: $E\left(u_i u_j'\right) = \begin{cases} \sigma_{ii} I_T & \text{if } i = j \\ \sigma_{ij} I_T & \text{if } i \neq j \end{cases} \quad i, j = 1, 2, ..., N.$

Under assumptions 1, 3, 4 and 5, the efficient and unbiased CP estimator of $\bar{\alpha}$ is:

$$\widehat{\bar{\alpha}}_{CP-SUR} = \left[X'(\Sigma_{sur} \otimes I_T)^{-1}X\right]^{-1}\left[X'(\Sigma_{sur} \otimes I_T)^{-1}Y\right],$$

where

$$\Sigma_{sur} = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{N1} & \sigma_{N2} & \cdots & \sigma_{NN} \end{pmatrix}.$$

To make this estimator ($\widehat{\bar{\alpha}}_{CP-SUR}$) a feasible, the $\sigma_{\mathrm{ij}}$ can be replaced with the following unbiased and consistent estimator:

$$\hat{\sigma}_{ij} = \frac{\hat{u}_i'\hat{u}_j}{T-K}; \quad \forall\ i,j = 1,2,...,N, \tag{2.5}$$

where $\hat{u}_i = y_i - X_i\hat{\alpha}_i$ is the residuals vector obtained from applying OLS to equation number $i$.[5]

## 3. RANDOM-COEFFICIENTS MODELS

This section reviews the standard random-coefficients model proposed by Swamy (1970), and presents the random-coefficients model in the general case, where the errors are allowed to be cross-sectional heteroskedastic and contemporaneously correlated as well as with the first-order autocorrelation of the time series errors.

### 3.1. RCR model

Suppose that each regression coefficient in (2.2) is now viewed as a random variable; that is the coefficients, $\alpha_i$, are viewed as invariant over time, but varying from one unit to another:

**Assumption 6** (for the stationary random-coefficients approach): The coefficient vector $\alpha_i$ is specified as:[6] $\alpha_i = \bar{\alpha} + \mu_i$, where $\bar{\alpha}$ is a $K \times 1$ vector of

[5]The $\hat{\sigma}_{ij}$ in (2.5) are unbiased estimators because, as assumed, the number of exogenous variables of each equation is equal, i.e., $K_i = K$ for $i = 1,2,...,N$. However, in the general case, $K_i \neq K_j$, the unbiased estimator is $\hat{u}_i'\hat{u}_j / [T - K_i - K_j + tr\,(P_{xx})]$ , where $P_{xx} = X_i(X_i'X_i)^{-1}X_i'X_j\left(X_j'X_j\right)^{-1}X_j'$. See Srivastava and Giles (1987, pp. 13–17) and Baltagi (2011, pp. 243–244).

[6]This means that the individuals in our database are drown from a population with a common regression parameter $\bar{\alpha}$, which is a fixed component, and a random component $\mu_i$, allowed to differ from unit to unit.

constants, and $\mu_i$ is a $K \times 1$ vector of stationary random variables with zero means and constant variance-covariances:

$$E(\mu_i) = 0 \text{ and } E(\mu_i \mu_j') = \begin{cases} \Psi & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \quad i, j = 1, 2, ..., N,$$

where $\Psi = diag\{\psi_k^2\}$; for $k = 1, 2, ..., K$, where $K < N$. Furthermore, $E(\mu_i \mathrm{x}_{jt}) = 0$ and $E(\mu_i u_{jt}) = 0 \; \forall \; i$ and $j$.

Also, Swamy (1970) assumed that the errors have different variances along individuals:

$$\textbf{Assumption 7}: \; E\left(u_i u_j'\right) = \begin{cases} \sigma_{ii} I_T & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \quad i, j = 1, 2, ..., N.$$

Under the assumption 6, the model in equation (2.2) can be rewritten as:

$$Y = X\bar{\alpha} + e; \quad e = D\mu + u, \tag{3.1}$$

where $Y, X, u,$ and $\bar{\alpha}$ are defined in (2.4), while $\mu = (\mu_1', \; ..., \; \mu_N')'$, and $D = diag\{X_i\}$; for $i = 1, 2, ..., N$.

The model in (3.1), under assumptions 1, 3, 6 and 7, called the 'RCR model', which was examined by Swamy (1970, 1971, 1973, and 1974), Youssef and Abonazel (2009), and Mousa *et al.* (2011). We will refer to assumptions 1, 3, 6 and 7 as RCR assumptions. Under these assumptions, the BLUE of $\bar{\alpha}$ in equation (3.1) is:

$$\hat{\bar{\alpha}}_{RCR} = \left(X'\Omega^{-1}X\right)^{-1} X'\Omega^{-1}Y,$$

where $\Omega$ is the variance-covariance matrix of $e$:

$$\Omega = (\Sigma_{rcr} \otimes I_T) + D(I_N \otimes \Psi) D',$$

where $\Sigma_{rcr} = diag\{\sigma_{ii}\}$; for $i = 1, 2, ..., N$. Swamy (1970) showed that the $\hat{\bar{\alpha}}_{RCR}$ estimator can be rewritten as:

$$\hat{\bar{\alpha}}_{RCR} = \left[\sum_{i=1}^{N} X_i'\left(X_i \Psi X_i' + \sigma_{ii} I_T\right)^{-1} X_i\right]^{-1} \sum_{i=1}^{N} X_i'\left(X_i \Psi X_i' + \sigma_{ii} I_T\right)^{-1} y_i.$$

The variance-covariance matrix of $\hat{\bar{\alpha}}_{RCR}$ under RCR assumptions is:

$$var\left(\hat{\bar{\alpha}}_{RCR}\right) = \left(X'\Omega^{-1}X\right)^{-1} = \left\{\sum_{i=1}^{N} \left[\Psi + \sigma_{ii}\left(X_i'X_i\right)^{-1}\right]^{-1}\right\}^{-1}.$$

To make the $\hat{\bar{\alpha}}_{RCR}$ estimator feasible, Swamy (1971) suggested using the estimator in (2.5) as an unbiased and consistent estimator of $\sigma_{ii}$, and the following unbiased estimator for $\Psi$:

$$\hat{\Psi} = \left[\frac{1}{N-1}\left(\sum_{i=1}^{N} \hat{\alpha}_i \, \hat{\alpha}_i' - \frac{1}{N}\sum_{i=1}^{N} \hat{\alpha}_i \, \sum_{i=1}^{N} \hat{\alpha}_i'\right)\right] - \left[\frac{1}{N}\sum_{i=1}^{N} \hat{\sigma}_{ii}\left(X_i'X_i\right)^{-1}\right]. \tag{3.2}$$

Swamy (1973, 1974) showed that the estimator $\widehat{\bar{\alpha}}_{RCR}$ is consistent as both $N, T \to \infty$ and is asymptotically efficient as $T \to \infty$.[7]

It is worth noting that, just as in the error-components model, the estimator (3.2) is not necessarily non-negative definite. Mousa *et al.* (2011) explained that it is possible to obtain negative estimates of Swamy's estimator in (3.2) in case of small samples and if some/all coefficients are fixed. But in medium and large samples, the negative variance estimates does not appear even if all coefficients are fixed. To solve this problem, Swamy has suggested replacing (3.2) by:[8]

$$\hat{\Psi}^{+} = \frac{1}{N-1}\left(\sum_{i=1}^{N} \hat{\alpha}_i\, \hat{\alpha}_i' - \frac{1}{N}\sum_{i=1}^{N} \hat{\alpha}_i\, \sum_{i=1}^{N} \hat{\alpha}_i'\right).$$

This estimator, although biased, is non-negative definite and consistent when $T \to \infty$. See Judge *et al.* (1985, p. 542).

## 3.2. Generalized RCR model

To generalize RCR model so that it would be more suitable for most economic models, we assume that the errors are cross-sectional heteroskedastic and contemporaneously correlated, as in assumption 5, as well as with the first-order autocorrelation of the time series errors. Therefore, we add the following assumption to assumption 5:

**Assumption 8**: $u_{it} = \rho_i u_{i,t-1} + \varepsilon_{it};\ |\rho_i| < 1,$ where $\rho_i\ (i = 1, 2, ..., N)$ are fixed first-order autocorrelation coefficients. Assume that: $E(\varepsilon_{it}) = 0$, $E(u_{i,t-1}\varepsilon_{jt}) = 0;\ \forall\, i$ and $j$, and

$$E\left(\varepsilon_i \varepsilon_j'\right) = \begin{cases} \sigma_{\varepsilon_{ii}} I_T & \text{if } i = j \\ \sigma_{\varepsilon_{ij}} I_T & \text{if } i \neq j \end{cases} \qquad i, j = 1, 2, ..., N.$$

This means that the initial time period of the errors have the same properties as in subsequent periods, i.e., $E\left(u_{i0}^2\right) = \sigma_{\varepsilon_{ii}} / \left(1 - \rho_i^2\right)$ and $E\left(u_{i0} u_{j0}\right) = \sigma_{\varepsilon_{ij}} / \left(1 - \rho_i \rho_j\right)\ \forall\, i$ and $j$.

We will refer to assumptions 1, 3, 5, 6, and 8 as the general RCR assumptions. Under these assumptions, the BLUE of $\bar{\alpha}$ is:

$$\widehat{\bar{\alpha}}_{GRCR} = \left(X' \Omega^{*-1} X\right)^{-1} X' \Omega^{*-1} Y,$$

[7] The statistical properties of $\widehat{\bar{\alpha}}_{RCR}$ have been examined by Swamy (1971), of course, under RCR assumptions.

[8] This suggestion has been used by Stata program, specifically in xtrchh and xtrchh2 Stata's commands. See Poi (2003).

where

$$\Omega^* = \begin{pmatrix} X_1\Psi X_1' + \sigma_{\varepsilon_{11}}\omega_{11} & \sigma_{\varepsilon_{12}}\omega_{12} & \cdots & \sigma_{\varepsilon_{1N}}\omega_{1N} \\ \sigma_{\varepsilon_{21}}\omega_{21} & X_2\Psi X_2' + \sigma_{\varepsilon_{22}}\omega_{22} & \cdots & \sigma_{\varepsilon_{2N}}\omega_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_{\varepsilon_{N1}}\omega_{N1} & \sigma_{\varepsilon_{N2}}\omega_{N2} & \cdots & X_N\Psi X_N' + \sigma_{\varepsilon_{NN}}\omega_{NN} \end{pmatrix}, \tag{3.3}$$

with

$$\omega_{ij} = \frac{1}{1-\rho_i\rho_j}\begin{pmatrix} 1 & \rho_i & \rho_i^2 & \cdots & \rho_i^{T-1} \\ \rho_j & 1 & \rho_i & \cdots & \rho_i^{T-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_j^{T-1} & \rho_j^{T-2} & \rho_j^{T-3} & \cdots & 1 \end{pmatrix}. \tag{3.4}$$

Since the elements of $\Omega^*$ are usually unknown, we develop a feasible Aitken estimator of $\bar{\alpha}$ based on consistent estimators of the elements of $\Omega^*$:

$$\hat{\rho}_i = \frac{\sum_{t=2}^{T}\hat{u}_{it}\hat{u}_{i,t-1}}{\sum_{t=2}^{T}\hat{u}_{i,t-1}^2}, \tag{3.5}$$

where $\hat{u}_i = (\hat{u}_{i1}, \ldots, \hat{u}_{iT})'$ is the residuals vector obtained from applying OLS to equation number $i$,

$$\hat{\sigma}_{\varepsilon_{ij}} = \frac{\hat{\varepsilon}_i'\hat{\varepsilon}_j}{T-K},$$

where $\hat{\varepsilon}_i = (\hat{\varepsilon}_{i1}, \ldots, \hat{\varepsilon}_{iT})'$; $\hat{\varepsilon}_{i1} = \hat{u}_{i1}\sqrt{1-\hat{\rho}_i^2}$ , and $\hat{\varepsilon}_{it} = \hat{u}_{it} - \hat{\rho}_i\hat{u}_{i,t-1}$ for $t = 2, \ldots, T$.

Replacing $\rho_i$ by $\hat{\rho}_i$ in (3.4), yields consistent estimators of $\omega_{ij}$, say $\hat{\omega}_{ij}$, which leads together with $\hat{\sigma}_{\varepsilon_{ij}}$ and $\hat{\omega}_{ij}$ to a consistent estimator of $\Psi$:[9]

$$\begin{aligned} \hat{\Psi}^* = {} & \tfrac{1}{N-1}\left(\sum_{i=1}^{N}\hat{\alpha}_i^*\hat{\alpha}_i^{*\prime} - \tfrac{1}{N}\sum_{i=1}^{N}\hat{\alpha}_i^*\sum_{i=1}^{N}\hat{\alpha}_i^{*\prime}\right) - \tfrac{1}{N}\sum_{i=1}^{N}\hat{\sigma}_{\varepsilon_{ii}}\left(X_i'\hat{\omega}_{ii}^{-1}X_i\right)^{-1} \\ & + \left[\tfrac{1}{N(N-1)}\sum_{\substack{i \neq j \\ i,j=1}}^{N}\hat{\sigma}_{\varepsilon_{ij}}\left(X_i'\hat{\omega}_{ii}^{-1}X_i\right)^{-1} X_i'\hat{\omega}_{ii}^{-1}\hat{\omega}_{ij}\hat{\omega}_{jj}^{-1}X_j\left(X_j'\hat{\omega}_{jj}^{-1}X_j\right)^{-1}\right], \end{aligned} \tag{3.6}$$

where

$$\hat{\alpha}_i^* = \left(X_i'\hat{\omega}_{ii}^{-1}X_i\right)^{-1}X_i'\hat{\omega}_{ii}^{-1}y_i. \tag{3.7}$$

By using the consistent estimators ($\hat{\sigma}_{\varepsilon_{ij}}$, $\hat{\omega}_{ij}$, and $\hat{\Psi}^*$) in (3.3), and proceed a consistent estimator of $\Omega^*$ is obtained, say $\hat{\Omega}^*$, that leads to get the generalized

[9]The estimator of $\rho_i$ in (3.5) is consistent, but it is not unbiased. See Srivastava and Giles (1987, p. 211) for other suitable consistent estimators of $\rho_i$ that are often used in practice.

RCR (GRCR) estimator of $\bar{\alpha}$:

$$\widehat{\bar{\alpha}}_{GRCR} = \left(X'\hat{\Omega}^{*-1}X\right)^{-1} X'\hat{\Omega}^{*-1}Y.$$

The estimated variance-covariance matrix of $\widehat{\bar{\alpha}}_{GRCR}$ is:

$$\widehat{var}\left(\widehat{\bar{\alpha}}_{GRCR}\right) = \left(X'\hat{\Omega}^{*-1}X\right)^{-1}. \tag{3.8}$$

## 4. EFFICIENCY GAINS

In this section, we examine the efficiency gains from the use of GRCR estimator. Under the general RCR assumptions, It is easy to verify that the classical pooling estimators ($\widehat{\bar{\alpha}}_{CP-OLS}$ and $\widehat{\bar{\alpha}}_{CP-SUR}$) and Swamy's estimator ($\widehat{\bar{\alpha}}_{RCR}$) are unbiased for $\bar{\alpha}$ and with variance-covariance matrices:

$$\begin{aligned} var\left(\widehat{\bar{\alpha}}_{CP-OLS}\right) &= G_1\Omega^* G_1'; \\ var\left(\widehat{\bar{\alpha}}_{CP-SUR}\right) &= G_2\Omega^* G_2'; \\ var\left(\widehat{\bar{\alpha}}_{RCR}\right) &= G_3\Omega^* G_3', \end{aligned}$$

where

$$\begin{aligned} G_1 &= \left(X'X\right)^{-1}X'; \\ G_2 &= \left[X'\left(\Sigma_{sur}^{-1}\otimes I_T\right)X\right]^{-1}X'\left(\Sigma_{sur}^{-1}\otimes I_T\right); \\ G_3 &= \left(X'\Omega^{-1}X\right)^{-1}X'\Omega^{-1}. \end{aligned} \tag{4.1}$$

The efficiency gains, from the use of GRCR estimator, can be summarized in the following equation:

$$EG_\gamma = var\left(\widehat{\bar{\alpha}}_\gamma\right) - var\left(\widehat{\bar{\alpha}}_{GRCR}\right) = \left(G_h - G_0\right)\Omega^*(G_h - G_0)'; \text{ for } h = 1,2,3,$$

where the subscript $\gamma$ indicates the estimator that is used (CP-OLS, CP-SUR, or RCR), $G_0 = \left(X'\Omega^{*-1}X\right)^{-1}X'\Omega^{*-1}$, and $G_h$ (for $h = 1,2,3$) matrices are defined in (4.1).

Since $\Omega^*$, $\Sigma_{rcr}$, $\Sigma_{sur}$ and $\Omega$ are positive definite matrices, then $EG_\gamma$ matrices are positive semi-definite matrices. In other words, the GRCR estimator is more efficient than CP-OLS, CP-SUR, and RCR estimators. These efficiency gains increase when $|\rho_i|$, $\sigma_{\varepsilon_{ij}}$, and $\psi_k^2$ increase. However, it is not clear to what extent these efficiency gains hold in small samples. Therefore, this will be examined in a simulation study.

## 5. ALTERNATIVE ESTIMATORS

A consistent estimator of $\bar{\alpha}$ can also be obtained under more general assumptions concerning $\alpha_i$ and the regressors. One such possible estimator is the mean group (MG) estimator, proposed by Pesaran and Smith (1995) for estimation of dynamic panel data (DPD) models with random coefficients.[10] The MG estimator is defined as the simple average of the OLS estimators:

$$\widehat{\bar{\alpha}}_{MG} = \frac{1}{N}\sum_{i=1}^{N}\hat{\alpha}_i. \tag{5.1}$$

Even though the MG estimator has been used in DPD models with random coefficients, it will be used here as one of alternative estimators of static panel data models with random coefficients. Note that the simple MG estimator in (5.1) is more suitable for the RCR Model. But to make it suitable for the GRCR model, we suggest a general mean group (GMG) estimator as:

$$\widehat{\bar{\alpha}}_{GMG} = \frac{1}{N}\sum_{i=1}^{N}\hat{\alpha}_i^*, \tag{5.2}$$

where $\hat{\alpha}_i^*$ is defined in (3.7).

**Lemma 5.1.** *If the general RCR assumptions are satisfied, then $\widehat{\bar{\alpha}}_{MG}$ and $\widehat{\bar{\alpha}}_{GMG}$ are unbiased estimators of $\bar{\alpha}$, with the estimated variance-covariance matrices of $\widehat{\bar{\alpha}}_{MG}$ and $\widehat{\bar{\alpha}}_{GMG}$ are:*

$$\begin{aligned}\widehat{var}\left(\widehat{\bar{\alpha}}_{MG}\right) = {} & \frac{1}{N}\hat{\Psi}^* + \frac{1}{N^2}\sum_{i=1}^{N}\hat{\sigma}_{\varepsilon_{ii}}\left(X_i'X_i\right)^{-1}X_i'\hat{\omega}_{ii}X_i\left(X_i'X_i\right)^{-1} \\ & + \frac{1}{N^2}\sum_{\substack{i\neq j\\ i,j=1}}^{N}\hat{\sigma}_{\varepsilon_{ij}}\left(X_i'X_i\right)^{-1}X_i'\hat{\omega}_{ij}X_j\left(X_j'X_j\right)^{-1},\end{aligned} \tag{5.3}$$

$$\widehat{var}\left(\widehat{\bar{\alpha}}_{GMG}\right) = \frac{1}{N(N-1)}\begin{bmatrix}\sum_{i=1}^{N}\hat{\alpha}_i^*\hat{\alpha}_i^{*\prime} - \frac{1}{N}\sum_{i=1}^{N}\hat{\alpha}_i^*\sum_{i=1}^{N}\hat{\alpha}_i^{*\prime} \\ + \sum_{\substack{i\neq j\\ i,j=1}}^{N}\hat{\sigma}_{\varepsilon_{ij}}\left(X_i'\hat{\omega}_{ii}^{-1}X_i\right)^{-1} \\ X_i'\hat{\omega}_{ii}^{-1}\hat{\omega}_{ij}\hat{\omega}_{jj}^{-1}X_j\left(X_j'\hat{\omega}_{jj}^{-1}X_j\right)^{-1}\end{bmatrix}. \tag{5.4}$$

[10] For more information about the estimation methods for DPD models, see, e.g., Baltagi (2013), Abonazel (2014, 2017), Youssef *et al.* (2014a,b), and Youssef and Abonazel (2017).

**Proof of Lemma 5.1:**

**A. Unbiasedness property of MG and GMG estimators:**

**Proof:** By substituting (3.7) and (2.2) into (5.2):

$$\begin{aligned}\widehat{\bar{\alpha}}_{GMG} &= \frac{1}{N}\sum_{i=1}^{N}\left(X_i'\omega_{ii}^{-1}X_i\right)^{-1}X_i'\omega_{ii}^{-1}\left(X_i\alpha_i+u_i\right)\\ &= \frac{1}{N}\sum_{i=1}^{N}\alpha_i+\left(X_i'\omega_{ii}^{-1}X_i\right)^{-1}X_i'\omega_{ii}^{-1}u_i.\end{aligned} \tag{5.5}$$

Similarly, we can rewrite $\widehat{\bar{\alpha}}_{MG}$ in (5.1) as:

$$\widehat{\bar{\alpha}}_{MG} = \frac{1}{N}\sum_{i=1}^{N}\alpha_i+\left(X_i'X_i\right)^{-1}X_i'u_i. \tag{5.6}$$

Taking the expectation for (5.5) and (5.6), and using assumptions 1 and 6:

$$E\left(\widehat{\bar{\alpha}}_{GMG}\right) = E\left(\widehat{\bar{\alpha}}_{MG}\right) = \frac{1}{N}\sum_{i=1}^{N}\bar{\alpha} = \bar{\alpha}. \qquad \square$$

**B. Derive the variance-covariance matrix of GMG:**

**Proof:** Note first that under assumption 6, $\alpha_i = \bar{\alpha}+\mu_i$. Add $\hat{\alpha}_i^*$ to the both sides:

$$\alpha_i+\hat{\alpha}_i^* = \bar{\alpha}+\mu_i+\hat{\alpha}_i^*,$$

$$\hat{\alpha}_i^* = \bar{\alpha}+\mu_i+\hat{\alpha}_i^*-\alpha_i = \bar{\alpha}+\mu_i+\tau_i, \tag{5.7}$$

where $\tau_i = \hat{\alpha}_i^*-\alpha_i = \left(X_i'\omega_{ii}^{-1}X_i\right)^{-1}X_i'\omega_{ii}^{-1}u_i$. From (5.7):

$$\frac{1}{N}\sum_{i=1}^{N}\hat{\alpha}_i^* = \bar{\alpha}+\frac{1}{N}\sum_{i=1}^{N}\mu_i+\frac{1}{N}\sum_{i=1}^{N}\tau_i,$$

which means that

$$\widehat{\bar{\alpha}}_{GMG} = \bar{\alpha}+\bar{\mu}+\bar{\tau}, \tag{5.8}$$

where $\bar{\mu} = \frac{1}{N}\sum_{i=1}^{N}\mu_i$ and $\bar{\tau} = \frac{1}{N}\sum_{i=1}^{N}\tau_i$. From (5.8) and using the general RCR assumptions:

$$\begin{aligned} var\left(\widehat{\bar{\alpha}}_{GMG}\right) &= var\left(\bar{\mu}\right) + var\left(\bar{\tau}\right) \\ &= \frac{1}{N}\Psi + \frac{1}{N^2}\sum_{i=1}^{N}\sigma_{\varepsilon_{ii}}\left(X_i'\omega_{ii}^{-1}X_i\right)^{-1} \\ &+ \frac{1}{N^2}\sum_{\substack{i \neq j \\ i,j=1}}^{N}\sigma_{\varepsilon_{ij}}\left(X_i'\omega_{ii}^{-1}X_i\right)^{-1}X_i'\omega_{ii}^{-1}\omega_{ij}\omega_{jj}^{-1}X_j\left(X_j'\omega_{jj}^{-1}X_j\right)^{-1}. \end{aligned}$$

Using the consistent estimators of $\Psi$, $\sigma_{\varepsilon_{ij}}$, and $\omega_{ij}$ defined above, then we get the formula of $\widehat{var}\left(\widehat{\bar{\alpha}}_{GMG}\right)$ as in equation (5.4). □

**C. Derive the variance-covariance matrix of MG:**

**Proof:** As above, equation (2.3) can be rewritten as follows:

$$\hat{\alpha}_i = \bar{\alpha} + \mu_i + \lambda_i, \tag{5.9}$$

where $\lambda_i = \hat{\alpha}_i - \alpha_i = \left(X_i'X_i\right)^{-1}X_i'u_i$. From (5.9):

$$\frac{1}{N}\sum_{i=1}^{N}\hat{\alpha}_i = \bar{\alpha} + \frac{1}{N}\sum_{i=1}^{N}\mu_i + \frac{1}{N}\sum_{i=1}^{N}\lambda_i,$$

which means that

$$\widehat{\bar{\alpha}}_{MG} = \bar{\alpha} + \bar{\mu} + \bar{\lambda}, \tag{5.10}$$

where $\bar{\mu} = \frac{1}{N}\sum_{i=1}^{N}\mu_i$, and $\bar{\lambda} = \frac{1}{N}\sum_{i=1}^{N}\lambda_i$ . From (5.10) and using the general RCR assumptions:

$$\begin{aligned} var\left(\widehat{\bar{\alpha}}_{MG}\right) &= var\left(\bar{\mu}\right) + var\left(\bar{\lambda}\right) \\ &= \frac{1}{N}\Psi + \frac{1}{N^2}\sum_{i=1}^{N}\sigma_{\varepsilon_{ii}}\left(X_i'X_i\right)^{-1}X_i'\omega_{ii}X_i\left(X_i'X_i\right)^{-1} \\ &+ \frac{1}{N^2}\sum_{\substack{i \neq j \\ i,j=1}}^{N}\sigma_{\varepsilon_{ij}}\left(X_i'X_i\right)^{-1}X_i'\omega_{ij}X_j\left(X_j'X_j\right)^{-1}. \end{aligned}$$

As in the GMG estimator, and by using the consistent estimators of $\Psi$, $\sigma_{\varepsilon_{ij}}$, and $\omega_{ij}$, then we get the formula of $\widehat{var}\left(\widehat{\bar{\alpha}}_{GM}\right)$ as in equation (5.3). □

It is noted from lemma 1 that the variance of the GMG estimator is less than the variance of the MG estimator when the general RCR assumptions are satisfied. In other words, the GMG estimator is more efficient than the MG estimator. But under RCR assumptions, we have:

$$var\left(\widehat{\bar{\alpha}}_{MG}\right) = var\left(\widehat{\bar{\alpha}}_{GMG}\right) = \frac{1}{N\left(N-1\right)}\left(\sum_{i=1}^{N}\alpha_i\alpha_i' - \frac{1}{N}\sum_{i=1}^{N}\alpha_i\sum_{i=1}^{N}\alpha_i'\right) = \frac{1}{N}\Psi^{+}.$$

The next lemma explains the asymptotic variances (as $T \to \infty$ with $N$ fixed) properties of GRCR, RCR, GMG, and MG estimators. In order to justify the derivation of the asymptotic variances, we must assume the following:

**Assumption 9**: $\operatorname*{plim}_{T\to\infty} T^{-1}X_i'X_i$ and $\operatorname*{plim}_{T\to\infty} T^{-1}X_i'\omega_{ii}^{-1}X_i$ are finite and positive definite for all $i$ and for $|\rho_i| < 1$.

**Lemma 5.2.** *If the general RCR assumptions and assumption 9 are satisfied, then the estimated asymptotic variance-covariance matrices of GRCR, RCR, GMG, and MG estimators are equal:*

$$\begin{aligned}\operatorname*{plim}_{T\to\infty}\widehat{var}\left(\widehat{\bar{\alpha}}_{GRCR}\right) &= \operatorname*{plim}_{T\to\infty}\widehat{var}\left(\widehat{\bar{\alpha}}_{RCR}\right) = \operatorname*{plim}_{T\to\infty}\widehat{var}\left(\widehat{\bar{\alpha}}_{GMG}\right)\\ &= \operatorname*{plim}_{T\to\infty}\widehat{var}\left(\widehat{\bar{\alpha}}_{MG}\right) = N^{-1}\Psi^{+}.\end{aligned}$$

**Proof of Lemma 5.2:**

Following the same argument as in Parks (1967) and utilizing assumption 9, we can show that:

$$\begin{aligned}&\operatorname*{plim}_{T\to\infty}\hat{\alpha}_i = \operatorname*{plim}_{T\to\infty}\hat{\alpha}_i^{*} = \alpha_i,\ \ \operatorname*{plim}_{T\to\infty}\hat{\rho}_{ij} = \rho_{ij},\\ &\operatorname*{plim}_{T\to\infty}\hat{\sigma}_{\varepsilon_{ij}} = \sigma_{\varepsilon_{ij}}, \text{and} \operatorname*{plim}_{T\to\infty}\hat{\omega}_{ij} = \omega_{ij},\end{aligned} \tag{5.11}$$

and then

$$\begin{aligned}\operatorname*{plim}_{T\to\infty}\frac{1}{T}\hat{\sigma}_{\varepsilon_{ii}}T\left(X_i'\hat{\omega}_{ii}^{-1}X_i\right)^{-1} &= \operatorname*{plim}_{T\to\infty}\frac{1}{T}\hat{\sigma}_{\varepsilon_{ii}}T\left(X_i'X_i\right)^{-1}X_i'\hat{\omega}_{ii}X_i\left(X_i'X_i\right)^{-1}\\ &= \operatorname*{plim}_{T\to\infty}\frac{1}{T}\hat{\sigma}_{\varepsilon_{ij}}T\left(X_i'X_i\right)^{-1}X_i'\hat{\omega}_{ij}X_j\left(X_j'X_j\right)^{-1}\\ &= \operatorname*{plim}_{T\to\infty}\frac{1}{T}\hat{\sigma}_{\varepsilon_{ij}}T\left(X_i'\hat{\omega}_{ii}^{-1}X_i\right)^{-1}X_i'\hat{\omega}_{ii}^{-1}\hat{\omega}_{ij}\hat{\omega}_{jj}^{-1}X_j\\ &\quad\left(X_j'\hat{\omega}_{jj}^{-1}X_j\right)^{-1} = 0.\end{aligned} \tag{5.12}$$

Substituting (5.11) and (5.12) in (3.6):

$$\operatorname*{plim}_{T\to\infty}\hat{\Psi}^{*} = \frac{1}{N-1}\left(\sum_{i=1}^{N}\alpha_i\alpha_i' - \frac{1}{N}\sum_{i=1}^{N}\alpha_i\sum_{i=1}^{N}\alpha_i'\right) = \Psi^{+}. \tag{5.13}$$

By substituting (5.11)–(5.13) into (5.3), (5.4), and (3.8):

(5.14)

$$\begin{aligned}\underset{T\to\infty}{\text{plim}}\,\widehat{var}\left(\widehat{\bar{\alpha}}_{MG}\right) = {} & \frac{1}{N}\underset{T\to\infty}{\text{plim}}\,\hat{\Psi}^{*} \\ & + \frac{1}{N^2}\sum_{i=1}^{N}\underset{T\to\infty}{\text{plim}}\,\frac{1}{T}\hat{\sigma}_{\varepsilon_{ii}}T\left(X_i'X_i\right)^{-1}X_i'\hat{\omega}_{ii}X_i\left(X_i'X_i\right)^{-1} \\ & + \frac{1}{N^2}\sum_{\substack{i\neq j\\ i,j=1}}^{N}\underset{T\to\infty}{\text{plim}}\,\frac{1}{T}\hat{\sigma}_{\varepsilon_{ij}}T\left(X_i'X_i\right)^{-1}X_i'\hat{\omega}_{ij}X_j\left(X_j'X_j\right)^{-1} \\ = {} & \frac{1}{N}\Psi^{+},\end{aligned}$$

(5.15)

$$\begin{aligned}\underset{T\to\infty}{\text{plim}}\,\widehat{var}\left(\widehat{\bar{\alpha}}_{GMG}\right) = {} & \tfrac{1}{N(N-1)}\underset{T\to\infty}{\text{plim}}\left(\sum_{i=1}^{N}\hat{\alpha}_i^{*}\hat{\alpha}_i^{*\prime} - \tfrac{1}{N}\sum_{i=1}^{N}\hat{\alpha}_i^{*}\sum_{i=1}^{N}\hat{\alpha}_i^{*\prime}\right) \\ & + \tfrac{1}{N(N-1)}\sum_{\substack{i\neq j\\ i,j=1}}^{N}\left[\begin{array}{l}\underset{T\to\infty}{\text{plim}}\,\tfrac{1}{T}\hat{\sigma}_{\varepsilon_{ij}}T\left(X_i'\hat{\omega}_{ii}^{-1}X_i\right)^{-1} \\ X_i'\hat{\omega}_{ii}^{-1}\hat{\omega}_{ij}\hat{\omega}_{jj}^{-1}X_j\left(X_j'\hat{\omega}_{jj}^{-1}X_j\right)^{-1}\end{array}\right] = \tfrac{1}{N}\Psi^{+},\end{aligned}$$

$$\text{(5.16)}\qquad \underset{T\to\infty}{\text{plim}}\,\widehat{var}\left(\widehat{\bar{\alpha}}_{GRCR}\right) = \underset{T\to\infty}{\text{plim}}\left(X'\hat{\Omega}^{*-1}X\right)^{-1} = \left[\sum_{i=1}^{N}\Psi^{+-1}\right]^{-1} = \frac{1}{N}\Psi^{+}.$$

Similarly, we will use the results in (5.11)–(5.13) in case of RCR estimator:

(5.17)

$$\begin{aligned}\underset{T\to\infty}{\text{plim}}\,\widehat{var}\left(\widehat{\bar{\alpha}}_{RCR}\right) &= \underset{T\to\infty}{\text{plim}}\left[\left(X'\hat{\Omega}^{-1}X\right)^{-1}X'\hat{\Omega}^{-1}\hat{\Omega}^{*}\,\hat{\Omega}^{-1}X\left(X'\hat{\Omega}^{-1}X\right)^{-1}\right] \\ &= \frac{1}{N}\Psi^{+}.\end{aligned}$$

From (5.14)–(5.17), we can conclude that:

$$\begin{aligned}\underset{T\to\infty}{\text{plim}}\,\widehat{var}\left(\widehat{\bar{\alpha}}_{GRCR}\right) &= \underset{T\to\infty}{\text{plim}}\,\widehat{var}\left(\widehat{\bar{\alpha}}_{RCR}\right) \\ &= \underset{T\to\infty}{\text{plim}}\,\widehat{var}\left(\widehat{\bar{\alpha}}_{GMG}\right) = \underset{T\to\infty}{\text{plim}}\,\widehat{var}\left(\widehat{\bar{\alpha}}_{MG}\right) = \frac{1}{N}\Psi^{+}. \qquad \square\end{aligned}$$

From Lemma 5.2, we can conclude that the means and the variance-covariance matrices of the limiting distributions of $\widehat{\bar{\alpha}}_{GRCR}$, $\widehat{\bar{\alpha}}_{RCR}$, $\widehat{\bar{\alpha}}_{GMG}$, and $\widehat{\bar{\alpha}}_{MG}$ are the same and are equal to $\bar{\alpha}$ and $N^{-1}\Psi$ respectively even if the errors are correlated as in assumption 8. it is not expected to increase the asymptotic efficiency of $\widehat{\bar{\alpha}}_{GRCR}$, $\widehat{\bar{\alpha}}_{RCR}$, $\widehat{\bar{\alpha}}_{GMG}$, and $\widehat{\bar{\alpha}}_{MG}$. This does not mean that the GRCR estimator cannot be more efficient than RCR, GMG, and MG in small samples when the errors are correlated as in assumption 8. This will be examined in our simulation study.

## 6. MONTE CARLO SIMULATION

In this section, the Monte Carlo simulation has been used for making comparisons between the behavior of the classical pooling estimators (CP-OLS and CP-SUR), random-coefficients estimators (RCR and GRCR), and mean group estimators (MG and GMG) in small and moderate samples. The program to set up the Monte Carlo simulation, written in the R language, is available upon request. Monte Carlo experiments were carried out based on the following data generating process:

$$y_{it} = \sum_{k=1}^{3} \alpha_{ki} x_{kit} + u_{it}, \qquad i = 1, 2, ..., N;\ t = 1, 2, ..., T. \tag{6.1}$$

To perform the simulation under the general RCR assumptions, the model in (6.1) was generated as follows:

1. The independent variables, $(x_{kit}; k = 1,\ 2,\ 3)$, were generated as independent standard normally distributed random variables. The values of $x_{kit}$ were allowed to differ for each cross-sectional unit. However, once generated for all $N$ cross-sectional units the values were held fixed over all Monte Carlo trials.
2. The errors, $u_{it}$, were generated as in assumption 8: $u_{it} = \rho u_{i,t-1} + \varepsilon_{it}$, where the values of $\varepsilon_i = (\varepsilon_{i1},\ ..., \varepsilon_{iT})'\ \forall\ i = 1, 2, ..., N$ were generated as multivariate normally distributed with means zeros and variance-covariance matrix:
$$\begin{pmatrix} \sigma_{\varepsilon_{ii}} & \sigma_{\varepsilon_{ij}} & \cdots & \sigma_{\varepsilon_{ij}} \\ \sigma_{\varepsilon_{ij}} & \sigma_{\varepsilon_{ii}} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \sigma_{\varepsilon_{ij}} \\ \sigma_{\varepsilon_{ij}} & \cdots & \sigma_{\varepsilon_{ij}} & \sigma_{\varepsilon_{ii}} \end{pmatrix},$$
where the values of $\sigma_{\varepsilon_{ii}}$, $\sigma_{\varepsilon_{ij}}$, and $\rho$ were chosen to be: $\sigma_{\varepsilon_{ii}} = 1$ or 100; $\sigma_{\varepsilon_{ij}} = 0, 0.75$, or 0.95, and $\rho = 0, 0.55$, or 0.85, where the values of $\sigma_{\varepsilon_{ii}}$, $\sigma_{\varepsilon_{ij}}$, and $\rho$ are constants for all $i,\ j = 1, 2, ..., N$ in each Monte Carlo trial. The initial values of $u_{it}$ are generated as $u_{i1} = \varepsilon_{i1}/\sqrt{1-\rho^2}\ \forall\ i = 1, 2, ..., N$. The values of errors were allowed to differ for each cross-sectional unit on a given Monte Carlo trial and were allowed to differ between trials. The errors are independent with all independent variables.
3. The coefficients, $\alpha_{ki}$, were generated as in assumption 6: $\alpha_i = \bar{\alpha} + \mu_i$, where $\bar{\alpha} = (1, 1, 1)'$, and $\mu_i$ were generated from two distributions. First, multivariate normal distribution with means zeros and variance-covariance matrix $\Psi = diag\left\{\psi_k^2\right\}; k = 1, 2, 3$. The values of $\Psi_k^2$ were chosen to be fixed for all $k$ and equal to 5 or 25. Second, multivariate student's $t$ distribution with degree of freedom ($df$): $df = 1$ or 5.

To include the case of fixed-coefficients models in our simulation study, we assume that $\mu_i = 0$.

**4.** The values of $N$ and $T$ were chosen to be 5, 8, 10, 12, 15, and 20 to represent small and moderate samples for the number of individuals and the time dimension. To compare the small and moderate samples performance for the different estimators, three different samplings schemes have been designed in our simulation, where each design contains four pairs of $N$ and $T$. The first two represent small samples while the moderate samples are represented by the second two pairs. These designs have been created as follows: First, case of $N < T$, the pairs of $N$ and $T$ were chosen to be $(N, T) = (5, 8), (5, 12), (10, 15)$, or (10, 20). Second, case of $N = T$, the pairs are $(N, T) = (5, 5), (10, 10), (15, 15)$, or (20, 20). Third, case of $N > T$, the pairs are $(N, T) = (8, 5), (12, 5), (15, 10)$, or (20, 10).

**5.** All Monte Carlo experiments involved 1000 replications and all the results of all separate experiments are obtained by precisely the same series of random numbers. To raise the efficiency of the comparison between these estimators, we calculate the average of total standard errors (ATSE) for each estimator by:

$$\text{ATSE} = \frac{1}{1000} \sum_{l=1}^{1000} \left\{ trace \left[ \widehat{var} \left( \widehat{\bar{\alpha}}_l \right) \right]^{0.5} \right\},$$

where $\widehat{\bar{\alpha}}_l$ is the estimated vector of $\bar{\alpha}$ in (6.1), and $\widehat{var}\left(\widehat{\bar{\alpha}}_l\right)$ is the estimated variance-covariance matrix of the estimator.

The Monte Carlo results are given in Tables 1–6. Specifically, Tables 1–3 present the ATSE values of the estimators when $\sigma_{\varepsilon_{ii}} = 1$, and in cases of $N < T, N = T$, and$N > T$, respectively. While case of $\sigma_{\varepsilon_{ii}} = 100$ is presented in Tables 4–6 in the same cases of $N$ and $T$. In our simulation study, the main factors that have an effect on the ATSE values of the estimators are $N$, $T$, $\sigma_{\varepsilon_{ii}}$, $\sigma_{\varepsilon_{ij}}$, $\rho$, $\psi_k^2$ (for normal distribution), and *df* (for student's $t$ distribution). From Tables 1–6, we can summarize some effects for all estimators in the following points:

- When the values of $N$ and $T$ are increased, the values of ATSE are decreasing for all simulation situations.
- When the value of $\sigma_{\varepsilon_{ii}}$ is increased, the values of ATSE are increasing in most situations.
- When the values of $(\rho, \sigma_{\varepsilon_{ij}})$ are increased, the values of ATSE are increasing in most situations.
- When the value of $\psi_k^2$ is increased, the values of ATSE are increasing for all situations.
- When the value of *df* is increased, the values of ATSE are decreasing for all situations.

**Table 1**: ATSE for various estimators when $\sigma_{\varepsilon_{ii}} = 1$ and $N < T$.

| $(\rho,\ \sigma_{\varepsilon_{ij}})$ | $(0, 0)$ | | | | $(0.55, 0.75)$ | | | | $(0.85, 0.95)$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $(N, T)$ | $(5, 8)$ | $(5, 12)$ | $(10, 15)$ | $(10, 20)$ | $(5, 8)$ | $(5, 12)$ | $(10, 15)$ | $(10, 20)$ | $(5, 8)$ | $(5, 12)$ | $(10, 15)$ | $(10, 20)$ |
| | | | | | | $\mu_i = 0$ | | | | | | |
| CP-OLS | 0.920 | 0.746 | 0.440 | 0.436 | 0.857 | 0.888 | 0.409 | 0.450 | 1.107 | 1.496 | 0.607 | 0.641 |
| CP-SUR | 0.958 | 0.767 | 0.419 | 0.417 | 0.829 | 0.880 | 0.381 | 0.384 | 0.947 | 1.469 | 0.453 | 0.532 |
| MG | 0.947 | 0.765 | 0.470 | 0.469 | 0.886 | 0.910 | 0.442 | 0.468 | 1.133 | 1.475 | 0.608 | 0.636 |
| GMG | 0.702 | 0.556 | 0.369 | 0.375 | 0.638 | 0.662 | 0.289 | 0.305 | 0.644 | 1.098 | 0.302 | 0.291 |
| RCR | 1.012 | 30.746 | 0.517 | 0.497 | 1.064 | 1.130 | 2.241 | 0.726 | 1.365 | 5.960 | 0.856 | 1.326 |
| GRCR | 0.754 | 0.624 | 0.352 | 0.357 | 0.634 | 0.703 | 0.302 | 0.295 | 0.735 | 1.141 | 0.324 | 0.388 |
| | | | | | | $\mu_i \sim N(0, 5)$ | | | | | | |
| CP-OLS | 4.933 | 4.682 | 2.320 | 2.742 | 2.588 | 2.902 | 2.598 | 2.130 | 3.627 | 5.079 | 2.165 | 2.935 |
| CP-SUR | 5.870 | 5.738 | 2.852 | 3.411 | 3.143 | 3.456 | 3.212 | 2.592 | 4.011 | 5.906 | 2.668 | 3.549 |
| MG | 4.057 | 4.112 | 2.086 | 2.494 | 2.173 | 2.478 | 2.352 | 1.888 | 3.094 | 4.040 | 1.938 | 2.626 |
| GMG | 4.057 | 4.110 | 2.084 | 2.494 | 2.176 | 2.479 | 2.348 | 1.879 | 3.052 | 4.024 | 1.908 | 2.606 |
| RCR | 4.053 | 4.114 | 2.083 | 2.493 | 2.632 | 3.304 | 2.352 | 1.888 | 3.287 | 6.422 | 2.052 | 2.648 |
| GRCR | 4.030 | 4.092 | 2.067 | 2.480 | 2.104 | 2.413 | 2.331 | 1.855 | 2.969 | 3.905 | 1.865 | 2.578 |
| | | | | | | $\mu_i \sim N(0, 25)$ | | | | | | |
| CP-OLS | 7.528 | 7.680 | 7.147 | 6.341 | 8.293 | 8.156 | 6.321 | 6.739 | 7.942 | 7.214 | 4.691 | 6.423 |
| CP-SUR | 8.866 | 9.439 | 8.935 | 8.046 | 10.104 | 9.880 | 8.028 | 8.402 | 9.074 | 8.482 | 5.739 | 7.937 |
| MG | 6.272 | 6.549 | 6.324 | 5.597 | 6.879 | 6.650 | 5.541 | 5.917 | 6.442 | 6.083 | 4.118 | 5.672 |
| GMG | 6.271 | 6.548 | 6.324 | 5.597 | 6.881 | 6.650 | 5.538 | 5.913 | 6.422 | 6.078 | 4.103 | 5.662 |
| RCR | 6.271 | 6.548 | 6.324 | 5.597 | 6.885 | 6.657 | 5.541 | 5.917 | 7.546 | 6.098 | 4.122 | 5.686 |
| GRCR | 6.251 | 6.539 | 6.319 | 5.590 | 6.857 | 6.626 | 5.530 | 5.906 | 6.389 | 6.010 | 4.082 | 5.649 |
| | | | | | | $\mu_i \sim t(5)$ | | | | | | |
| CP-OLS | 2.253 | 1.983 | 1.562 | 1.544 | 1.479 | 1.977 | 1.060 | 1.223 | 2.115 | 3.301 | 1.470 | 1.439 |
| CP-SUR | 2.626 | 2.419 | 1.925 | 1.912 | 1.694 | 2.266 | 1.275 | 1.454 | 2.403 | 3.903 | 1.717 | 1.643 |
| MG | 1.859 | 1.776 | 1.410 | 1.401 | 1.324 | 1.722 | 0.984 | 1.078 | 1.923 | 2.707 | 1.335 | 1.260 |
| GMG | 1.856 | 1.771 | 1.408 | 1.400 | 1.316 | 1.718 | 0.970 | 1.064 | 1.826 | 2.666 | 1.284 | 1.215 |
| RCR | 2.002 | 1.768 | 1.452 | 1.396 | 2.020 | 3.260 | 1.017 | 1.087 | 12.328 | 6.655 | 2.035 | 2.650 |
| GRCR | 1.788 | 1.727 | 1.377 | 1.375 | 1.215 | 1.655 | 0.926 | 1.019 | 1.786 | 2.552 | 1.221 | 1.155 |
| | | | | | | $\mu_i \sim t(1)$ | | | | | | |
| CP-OLS | 16.112 | 4.096 | 2.732 | 10.189 | 12.490 | 24.982 | 6.424 | 2.837 | 6.685 | 5.668 | 12.763 | 1.786 |
| CP-SUR | 19.483 | 5.046 | 3.365 | 12.976 | 14.940 | 29.854 | 8.009 | 3.555 | 7.807 | 7.043 | 15.947 | 2.126 |
| MG | 11.751 | 3.427 | 2.432 | 9.094 | 9.811 | 19.875 | 5.742 | 2.306 | 5.568 | 4.365 | 11.473 | 1.620 |
| GMG | 11.751 | 3.423 | 2.431 | 9.094 | 9.811 | 19.875 | 5.740 | 2.298 | 5.540 | 4.352 | 11.468 | 1.583 |
| RCR | 11.751 | 3.423 | 2.431 | 9.094 | 9.813 | 19.877 | 5.742 | 2.304 | 5.591 | 7.730 | 11.475 | 1.829 |
| GRCR | 11.739 | 3.403 | 2.417 | 9.090 | 9.795 | 19.868 | 5.733 | 2.271 | 5.498 | 4.228 | 11.462 | 1.530 |

**Table 2**: ATSE for various estimators when $\sigma_{\varepsilon_{ii}} = 1$ and $N = T$.

| $(\rho,\ \sigma_{\varepsilon_{ij}})$ | $(0,0)$ | | | | $(0.55, 0.75)$ | | | | $(0.85, 0.95)$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $(N,T)$ | $(5,5)$ | $(10,10)$ | $(15,15)$ | $(20,20)$ | $(5,5)$ | $(10,10)$ | $(15,15)$ | $(20,20)$ | $(5,5)$ | $(10,10)$ | $(15,15)$ | $(20,20)$ |
| | | | | | | $\mu_i = 0$ | | | | | | |
| CP-OLS | 1.671 | 0.461 | 0.259 | 0.174 | 2.081 | 0.424 | 0.274 | 0.207 | 3.351 | 0.678 | 0.394 | 0.276 |
| CP-SUR | 2.387 | 0.550 | 0.299 | 0.178 | 3.340 | 0.478 | 0.291 | 0.182 | 4.301 | 0.716 | 0.293 | 0.192 |
| MG | 1.686 | 0.486 | 0.280 | 0.183 | 2.058 | 0.474 | 0.300 | 0.210 | 3.093 | 0.668 | 0.377 | 0.255 |
| GMG | 1.174 | 0.395 | 0.234 | 0.159 | 1.669 | 0.363 | 0.209 | 0.149 | 2.028 | 0.370 | 0.190 | 0.115 |
| RCR | 1.905 | 0.557 | 0.314 | 0.179 | 1.997 | 0.953 | 0.411 | 0.502 | 3.249 | 1.982 | 0.471 | 0.458 |
| GRCR | 1.294 | 0.320 | 0.173 | 0.102 | 1.678 | 0.264 | 0.151 | 0.093 | 2.480 | 0.380 | 0.145 | 0.094 |
| | | | | | | $\mu_i \sim N(0,5)$ | | | | | | |
| CP-OLS | 4.119 | 3.404 | 1.982 | 1.651 | 4.593 | 2.002 | 1.517 | 1.474 | 5.023 | 2.926 | 1.847 | 1.740 |
| CP-SUR | 6.478 | 5.521 | 3.511 | 3.097 | 8.141 | 3.313 | 2.735 | 2.737 | 7.176 | 4.951 | 3.313 | 3.368 |
| MG | 3.480 | 2.750 | 1.744 | 1.520 | 4.015 | 1.671 | 1.295 | 1.341 | 4.284 | 2.531 | 1.633 | 1.608 |
| GMG | 3.481 | 2.750 | 1.743 | 1.520 | 4.008 | 1.664 | 1.289 | 1.337 | 4.034 | 2.515 | 1.615 | 1.599 |
| RCR | 5.955 | 2.749 | 1.743 | 1.520 | 4.232 | 1.666 | 1.295 | 1.342 | 12.312 | 2.574 | 1.651 | 1.617 |
| GRCR | 3.400 | 2.727 | 1.730 | 1.513 | 3.826 | 1.622 | 1.266 | 1.328 | 3.913 | 2.463 | 1.591 | 1.590 |
| | | | | | | $\mu_i \sim N(0,25)$ | | | | | | |
| CP-OLS | 8.056 | 6.265 | 4.022 | 3.637 | 7.976 | 5.496 | 4.240 | 3.968 | 10.264 | 6.615 | 4.558 | 3.733 |
| CP-SUR | 12.776 | 10.403 | 7.168 | 6.869 | 14.233 | 9.622 | 7.606 | 7.540 | 15.004 | 11.368 | 8.361 | 7.229 |
| MG | 6.474 | 5.145 | 3.558 | 3.348 | 6.491 | 4.599 | 3.692 | 3.623 | 6.798 | 5.597 | 4.042 | 3.464 |
| GMG | 6.476 | 5.145 | 3.558 | 3.348 | 6.498 | 4.596 | 3.690 | 3.622 | 6.822 | 5.589 | 4.036 | 3.460 |
| RCR | 6.469 | 5.145 | 3.558 | 3.348 | 6.457 | 4.597 | 3.692 | 3.624 | 10.576 | 5.614 | 4.050 | 3.468 |
| GRCR | 6.412 | 5.134 | 3.552 | 3.345 | 6.399 | 4.581 | 3.683 | 3.618 | 6.534 | 5.566 | 4.027 | 3.456 |
| | | | | | | $\mu_i \sim t(5)$ | | | | | | |
| CP-OLS | 2.017 | 1.444 | 1.054 | 0.818 | 2.719 | 2.306 | 1.452 | 1.202 | 3.512 | 1.374 | 1.130 | 0.866 |
| CP-SUR | 2.952 | 2.278 | 1.848 | 1.499 | 4.581 | 4.002 | 2.602 | 2.251 | 4.784 | 2.113 | 1.960 | 1.584 |
| MG | 1.900 | 1.215 | 0.933 | 0.759 | 2.435 | 1.892 | 1.228 | 1.113 | 3.241 | 1.209 | 1.017 | 0.800 |
| GMG | 1.752 | 1.214 | 0.933 | 0.759 | 2.369 | 1.886 | 1.221 | 1.108 | 2.635 | 1.177 | 0.989 | 0.780 |
| RCR | 2.987 | 1.209 | 0.931 | 0.758 | 2.862 | 1.886 | 1.229 | 1.114 | 11.891 | 1.760 | 1.527 | 0.815 |
| GRCR | 1.628 | 1.165 | 0.908 | 0.744 | 2.193 | 1.848 | 1.199 | 1.097 | 2.727 | 1.073 | 0.951 | 0.762 |
| | | | | | | $\mu_i \sim t(1)$ | | | | | | |
| CP-OLS | 2.946 | 4.082 | 36.296 | 32.249 | 170.833 | 4.983 | 7.221 | 5.545 | 5.447 | 14.094 | 27.076 | 2.245 |
| CP-SUR | 4.663 | 6.691 | 70.583 | 64.229 | 291.169 | 8.653 | 13.554 | 10.472 | 7.942 | 25.514 | 54.690 | 4.290 |
| MG | 2.569 | 3.337 | 23.288 | 26.932 | 92.236 | 4.064 | 5.831 | 5.069 | 4.403 | 11.428 | 20.763 | 2.085 |
| GMG | 2.565 | 3.337 | 23.288 | 26.932 | 92.238 | 4.060 | 5.829 | 5.068 | 4.362 | 11.420 | 20.759 | 2.078 |
| RCR | 5.160 | 3.337 | 23.288 | 26.932 | 92.238 | 4.061 | 5.831 | 5.069 | 7.663 | 11.440 | 20.767 | 2.091 |
| GRCR | 2.433 | 3.320 | 23.280 | 26.931 | 92.226 | 4.042 | 5.823 | 5.065 | 4.024 | 11.401 | 20.753 | 2.072 |

**Table 3**: ATSE for various estimators when $\sigma_{\varepsilon_{ii}} = 1$ and $N > T$.

| $(\rho,\ \sigma_{\varepsilon_{ij}})$ | $(0, 0)$ | | | | $(0.55, 0.75)$ | | | | $(0.85, 0.95)$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $(N, T)$ | $(8, 5)$ | $(12, 5)$ | $(15, 10)$ | $(20, 10)$ | $(8, 5)$ | $(12, 5)$ | $(15, 10)$ | $(20, 10)$ | $(8, 5)$ | $(12, 5)$ | $(15, 10)$ | $(20, 10)$ |
| $\mu_i = 0$ | | | | | | | | | | | | |
| CP-OLS | 1.763 | 3.198 | 0.510 | 0.438 | 1.254 | 1.399 | 0.436 | 0.536 | 1.218 | 1.350 | 0.688 | 0.591 |
| CP-SUR | 2.504 | 4.585 | 0.635 | 0.518 | 1.748 | 1.963 | 0.497 | 0.607 | 1.637 | 1.808 | 0.780 | 0.655 |
| MG | 1.856 | 2.927 | 0.576 | 0.475 | 1.434 | 1.455 | 0.501 | 0.618 | 1.528 | 1.523 | 0.830 | 0.631 |
| GMG | 1.288 | 1.767 | 0.452 | 0.391 | 1.017 | 0.995 | 0.350 | 0.417 | 1.014 | 0.982 | 0.468 | 0.433 |
| RCR | 7.356 | 2.702 | 0.567 | 0.573 | 1.353 | 1.333 | 0.693 | 1.625 | 1.490 | 1.468 | 2.432 | 1.605 |
| GRCR | 1.289 | 2.277 | 0.342 | 0.267 | 0.937 | 1.010 | 0.248 | 0.306 | 0.865 | 0.856 | 0.413 | 0.312 |
| $\mu_i \sim N(0, 5)$ | | | | | | | | | | | | |
| CP-OLS | 3.136 | 4.014 | 2.525 | 2.017 | 3.677 | 3.352 | 2.477 | 3.105 | 2.146 | 3.501 | 1.927 | 2.415 |
| CP-SUR | 4.590 | 5.845 | 3.576 | 2.888 | 5.279 | 4.824 | 3.485 | 4.396 | 3.080 | 4.935 | 2.687 | 3.393 |
| MG | 2.753 | 3.418 | 2.153 | 1.685 | 2.972 | 2.643 | 2.113 | 2.628 | 2.191 | 2.813 | 1.724 | 2.156 |
| GMG | 2.665 | 3.425 | 2.152 | 1.684 | 2.951 | 2.660 | 2.106 | 2.617 | 2.097 | 2.748 | 1.679 | 2.142 |
| RCR | 3.611 | 3.306 | 2.146 | 1.681 | 2.897 | 3.034 | 2.109 | 2.621 | 61.169 | 137.429 | 2.187 | 2.147 |
| GRCR | 2.400 | 2.982 | 2.103 | 1.636 | 2.774 | 2.399 | 2.066 | 2.572 | 1.852 | 2.550 | 1.532 | 2.075 |
| $\mu_i \sim N(0, 25)$ | | | | | | | | | | | | |
| CP-OLS | 6.919 | 6.434 | 6.179 | 5.259 | 6.442 | 5.639 | 4.972 | 4.460 | 6.279 | 7.428 | 5.480 | 5.366 |
| CP-SUR | 10.250 | 9.292 | 8.750 | 7.682 | 9.200 | 8.224 | 7.123 | 6.378 | 9.507 | 10.544 | 7.791 | 7.698 |
| MG | 5.090 | 5.029 | 5.092 | 4.381 | 4.987 | 4.505 | 4.167 | 3.688 | 5.353 | 5.689 | 4.545 | 4.756 |
| GMG | 5.046 | 5.031 | 5.092 | 4.380 | 4.971 | 4.512 | 4.163 | 3.680 | 5.316 | 5.677 | 4.530 | 4.749 |
| RCR | 4.986 | 4.735 | 5.091 | 4.380 | 4.939 | 4.466 | 4.165 | 3.683 | 5.303 | 6.219 | 4.538 | 4.753 |
| GRCR | 4.898 | 4.588 | 5.071 | 4.362 | 4.874 | 4.408 | 4.142 | 3.645 | 5.189 | 5.559 | 4.479 | 4.720 |
| $\mu_i \sim t(5)$ | | | | | | | | | | | | |
| CP-OLS | 1.779 | 2.367 | 1.151 | 1.080 | 1.780 | 2.464 | 1.986 | 1.308 | 2.157 | 2.848 | 1.473 | 1.283 |
| CP-SUR | 2.541 | 3.365 | 1.604 | 1.493 | 2.596 | 3.711 | 2.929 | 1.745 | 3.137 | 4.179 | 1.987 | 1.730 |
| MG | 1.839 | 1.989 | 1.010 | 0.943 | 1.647 | 2.276 | 1.603 | 1.074 | 2.109 | 2.401 | 1.260 | 1.467 |
| GMG | 1.577 | 1.974 | 1.008 | 0.942 | 1.563 | 2.245 | 1.586 | 1.076 | 1.730 | 2.362 | 1.235 | 1.255 |
| RCR | 2.573 | 2.327 | 0.991 | 0.960 | 2.785 | 2.945 | 1.591 | 1.097 | 3.523 | 3.020 | 3.322 | 3.509 |
| GRCR | 1.336 | 1.738 | 0.924 | 0.837 | 1.529 | 1.893 | 1.525 | 0.982 | 1.652 | 2.120 | 1.124 | 1.049 |
| $\mu_i \sim t(1)$ | | | | | | | | | | | | |
| CP-OLS | 23.572 | 9.953 | 1.708 | 9.638 | 9.612 | 3.030 | 5.400 | 4.609 | 6.932 | 8.340 | 25.666 | 4.259 |
| CP-SUR | 35.133 | 13.767 | 2.466 | 14.035 | 15.207 | 4.429 | 8.027 | 6.816 | 9.309 | 12.412 | 39.880 | 6.199 |
| MG | 17.304 | 6.568 | 1.410 | 6.014 | 7.568 | 2.654 | 4.164 | 3.451 | 4.802 | 6.004 | 16.848 | 3.318 |
| GMG | 17.295 | 6.563 | 1.409 | 6.014 | 7.580 | 2.629 | 4.155 | 3.452 | 4.781 | 5.991 | 16.840 | 3.267 |
| RCR | 17.295 | 6.535 | 1.398 | 6.012 | 7.546 | 2.499 | 4.158 | 3.456 | 6.130 | 5.997 | 16.849 | 4.158 |
| GRCR | 17.263 | 6.483 | 1.345 | 5.979 | 7.492 | 2.345 | 4.128 | 3.407 | 4.593 | 5.877 | 16.779 | 3.081 |

**Table 4**: ATSE for various estimators when $\sigma_{\varepsilon_{ii}} = 100$ and $N < T$.

| $(\rho,\ \sigma_{\varepsilon_{ij}})$ | $(0, 0)$ | | | | $(0.55, 0.75)$ | | | | $(0.85, 0.95)$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $(N, T)$ | $(5, 8)$ | $(5, 12)$ | $(10, 15)$ | $(10, 20)$ | $(5, 8)$ | $(5, 12)$ | $(10, 15)$ | $(10, 20)$ | $(5, 8)$ | $(5, 12)$ | $(10, 15)$ | $(10, 20)$ |
| $\mu_i = 0$ | | | | | | | | | | | | |
| CP-OLS | 2.908 | 2.357 | 1.389 | 1.379 | 2.756 | 2.863 | 1.414 | 1.395 | 3.798 | 5.179 | 2.042 | 2.208 |
| CP-SUR | 3.028 | 2.422 | 1.323 | 1.316 | 2.806 | 2.997 | 1.335 | 1.302 | 3.520 | 5.316 | 1.692 | 1.989 |
| MG | 2.993 | 2.419 | 1.486 | 1.483 | 2.830 | 2.984 | 1.492 | 1.503 | 3.850 | 4.907 | 2.010 | 2.292 |
| GMG | 2.221 | 1.759 | 1.168 | 1.187 | 1.975 | 2.180 | 1.027 | 1.004 | 2.132 | 3.466 | 1.022 | 1.191 |
| RCR | 3.199 | 97.225 | 1.634 | 1.570 | 3.205 | 6.691 | 2.576 | 2.846 | 4.711 | 7.169 | 2.708 | 3.170 |
| GRCR | 2.381 | 1.970 | 1.111 | 1.128 | 2.188 | 2.399 | 1.061 | 1.029 | 2.667 | 3.872 | 1.220 | 1.429 |
| $\mu_i \sim N(0, 5)$ | | | | | | | | | | | | |
| CP-OLS | 5.096 | 4.872 | 2.481 | 2.890 | 3.298 | 3.570 | 2.732 | 2.260 | 4.432 | 6.390 | 2.479 | 3.180 |
| CP-SUR | 5.787 | 5.751 | 2.856 | 3.437 | 3.573 | 3.960 | 3.305 | 2.557 | 4.449 | 6.946 | 2.463 | 3.524 |
| MG | 4.533 | 4.450 | 2.361 | 2.737 | 3.193 | 3.448 | 2.575 | 2.172 | 4.327 | 5.642 | 2.363 | 3.076 |
| GMG | 4.507 | 4.427 | 2.349 | 2.734 | 2.869 | 3.165 | 2.539 | 2.101 | 3.695 | 5.110 | 2.150 | 2.849 |
| RCR | 11.579 | 5.572 | 2.500 | 2.702 | 3.871 | 8.045 | 3.278 | 3.489 | 7.748 | 9.539 | 5.301 | 22.220 |
| GRCR | 4.179 | 4.294 | 2.166 | 2.576 | 2.755 | 3.026 | 2.378 | 1.911 | 3.456 | 5.004 | 1.879 | 2.560 |
| $\mu_i \sim N(0, 25)$ | | | | | | | | | | | | |
| CP-OLS | 7.670 | 7.803 | 7.209 | 6.407 | 8.362 | 8.314 | 6.380 | 6.781 | 7.971 | 7.887 | 4.852 | 6.554 |
| CP-SUR | 8.833 | 9.460 | 8.952 | 8.050 | 10.073 | 10.032 | 8.245 | 8.508 | 9.153 | 9.160 | 5.890 | 8.277 |
| MG | 6.570 | 6.760 | 6.431 | 5.714 | 7.118 | 7.016 | 5.653 | 6.018 | 6.812 | 7.017 | 4.338 | 5.913 |
| GMG | 6.556 | 6.749 | 6.426 | 5.713 | 7.116 | 7.013 | 5.625 | 5.991 | 6.658 | 6.996 | 4.240 | 5.795 |
| RCR | 10.949 | 6.908 | 6.423 | 5.706 | 7.103 | 7.629 | 5.647 | 6.008 | 11.120 | 16.814 | 9.260 | 6.478 |
| GRCR | 6.400 | 6.633 | 6.370 | 5.646 | 6.945 | 6.826 | 5.558 | 5.932 | 6.286 | 6.595 | 4.057 | 5.661 |
| $\mu_i \sim t(5)$ | | | | | | | | | | | | |
| CP-OLS | 3.227 | 2.672 | 1.820 | 1.804 | 2.894 | 3.067 | 1.534 | 1.558 | 4.052 | 5.630 | 2.112 | 2.299 |
| CP-SUR | 3.432 | 2.879 | 1.975 | 1.959 | 3.045 | 3.327 | 1.529 | 1.560 | 3.998 | 6.065 | 1.838 | 2.099 |
| MG | 3.186 | 2.654 | 1.829 | 1.810 | 2.924 | 3.097 | 1.588 | 1.617 | 4.042 | 5.146 | 2.071 | 2.318 |
| GMG | 2.816 | 2.405 | 1.799 | 1.782 | 2.296 | 2.690 | 1.394 | 1.435 | 2.792 | 4.288 | 1.603 | 1.692 |
| RCR | 3.665 | 3.442 | 2.592 | 2.462 | 4.922 | 4.147 | 3.057 | 4.985 | 9.667 | 14.064 | 3.871 | 6.113 |
| GRCR | 2.666 | 2.317 | 1.625 | 1.543 | 2.374 | 2.662 | 1.232 | 1.233 | 3.045 | 4.365 | 1.456 | 1.604 |
| $\mu_i \sim t(1)$ | | | | | | | | | | | | |
| CP-OLS | 16.193 | 4.345 | 2.882 | 10.228 | 12.527 | 25.028 | 6.481 | 2.957 | 6.842 | 6.962 | 12.819 | 2.363 |
| CP-SUR | 19.488 | 5.071 | 3.383 | 12.975 | 14.929 | 30.583 | 8.213 | 3.571 | 7.803 | 7.838 | 16.626 | 2.317 |
| MG | 11.990 | 3.871 | 2.673 | 9.164 | 9.996 | 19.985 | 5.841 | 2.595 | 6.095 | 5.929 | 11.548 | 2.434 |
| GMG | 11.990 | 3.832 | 2.665 | 9.163 | 9.979 | 19.993 | 5.819 | 2.524 | 5.898 | 5.591 | 11.512 | 1.988 |
| RCR | 11.965 | 4.529 | 2.625 | 9.162 | 9.966 | 19.996 | 5.839 | 3.527 | 13.705 | 59.015 | 11.574 | 14.464 |
| GRCR | 11.840 | 3.650 | 2.507 | 9.122 | 9.862 | 19.940 | 5.762 | 2.360 | 5.434 | 5.506 | 11.460 | 1.773 |

**Table 5**: ATSE for various estimators when $\sigma_{\varepsilon_{ii}} = 100$ and $N = T$.

| $(\rho,\ \sigma_{\varepsilon_{ij}})$ | $(0,0)$ | | | | $(0.55, 0.75)$ | | | | $(0.85, 0.95)$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $(N,T)$ | $(5,5)$ | $(10,10)$ | $(15,15)$ | $(20,20)$ | $(5,5)$ | $(10,10)$ | $(15,15)$ | $(20,20)$ | $(5,5)$ | $(10,10)$ | $(15,15)$ | $(20,20)$ |
| | | | | | | $\mu_i = 0$ | | | | | | |
| CP-OLS | 5.284 | 1.456 | 0.818 | 0.548 | 6.920 | 1.339 | 0.904 | 0.629 | 11.353 | 2.314 | 1.215 | 0.871 |
| CP-SUR | 7.548 | 1.737 | 0.942 | 0.559 | 10.528 | 1.580 | 0.977 | 0.589 | 15.654 | 2.573 | 0.987 | 0.625 |
| MG | 5.331 | 1.537 | 0.886 | 0.577 | 6.606 | 1.417 | 0.998 | 0.658 | 10.554 | 2.362 | 1.238 | 0.839 |
| GMG | 3.712 | 1.250 | 0.741 | 0.503 | 5.470 | 1.105 | 0.693 | 0.466 | 6.959 | 1.419 | 0.602 | 0.410 |
| RCR | 6.023 | 1.759 | 0.990 | 0.564 | 8.315 | 2.026 | 2.034 | 1.388 | 10.978 | 3.817 | 2.088 | 1.241 |
| GRCR | 4.090 | 1.007 | 0.545 | 0.318 | 5.497 | 0.907 | 0.527 | 0.318 | 8.037 | 1.363 | 0.525 | 0.325 |
| | | | | | | $\mu_i \sim N(0,5)$ | | | | | | |
| CP-OLS | 5.580 | 3.519 | 2.061 | 1.705 | 7.429 | 2.182 | 1.629 | 1.543 | 10.993 | 3.155 | 1.991 | 1.859 |
| CP-SUR | 8.237 | 5.479 | 3.497 | 3.091 | 11.726 | 3.255 | 2.651 | 2.742 | 15.414 | 4.585 | 3.080 | 3.221 |
| MG | 5.622 | 2.996 | 1.876 | 1.592 | 6.993 | 1.987 | 1.522 | 1.438 | 10.338 | 3.017 | 1.864 | 1.733 |
| GMG | 4.959 | 2.994 | 1.876 | 1.591 | 6.571 | 1.968 | 1.459 | 1.406 | 7.682 | 2.893 | 1.712 | 1.649 |
| RCR | 8.572 | 3.064 | 1.861 | 1.588 | 8.773 | 2.645 | 2.696 | 1.435 | 10.818 | 6.531 | 3.172 | 1.779 |
| GRCR | 4.679 | 2.764 | 1.747 | 1.520 | 6.313 | 1.727 | 1.249 | 1.322 | 8.234 | 2.397 | 1.489 | 1.558 |
| | | | | | | $\mu_i \sim N(0,25)$ | | | | | | |
| CP-OLS | 8.220 | 6.333 | 4.056 | 3.661 | 9.384 | 5.567 | 4.285 | 3.991 | 12.808 | 6.724 | 4.618 | 3.788 |
| CP-SUR | 12.685 | 10.388 | 7.152 | 6.865 | 15.219 | 9.557 | 7.574 | 7.573 | 18.954 | 11.401 | 8.194 | 7.215 |
| MG | 7.404 | 5.282 | 3.620 | 3.380 | 8.388 | 4.740 | 3.779 | 3.657 | 11.236 | 5.845 | 4.138 | 3.523 |
| GMG | 7.257 | 5.281 | 3.620 | 3.380 | 8.438 | 4.728 | 3.754 | 3.645 | 9.858 | 5.787 | 4.073 | 3.482 |
| RCR | 12.035 | 5.272 | 3.618 | 3.380 | 9.526 | 4.731 | 3.774 | 3.658 | 12.921 | 6.137 | 4.153 | 3.545 |
| GRCR | 6.703 | 5.166 | 3.556 | 3.347 | 7.863 | 4.608 | 3.688 | 3.613 | 9.475 | 5.537 | 3.995 | 3.440 |
| | | | | | | $\mu_i \sim t(5)$ | | | | | | |
| CP-OLS | 5.268 | 1.758 | 1.205 | 0.930 | 6.905 | 2.466 | 1.566 | 1.289 | 11.183 | 2.322 | 1.363 | 1.078 |
| CP-SUR | 7.487 | 2.302 | 1.826 | 1.505 | 10.462 | 3.902 | 2.518 | 2.232 | 15.445 | 2.648 | 1.486 | 1.354 |
| MG | 5.301 | 1.734 | 1.173 | 0.901 | 6.588 | 2.197 | 1.457 | 1.231 | 10.371 | 2.363 | 1.359 | 1.024 |
| GMG | 3.914 | 1.688 | 1.171 | 0.900 | 5.741 | 2.170 | 1.392 | 1.193 | 7.036 | 1.810 | 1.138 | 0.874 |
| RCR | 6.313 | 2.356 | 1.226 | 0.885 | 8.980 | 4.088 | 1.806 | 1.224 | 10.384 | 6.372 | 4.418 | 4.574 |
| GRCR | 4.238 | 1.313 | 0.937 | 0.764 | 5.796 | 1.894 | 1.179 | 1.094 | 8.124 | 1.489 | 0.823 | 0.688 |
| | | | | | | $\mu_i \sim t(1)$ | | | | | | |
| CP-OLS | 5.492 | 4.176 | 36.310 | 32.254 | 170.969 | 5.046 | 7.246 | 5.564 | 11.208 | 14.166 | 27.093 | 2.332 |
| CP-SUR | 8.085 | 6.670 | 70.596 | 64.232 | 277.362 | 8.718 | 13.502 | 10.390 | 15.450 | 26.068 | 54.457 | 4.185 |
| MG | 5.469 | 3.529 | 23.379 | 26.943 | 92.536 | 4.228 | 5.898 | 5.095 | 10.448 | 11.655 | 20.834 | 2.180 |
| GMG | 4.346 | 3.528 | 23.378 | 26.943 | 92.558 | 4.213 | 5.878 | 5.086 | 7.748 | 11.603 | 20.786 | 2.114 |
| RCR | 7.220 | 3.503 | 23.365 | 26.943 | 92.513 | 4.383 | 5.895 | 5.096 | 13.141 | 12.397 | 20.840 | 2.210 |
| GRCR | 4.471 | 3.354 | 23.296 | 26.932 | 92.445 | 4.050 | 5.822 | 5.064 | 8.345 | 11.384 | 20.731 | 2.046 |

**Table 6**: ATSE for various estimators when $\sigma_{\varepsilon_{ii}} = 100$ and $N > T$.

| $(\rho,\ \sigma_{\varepsilon_{ij}})$ | $(0, 0)$ | | | | $(0.55, 0.75)$ | | | | $(0.85, 0.95)$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $(N, T)$ | $(8, 5)$ | $(12, 5)$ | $(15, 10)$ | $(20, 10)$ | $(8, 5)$ | $(12, 5)$ | $(15, 10)$ | $(20, 10)$ | $(8, 5)$ | $(12, 5)$ | $(15, 10)$ | $(20, 10)$ |
| | | | | | | $\mu_i = 0$ | | | | | | |
| CP-OLS | 5.574 | 3.501 | 1.511 | 1.493 | 5.616 | 4.178 | 1.764 | 1.546 | 8.088 | 9.255 | 2.325 | 2.474 |
| CP-SUR | 7.919 | 4.835 | 1.798 | 1.840 | 7.780 | 5.841 | 2.229 | 1.813 | 11.886 | 12.804 | 2.723 | 2.975 |
| MG | 5.868 | 3.453 | 1.659 | 1.676 | 5.678 | 4.306 | 1.908 | 1.629 | 9.127 | 8.473 | 2.678 | 2.773 |
| GMG | 4.073 | 2.490 | 1.349 | 1.337 | 3.643 | 3.717 | 1.515 | 1.219 | 5.788 | 7.373 | 1.382 | 1.581 |
| RCR | 23.253 | 3.498 | 1.759 | 1.808 | 5.403 | 6.417 | 5.387 | 2.286 | 8.172 | 11.799 | 2.744 | 4.156 |
| GRCR | 4.072 | 2.397 | 0.931 | 0.972 | 3.998 | 3.241 | 1.142 | 0.872 | 5.937 | 6.519 | 1.267 | 1.352 |
| | | | | | | $\mu_i \sim N(0, 5)$ | | | | | | |
| CP-OLS | 5.574 | 4.258 | 2.867 | 2.692 | 5.221 | 5.014 | 2.744 | 2.396 | 8.256 | 9.261 | 2.333 | 3.037 |
| CP-SUR | 7.899 | 5.954 | 3.858 | 3.725 | 7.202 | 7.096 | 3.802 | 3.166 | 12.049 | 12.885 | 2.782 | 4.092 |
| MG | 5.793 | 3.775 | 2.616 | 2.509 | 5.407 | 4.904 | 2.622 | 2.241 | 9.299 | 8.462 | 2.682 | 3.135 |
| GMG | 4.753 | 3.635 | 2.615 | 2.503 | 4.022 | 4.657 | 2.663 | 2.226 | 6.423 | 7.531 | 2.230 | 2.815 |
| RCR | 7.585 | 5.340 | 2.525 | 2.569 | 25.633 | 6.314 | 8.404 | 2.808 | 10.171 | 10.268 | 15.344 | 8.355 |
| GRCR | 4.220 | 3.123 | 2.206 | 2.063 | 3.901 | 3.925 | 2.101 | 1.771 | 6.533 | 6.464 | 1.443 | 2.026 |
| | | | | | | $\mu_i \sim N(0, 25)$ | | | | | | |
| CP-OLS | 7.383 | 6.000 | 5.791 | 4.700 | 6.808 | 7.512 | 4.220 | 6.284 | 7.648 | 11.202 | 4.729 | 4.463 |
| CP-SUR | 10.777 | 8.636 | 8.118 | 6.667 | 9.409 | 11.012 | 5.987 | 8.667 | 11.213 | 16.010 | 6.596 | 6.367 |
| MG | 6.876 | 4.940 | 4.816 | 4.146 | 6.287 | 6.642 | 3.722 | 5.162 | 8.635 | 9.623 | 4.346 | 4.168 |
| GMG | 6.442 | 4.902 | 4.815 | 4.143 | 6.205 | 6.532 | 3.765 | 5.156 | 7.205 | 9.360 | 4.171 | 3.961 |
| RCR | 11.741 | 5.730 | 4.792 | 4.090 | 11.299 | 7.379 | 3.776 | 5.160 | 12.146 | 12.980 | 13.643 | 7.505 |
| GRCR | 5.510 | 4.310 | 4.615 | 3.915 | 5.288 | 5.902 | 3.379 | 4.983 | 6.356 | 8.403 | 3.669 | 3.352 |
| | | | | | | $\mu_i \sim t(5)$ | | | | | | |
| CP-OLS | 5.373 | 3.666 | 1.719 | 1.726 | 5.575 | 4.294 | 1.789 | 1.805 | 8.085 | 9.347 | 2.373 | 2.455 |
| CP-SUR | 7.646 | 5.136 | 2.115 | 2.217 | 7.757 | 5.989 | 2.248 | 2.223 | 11.901 | 13.041 | 2.803 | 2.974 |
| MG | 5.706 | 3.482 | 1.779 | 1.837 | 5.623 | 4.394 | 1.926 | 1.802 | 9.133 | 8.456 | 2.695 | 2.784 |
| GMG | 4.249 | 3.082 | 1.722 | 1.759 | 3.683 | 3.907 | 1.647 | 1.727 | 5.933 | 7.429 | 1.691 | 1.879 |
| RCR | 9.861 | 5.223 | 2.501 | 2.758 | 5.421 | 5.238 | 3.195 | 3.158 | 13.392 | 14.875 | 4.908 | 6.298 |
| GRCR | 3.915 | 2.670 | 1.150 | 1.268 | 4.044 | 3.334 | 1.188 | 1.170 | 6.032 | 6.570 | 1.342 | 1.415 |
| | | | | | | $\mu_i \sim t(1)$ | | | | | | |
| CP-OLS | 5.821 | 3.703 | 4.328 | 6.252 | 6.016 | 5.931 | 31.442 | 4.149 | 11.344 | 10.999 | 5.576 | 3.013 |
| CP-SUR | 8.533 | 5.188 | 6.188 | 9.132 | 8.500 | 8.555 | 47.659 | 5.806 | 17.261 | 15.893 | 8.562 | 3.969 |
| MG | 5.986 | 3.550 | 3.544 | 5.182 | 5.876 | 5.420 | 21.165 | 3.416 | 11.058 | 9.507 | 4.826 | 3.140 |
| GMG | 4.941 | 3.242 | 3.537 | 5.179 | 5.579 | 5.219 | 21.177 | 3.402 | 8.986 | 9.203 | 4.557 | 2.831 |
| RCR | 8.791 | 13.034 | 13.254 | 5.140 | 7.133 | 6.561 | 21.171 | 3.896 | 13.086 | 12.317 | 10.078 | 10.717 |
| GRCR | 4.403 | 2.740 | 3.115 | 4.987 | 4.936 | 4.559 | 21.041 | 3.093 | 8.697 | 7.876 | 3.877 | 2.021 |

For more deeps in simulation results, we can conclude the following results:

**1**. Generally, the performance of all estimators in cases of $N \leqslant T$ is better than their performance in case of $N > T$. Similarly, their performance in cases of $\sigma_{\varepsilon_{ii}} = 1$ is better than the performance in case of $\sigma_{\varepsilon_{ii}} = 100$, but not as significantly better as in $N$ and $T$.

**2**. When $\sigma_{\varepsilon_{ij}} = \rho = \mu_i = 0$, the ATSE values of the classical pooling estimators (CP-OLS and CP-SUR) are approximately equivalent, especially when the sample size is moderate and/or $N \leqslant T$. However, the ATSE values of GMG and GRCR estimators are smaller than those of the classical pooling estimators in this situation ($\sigma_{\varepsilon_{ij}} = \rho = \mu_i = 0$) and other simulation situations (case of $\sigma_{\varepsilon_{ii}}$, $\sigma_{\varepsilon_{ij}}, \rho, \psi_k^2$ are increasing, and *df* is decreasing). In other words, GMG and GRCR are more efficient than CP-OLS and CP-SUR whether the regression coefficients are fixed or random.

**3**. If $T \geq 15$, the values of ATSE for the MG and GMG estimators are approximately equivalent. This result is consistent with Lemma 5.2. According to our study, this case ($T \geq 15$) is achieved when the sample size is moderate in Tables 1, 2, 4, and 5. Moreover, convergence slows down if $\sigma_{\varepsilon_{ii}}$, $\sigma_{\varepsilon_{ij}}$, and $\rho$ are increased. But the situation for the RCR and GRCR estimators is different; the convergence between them is very slow even if $T = 20$. So the MG and GMG estimators are more efficient than RCR in all simulation situations.

**4**. When the coefficients are random (whether they are distributed as normal or student's $t$), the values of ATSE for GMG and GRCR are smaller than those of MG and RCR in all simulation situations (for any $N$, $T$, $\sigma_{\varepsilon_{ii}}$, $\sigma_{\varepsilon_{ij}}$, and $\rho$). However, the ATSE values of GRCR are smaller than those of GMG estimator in most situations, especially when the sample size is moderate. In other words, the GRCR estimator performs better than all other estimators as long as the sample size is moderate regardless of other simulation factors.

## 7. CONCLUSION

In this article, the classical pooling (CP-OLS and CP-SUR), random-coefficients (RCR and GRCR), and mean group (MG and GMG) estimators of stationary RCPD models were examined in different sample sizes for the case where the errors are cross-sectionally and serially correlated. Analytical efficiency comparisons for these estimators indicate that the mean group and random-coefficients estimators are equivalent when $T$ is sufficiently large. Furthermore, the Monte Carlo simulation results show that the classical pooling estimators are absolutely not suitable for random-coefficients models. And, the MG and GMG estimators are more efficient than the RCR estimator for random- and fixed-coefficients models, especially when $T$ is small ($T \leq 12$). But when $T \geq 20$, the MG, GMG, and GRCR estimators are approximately equivalent. However, the GRCR estimator performs better than the MG and GMG estimators in most situations, especially in moderate samples. Therefore, we conclude that the GRCR estimator is suitable to stationary RCPD models whether the coefficients are random or fixed.

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