REVSTAT – Statistical Journal
Volume 14, Number 1, February 2016, 89–100

SHAPIRO–WILK TEST WITH KNOWN MEAN

Authors: Zofia Hanusz
– Department of Applied Mathematics and Computer Science,
University of Life Sciences in Lublin, Poland
zofia.hanusz@up.lublin.pl

Joanna Tarasinska
– Department of Applied Mathematics and Computer Science,
University of Life Sciences in Lublin, Poland
joanna.tarasinska@up.lublin.pl

Wojciech Zielinski
– Department of Econometrics and Statistics,
Warsaw University of Life Sciences, Poland
wojtek.zielinski@statystyka.info

Received: August 2014 Revised: October 2014 Accepted: November 2014

Abstract:

- An adaptation of the Shapiro–Wilk W test to the case of normality with a known mean is considered. The table of critical values for different sample sizes and several significance levels is given. The power of the test is investigated and compared with the Kolmogorov test and the two-step procedure consisting of the Shapiro–Wilk W and t tests. Additionally, the normalizing coefficients for the test statistic are given.

Key-Words:

- *normality; Shapiro–Wilk test; Kolmogorov test; Student t test; power.*

AMS Subject Classification:

- 62F03; 62E20.

1. INTRODUCTION AND MOTIVATION

One of the common problems in applications is to check whether the mean value of an investigated phenomena equals a given number, i.e. testing the hypothesis $H_0\colon \mu = \mu_0$. For example, for econometrical applications see [5], for biological applications [8], for engineering [11], for medical applications [7]. See also [1], [2], [4], [6].

To test the hypothesis H_0, the classical t test is used. However, this test requires the assumption of normality of the phenomena, so it is advised (see statistical packages such as SAS, Statistica, Statgraphics) to check normality first, for example with the Shapiro–Wilk W test. If normality is rejected, tests other than t are recommended (e.g. the sign test). So, the procedure of testing the hypothesis $H_0\colon \mu = \mu_0$ becomes a little complicated, and should be conducted in two steps:

1. check normality with the W test,

2. if normality is not rejected then use the t test else use the sign test.

In this paper we propose a modification of the Shapiro–Wilk W test, dedicated to checking normality with known mean value μ_0, i.e. to testing the hypothesis $H_0\colon X \sim N\left(\mu_0, \sigma^2\right)$, where X is the random variable of interest. This test could have very wide applications. For example, when we apply the paired t-test, the differences are assumed to be normally distributed with a given mean value $\mu_0 = \mu_1 - \mu_2$. The other application can be measurement errors which should be distributed as $N(0, \sigma^2)$, i.e. a measurement should be unbiased and normally distributed. Also, dimensions or weight of manufactured products should be normally distributed with given mean value. Another application is in the analysis of linear models, where one has to verify that residuals are normally distributed with null mean.

The modification of the W test and its properties are described in Section 2. The simulation results on its power are given in Section 3. Some concluding remarks are given in Section 4.

2. DERIVATION OF THE W_0 STATISTIC AND ITS PROPERTIES

Suppose that a random variable X is observed and we are interested in testing the hypothesis

$$H_0\colon\; X \sim N(\mu, \sigma^2)\;.$$

Shapiro and Wilk ([12]) proposed the W test based on the statistic

$$W = \frac{\left(\sum_{i=1}^{n} a_i X_{(i)}\right)^2}{\sum_{i=1}^{n} \left(X_i - \bar{X}\right)^2} , \tag{2.1}$$

where $X_{(1)} \le X_{(2)} \le \cdots \le X_{(n)}$ are the ordered values of a sample $X_1, X_2, ..., X_n$, and a_i are tabulated coefficients. A lower tail of W indicates nonnormality.

Now, let us assume that the expected value μ, say μ_0, is known. Thus it is of interest to test the null hypothesis

$$H_0\colon\ X \sim N(\mu_0, \sigma^2) . \tag{2.2}$$

Application of Shapiro and Wilk's technique to the problem of testing (2.2) gives the statistic

$$W_0 = \frac{\left(\sum_{i=1}^{n} a_i X_{(i)}\right)^2}{\sum_{i=1}^{n} (X_i - \mu_0)^2} .$$

The null hypothesis (2.2) is rejected when $W_0 < W_0(\alpha, n)$, where $W_0(\alpha, n)$ is the critical value at significance level α.

The statistic W_0 has properties similar to the W statistic, namely, W_0 is scale invariant and the maximum value of W_0 is one. As it is known, the minimum value of W is $\varepsilon = \dfrac{n\, a_1^2}{n-1}$ ([12]).

Lemma 2.1. *The minimum value of* W_0 *is zero.*

Proof: Since W_0 is scale invariant it suffices to consider the maximization of $\sum_{i=1}^{n} (X_i - \mu_0)^2$ subject to $\sum_{i=1}^{n} a_i X_{(i)} = 1$. The lemma follows from the fact that $\sum_{i=1}^{n} (X_i - \mu_0)^2$ may be arbitrarily large. □

Shapiro and Wilk ([12]) gave an analytic form of the probability density function for the W statistic in the case of sample size $n = 3$. It is

$$g(w) = \frac{3}{\pi} (1-w)^{-\frac{1}{2}}\, w^{-\frac{1}{2}} \qquad \text{for} \quad \frac{3}{4} \le w \le 1 . \tag{2.3}$$

They also establish that W is statistically independent of $\bar{X}$ and of $\sum_{i=1}^{n} \left(X_i - \bar{X}\right)^2$ for samples from a normal distribution.

Thus, it is easy to obtain the probability density function of W_0 for samples of size $n=3$. Let us notice that $W_0 = W\cdot C$, where

$$C \;=\; \frac{\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2}{\sum_{i=1}^{n}\left(X_i-\mu_0\right)^2} \;=\; \frac{\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2}{\sum_{i=1}^{n}\left(X_i-\bar{X}\right)^2+n\left(\bar{X}-\mu_0\right)^2}$$

is a random variable distributed as $Beta\left(\frac{n-1}{2},\frac{1}{2}\right)$, independent of W. Thus in the case of $n=3$, under H_0, we have the probability density function of C, namely,

$$f(c) \;=\; \frac{1}{2}\,(1-c)^{-\frac{1}{2}} \qquad \text{for} \quad 0\le c\le 1\ .$$

Taking the new variable $W_0 = W\cdot C$ in the joint probability density function $g(w)\,f(c)$ and integrating this function over c, we get the probability density function for W_0 in the following form

$$\varphi(w_0) \;=\; \begin{cases} \dfrac{3}{2\pi}\cdot w_0^{-\frac{1}{2}}\cdot \displaystyle\int_{w_0}^{\frac{4}{3}w_0}(1-c)^{-\frac{1}{2}}\,(c-w_0)^{-\frac{1}{2}}\;dc & \text{for} \quad 0\le w_0\le \frac{3}{4}\ , \\ \dfrac{3}{2\pi}\cdot w_0^{-\frac{1}{2}}\cdot \displaystyle\int_{w_0}^{1}(1-c)^{-\frac{1}{2}}\,(c-w_0)^{-\frac{1}{2}}\;dc & \text{for} \quad \frac{3}{4}\le w_0\le 1\ . \end{cases}$$

Finally, after integrating, we get

$$\varphi\,(w_0) = \begin{cases} \dfrac{3}{2\pi}\cdot w_0^{-\frac{1}{2}}\cdot\left(\arcsin\dfrac{5\,w_0-3}{3\,(1-w_0)}+\dfrac{\pi}{2}\right) & \text{for} \quad 0\le w_0\le \frac{3}{4}\ , \\ \dfrac{3}{2}\cdot w_0^{-\frac{1}{2}} & \text{for} \quad \frac{3}{4}\le w_0\le 1\ . \end{cases}$$

The plot of $\varphi\,(w_0)$ is shown in Figure 1.

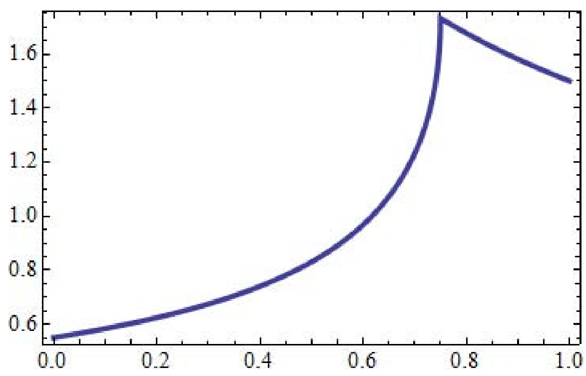

Figure 1: Plot of probability density function of W_0 for $n=3$.

For sample size $n > 3$ the analytical form of the null distribution of W_0 is not available. Hence, to obtain any information about the distribution, Monte Carlo simulations were performed. In simulations, for each $n = 3, 4, ..., 50$, $N =$ 1,000,000 samples from the distribution $N(0, 1)$ were drawn and for each sample the value of W_0 was calculated, thus the sample $w_1, w_2, ..., w_N$ of values of the W_0 statistic was obtained. The critical value $W_0(\alpha, n)$ was taken as the α-th quantile of $w_1, w_2, ..., w_N$. All calculations were done in the R program ([9]) using the procedure *shapiro.test* in which Royston's procedure is used ([10]). The same calculations were also done independently in Mathematica. The results are given in Table 1.

Table 1: Critical values of W_0 statistic for sample sizes n and significance level α.

n	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.1$	n	$\alpha = 0.01$	$\alpha = 0.05$	$\alpha = 0.1$
3	0.0184	0.0881	0.1714	27	0.7379	0.8232	0.8601
4	0.0721	0.2037	0.3127	28	0.7463	0.8287	0.8645
5	0.1419	0.3086	0.4190	29	0.7539	0.8340	0.8688
6	0.2090	0.3867	0.4952	30	0.7611	0.8394	0.8730
7	0.2742	0.4525	0.5543	31	0.7677	0.8437	0.8765
8	0.3299	0.5051	0.5998	32	0.7746	0.8482	0.8800
9	0.3785	0.5493	0.6374	33	0.7804	0.8524	0.8834
10	0.4233	0.5852	0.6682	34	0.7871	0.8565	0.8863
11	0.4606	0.6165	0.6935	35	0.7917	0.8602	0.8894
12	0.4940	0.6431	0.7154	36	0.7969	0.8634	0.8921
13	0.5246	0.6661	0.7346	37	0.8008	0.8670	0.8947
14	0.5494	0.6862	0.7504	38	0.8063	0.8701	0.8972
15	0.5739	0.7038	0.7651	39	0.8109	0.8731	0.8996
16	0.5954	0.7196	0.7778	40	0.8145	0.8760	0.9018
17	0.6126	0.7337	0.7890	41	0.8194	0.8787	0.9040
18	0.6319	0.7476	0.7998	42	0.8227	0.8816	0.9061
19	0.6478	0.7590	0.8088	43	0.8271	0.8839	0.9081
20	0.6626	0.7696	0.8176	44	0.8301	0.8862	0.9100
21	0.6761	0.7792	0.8250	45	0.8343	0.8887	0.9120
22	0.6876	0.7875	0.8319	46	0.8374	0.8911	0.9138
23	0.7008	0.7965	0.8390	47	0.8403	0.8931	0.9154
24	0.7104	0.8034	0.8446	48	0.8433	0.8951	0.9169
25	0.7205	0.8103	0.8501	49	0.8470	0.8974	0.9187
26	0.7296	0.8170	0.8553	50	0.8491	0.8989	0.9200

Shapiro and Wilk ([13]) approximated the distribution of the W statistic by a Johnson curve. For each n they made the least squares regression of the empirical sampling value of

$$u(p) = \ln \frac{W(p) - \varepsilon}{1 - W(p)}$$

on the p-th quantile of the standard normal distribution z_p, where ε was the minimum value of the W statistic and $W(p)$ was the p-th empirical sampling quantile. They took the following values of p:

$$p = 0.01,\ 0.02,\ 0.05\ (0.05)\ 0.25,\ 0.5,\ 0.75\ (0.05)\ 0.95,\ 0.98,\ 0.99\ ,$$

and gave the tables for ε, γ and δ such that $z = \gamma + \delta \ln \dfrac{W - \varepsilon}{1 - W}$ has approximately standard normal distribution.

In our study, a similar approach was applied for the W_0 statistic for sample sizes $n = 3, 4, ..., 50$. As $\varepsilon = 0$ (see Lemma 2.1), the least squares regression of $\ln \dfrac{W_0(p)}{1 - W_0(p)}$ on z_p was based on 1,000,000 pseudorandom samples from $N(0,1)$. The values of δ and γ, such that $Z = \gamma + \delta \ln \dfrac{W_0}{1 - W_0}$ has approximately standard normal distribution are listed in Table 2. The lower tail of Z's indicates nonnormality.

Table 2: The normalizing constants for W_0 for sample sizes n.

n	γ	δ	n	γ	δ	n	γ	δ
3	−0.3137	0.5551	19	−3.2563	1.3698	35	−4.4593	1.5241
4	−0.6479	0.7282	20	−3.3584	1.3847	36	−4.5088	1.5272
5	−0.9586	0.8510	21	−3.4511	1.3983	37	−4.5621	1.5336
6	−1.2299	0.9384	22	−3.5365	1.4095	38	−4.6152	1.5382
7	−1.4778	1.0092	23	−3.6320	1.4236	39	−4.6749	1.5467
8	−1.6950	1.0671	24	−3.7067	1.4319	40	−4.7186	1.5495
9	−1.8960	1.1157	25	−3.7869	1.4431	41	−4.7771	1.5574
10	−2.0790	1.1573	26	−3.8624	1.4520	42	−4.8195	1.5597
11	−2.2470	1.1929	27	−3.9346	1.4606	43	−4.8711	1.5659
12	−2.4039	1.2238	28	−4.0077	1.4703	44	−4.9137	1.5693
13	−2.5513	1.2517	29	−4.0770	1.4783	45	−4.9706	1.5769
14	−2.6821	1.2755	30	−4.1538	1.4891	46	−5.0118	1.5797
15	−2.8104	1.2979	31	−4.2084	1.4935	47	−5.0512	1.5826
16	−2.9320	1.3181	32	−4.2782	1.5030	48	−5.0908	1.5858
17	−3.0400	1.3350	33	−4.3354	1.5086	49	−5.1470	1.5935
18	−3.1553	1.3542	34	−4.4017	1.5172	50	−5.1795	1.5954

To check the goodness of approximation, another $N = 1{,}000{,}000$ pseudorandom samples from $N(0,1)$ were generated. For each of them W_{0i} and $Z_i = \gamma + \delta \ln \dfrac{W_{0i}}{1 - W_{0i}}$ were calculated ($i = 1, 2, ..., N$). The ratios $\dfrac{\#\{Z_i : Z_i \le z_p\}}{N}$ with $p = 0.01,\ 0.02,\ 0.05,\ 0.1,\ 0.5,\ 0.9,\ 0.95,\ 0.98,\ 0.99$ are given in Table 3.

Table 3: The simulated probabilities $P\left(\gamma+\delta\ \ln\frac{W_0}{1-W_0}\le z_p\right)$ for sample sizes n.

n	Probability								
	0.01	0.02	0.05	0.10	0.5	0.90	0.95	0.98	0.99
3	0.015	0.023	0.047	0.06	0.458	0.919	0.957	0.979	0.987
4	0.014	0.024	0.049	0.091	0.453	0.912	0.957	0.981	0.989
5	0.014	0.024	0.051	0.094	0.453	0.908	0.955	0.982	0.990
6	0.013	0.024	0.051	0.095	0.454	0.906	0.956	0.983	0.991
7	0.013	0.024	0.052	0.096	0.456	0.905	0.956	0.983	0.991
8	0.013	0.024	0.053	0.097	0.457	0.903	0.955	0.983	0.992
9	0.013	0.024	0.052	0.097	0.457	0.902	0.955	0.983	0.992
10	0.013	0.024	0.053	0.098	0.457	0.90	0.955	0.983	0.992
11	0.013	0.024	0.054	0.099	0.456	0.900	0.954	0.983	0.992
12	0.013	0.024	0.054	0.099	0.458	0.900	0.954	0.984	0.992
13	0.013	0.024	0.054	0.100	0.459	0.900	0.954	0.984	0.993
14	0.013	0.024	0.054	0.099	0.458	0.899	0.954	0.984	0.992
15	0.013	0.024	0.053	0.099	0.456	0.898	0.954	0.984	0.993
16	0.013	0.024	0.054	0.100	0.458	0.899	0.954	0.984	0.993
17	0.013	0.024	0.054	0.099	0.457	0.898	0.954	0.984	0.993
18	0.013	0.024	0.054	0.099	0.457	0.897	0.953	0.984	0.993
19	0.013	0.024	0.054	0.100	0.457	0.897	0.953	0.984	0.993
20	0.013	0.024	0.054	0.100	0.458	0.897	0.953	0.984	0.993
21	0.013	0.024	0.054	0.100	0.457	0.897	0.953	0.984	0.993
22	0.013	0.024	0.054	0.100	0.457	0.897	0.953	0.984	0.993
23	0.013	0.024	0.055	0.100	0.458	0.897	0.953	0.984	0.993
24	0.013	0.024	0.054	0.101	0.459	0.897	0.954	0.984	0.993
25	0.013	0.024	0.054	0.100	0.458	0.897	0.954	0.984	0.993
26	0.013	0.024	0.054	0.100	0.459	0.898	0.953	0.984	0.993
27	0.013	0.024	0.055	0.100	0.458	0.897	0.954	0.985	0.993
28	0.013	0.024	0.054	0.100	0.457	0.897	0.953	0.984	0.993
29	0.013	0.024	0.054	0.100	0.458	0.898	0.954	0.984	0.993
30	0.013	0.024	0.055	0.101	0.458	0.897	0.953	0.984	0.993
31	0.013	0.024	0.054	0.100	0.458	0.897	0.954	0.985	0.993
32	0.013	0.024	0.055	0.101	0.459	0.900	0.953	0.984	0.993
33	0.013	0.024	0.054	0.100	0.457	0.897	0.953	0.984	0.993
34	0.013	0.024	0.055	0.101	0.459	0.897	0.954	0.984	0.993
35	0.013	0.024	0.054	0.101	0.458	0.896	0.953	0.984	0.993
36	0.013	0.024	0.054	0.100	0.458	0.897	0.954	0.985	0.993
37	0.013	0.024	0.054	0.100	0.457	0.896	0.953	0.984	0.993
38	0.013	0.024	0.054	0.101	0.458	0.897	0.953	0.985	0.993
39	0.013	0.024	0.054	0.100	0.456	0.896	0.953	0.984	0.993
40	0.013	0.024	0.055	0.100	0.457	0.897	0.954	0.985	0.993
41	0.013	0.024	0.054	0.100	0.456	0.896	0.953	0.984	0.993
42	0.013	0.024	0.055	0.101	0.458	0.897	0.954	0.985	0.993
43	0.013	0.024	0.054	0.100	0.457	0.896	0.954	0.985	0.993
44	0.013	0.024	0.054	0.099	0.456	0.896	0.953	0.984	0.993
45	0.013	0.024	0.055	0.100	0.457	0.896	0.953	0.984	0.993
46	0.013	0.024	0.055	0.100	0.458	0.897	0.954	0.984	0.993
47	0.013	0.024	0.055	0.101	0.458	0.897	0.954	0.985	0.994
48	0.013	0.024	0.054	0.100	0.457	0.897	0.954	0.985	0.994
49	0.013	0.025	0.055	0.101	0.458	0.896	0.954	0.985	0.994
50	0.013	0.024	0.054	0.100	0.457	0.896	0.953	0.984	0.993

3. POWER COMPARISONS

Suppose that the hypothesis $H_0\colon X \sim N(\mu_0, \sigma^2)$ is verified using the W_0 test. Three kinds of alternative hypothesis are considered:

a) $X \sim N(\mu, \sigma^2)$ with $\mu \neq \mu_0$;

b) X is not normal with $\mu = \mu_0$;

c) X is not normal with $\mu \neq \mu_0$.

We focus on the power of the W_0 test. The Shapiro–Wilk W test was investigated against different nonnormal alternatives. Very exhaustive research was done by Shapiro *et al.* ([14]) and Chen ([3]). It was showed that the W test is very powerful in comparison to other normality tests such as Kolmogorov, chi-square, β_1, β_2 and against very different distributions including Student's t, Gamma, Beta or Uniform.

As the construction of W_0 is similar to the W test, it may be expected that the W_0 test will also be powerful against alternatives of kind b) and c). Hence, in our study we confine ourselves to the a) alternative, i.e. when the true distribution is normal with a mean other than μ_0. The W_0 test is compared with two other procedures. The first one is the Kolmogorov test (modified to the case of known mean). The test statistic of the Kolmogorov test is given by

$$\max_{1\leq i\leq n}\left\{\left|F\bigl(X_{(i)}\bigr)-\frac{i-1}{n}\right|,\ \left|F\bigl(X_{(i)}\bigr)-\frac{i}{n}\right|\right\},$$

where $F\bigl(X_{(i)}\bigr) = \Phi\Bigl(\frac{X_{(i)}-\mu_0}{S}\Bigr)$, $S = \frac{1}{n}\sqrt{\sum_{i=1}^{n}(X_i-\mu_0)^2}$ and Φ is the CDF of the standard normal distribution.

The second procedure, denoted by $W+t$, is a two-step one. In the first step the normality is verified by the classical W test. If normality is not rejected, then the hypothesis of equality of the mean to a given number μ_0 is verified by the t test.

All three tests are calculated at the significance level α. In the case of the $W+t$ test we need to apply two significance levels α_W and α_t for both tests. Those numbers were chosen in such a way that the overall significance level is α, i.e.

$$P_{H_0}\Bigl\{W \text{ accepts normality and } t \text{ accepts mean } \mu_0\Bigr\} \;\geq\; 1-(\alpha_W+\alpha_t) \;=\; 1-\alpha\ .$$

Because there are no preferences to the W or t test, $\alpha_W = \alpha_t = \frac{\alpha}{2}$ were taken. The power comparison of the three tests was performed by the Monte Carlo method. A sample of size n from the standard normal distribution was generated and this sample was used in all tests. The sample was then shifted to different values of μ and then each of the tests was applied to the shifted sample.

This procedure was repeated 10,000 times. The number of rejections of the hypothesis $H_0\colon X \sim N(\mu_0, \sigma^2)$ was calculated. It should be added that critical points for Kolmogorov test were also determined by Monte Carlo method. In the simulations the hypothesis H_0 was verified for samples of sizes $10, 20, 30, 40, 50$ and significance levels $\alpha = 0.01$, 0.05, 0.1. The simulated powers are given in Table 4.

Table 4: Power of W_0, Kolmogorov and $W+t$ tests.

n	μ	$\alpha=0.01$			$\alpha=0.05$			$\alpha=0.1$		
		W_0	K	$W+t$	W_0	K	$W+t$	W_0	K	$W+t$
10	0.0	0.011	0.010	0.010	0.050	0.049	0.048	0.101	0.100	0.098
	0.3	0.034	0.030	0.023	0.129	0.112	0.097	0.216	0.198	0.176
	0.6	0.148	0.115	0.097	0.386	0.326	0.287	0.526	0.464	0.422
	0.9	0.392	0.310	0.287	0.705	0.610	0.585	0.829	0.746	0.727
	1.2	0.687	0.565	0.560	0.916	0.845	0.847	0.964	0.925	0.924
	1.5	0.892	0.789	0.813	0.987	0.957	0.965	0.996	0.985	0.989
	1.8	0.974	0.920	0.943	0.998	0.991	0.995	1.000	0.998	0.999
	2.1	0.996	0.978	0.990	1.000	0.999	1.000	1.000	1.000	1.000
	2.4	1.000	0.993	0.997	1.000	1.000	1.000	1.000	1.000	1.000
20	0.0	0.011	0.011	0.010	0.049	0.050	0.051	0.096	0.099	0.096
	0.2	0.036	0.031	0.027	0.131	0.113	0.106	0.214	0.189	0.179
	0.4	0.165	0.123	0.121	0.383	0.313	0.297	0.515	0.442	0.424
	0.6	0.435	0.329	0.344	0.709	0.602	0.610	0.808	0.732	0.733
	0.8	0.741	0.601	0.656	0.916	0.842	0.861	0.960	0.913	0.955
	1.0	0.924	0.835	0.879	0.988	0.959	0.976	0.995	0.984	0.991
	1.2	0.988	0.950	0.978	0.999	0.994	0.997	1.000	0.998	0.999
	1.4	0.999	0.991	0.997	1.000	1.000	1.000	0.999	0.991	1.000
	1.6	1.000	0.999	1.000	1.000	1.000	1.000	1.000	0.999	1.000
	1.8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
30	0.00	0.012	0.009	0.012	0.054	0.053	0.051	0.105	0.103	0.097
	0.15	0.039	0.030	0.028	0.124	0.107	0.099	0.201	0.182	0.165
	0.30	0.150	0.111	0.106	0.350	0.285	0.267	0.478	0.408	0.388
	0.45	0.398	0.290	0.312	0.657	0.550	0.565	0.762	0.679	0.683
	0.60	0.693	0.540	0.608	0.876	0.793	0.817	0.930	0.878	0.889
	0.75	0.891	0.777	0.844	0.972	0.932	0.952	0.987	0.968	0.977
	0.90	0.975	0.922	0.959	0.996	0.985	0.992	0.999	0.994	0.997
	1.05	0.997	0.981	0.994	1.000	0.998	1.000	1.000	0.999	1.000
	1.20	1.000	0.997	0.999	1.000	1.000	1.000	1.000	1.000	1.000
	1.35	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.50	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
40	0.00	0.011	0.009	0.010	0.051	0.051	0.050	0.098	0.101	0.101
	0.15	0.042	0.034	0.033	0.144	0.125	0.114	0.236	0.206	0.196
	0.30	0.213	0.152	0.159	0.440	0.364	0.357	0.565	0.486	0.486
	0.45	0.539	0.402	0.459	0.776	0.672	0.702	0.861	0.789	0.802
	0.60	0.837	0.706	0.785	0.953	0.900	0.924	0.978	0.951	0.963
	0.75	0.972	0.910	0.955	0.995	0.981	0.991	0.998	0.993	0.996
	0.90	0.997	0.981	0.995	1.000	0.998	1.000	1.000	1.000	1.000
	1.05	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.20	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
50	0.00	0.010	0.008	0.011	0.050	0.050	0.050	0.099	0.100	0.099
	0.15	0.064	0.048	0.044	0.179	0.150	0.141	0.272	0.237	0.225
	0.30	0.301	0.212	0.224	0.538	0.444	0.452	0.661	0.579	0.578
	0.45	0.691	0.542	0.609	0.872	0.784	0.817	0.927	0.872	0.891
	0.60	0.937	0.840	0.905	0.983	0.956	0.973	0.993	0.978	0.986
	0.75	0.994	0.968	0.988	0.999	0.996	0.999	1.000	0.999	1.000
	0.90	1.000	0.998	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	1.05	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

The relative powers of W_0 with respect to the Kolmogorov and $W+t$ tests are shown in Figure 2. On the x-axis there are values of $\mu \geq 0$ and on the y-axis there are values of

$$\frac{\text{power of } W_0 \text{ test}}{\text{power of Kolmogorov test}} \text{ (solid line)} \quad \text{and} \quad \frac{\text{power of } W_0 \text{ test}}{\text{power of } W+t \text{ test}} \text{ (dashed line)} .$$

One can see that generally the lines are above 1, which shows that W_0 is more powerful than the other two tests.

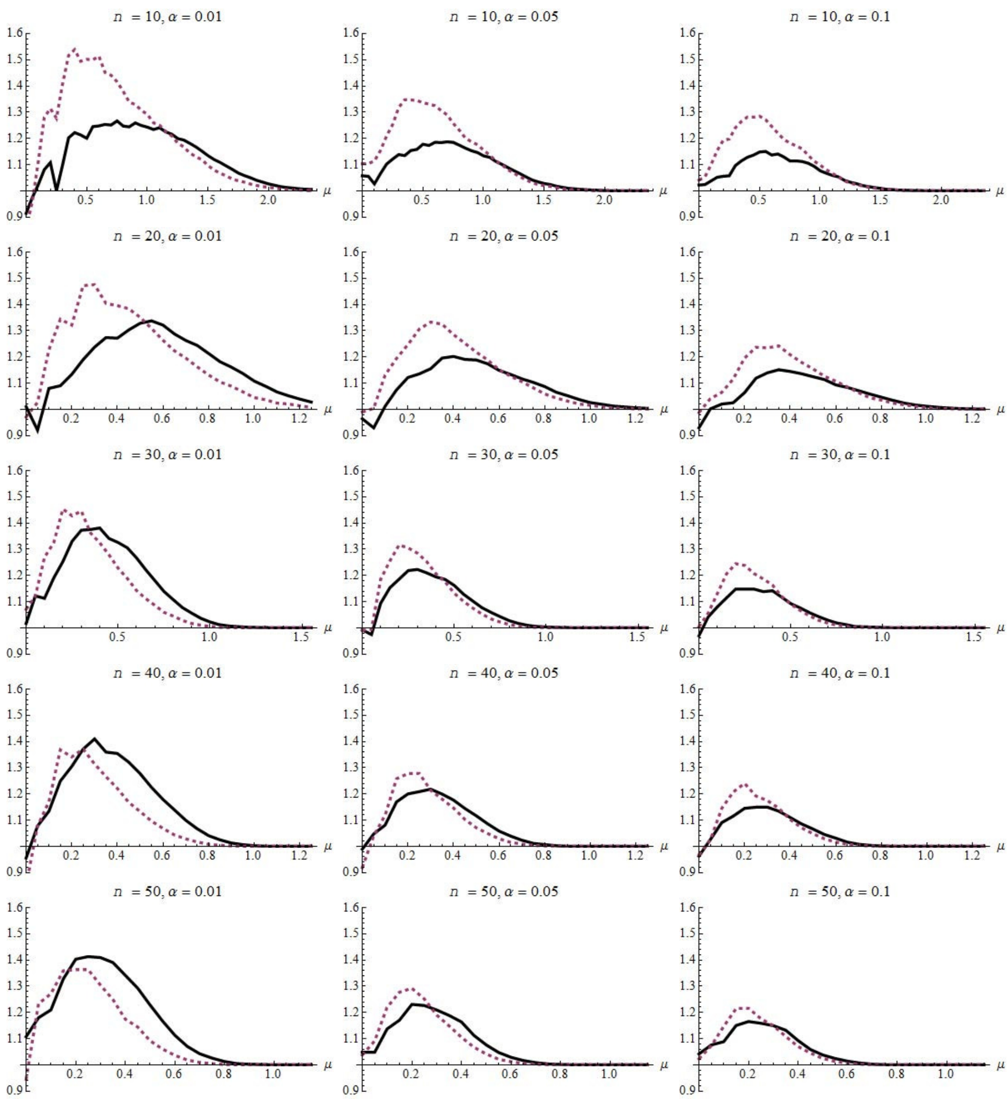

Figure 2: Relative power of W_0 with respect to Kolmogorov and $W+t$ tests.

4. CONCLUDING REMARKS

In many statistical models it is assumed that random variables are normally distributed with known mean. Thus the W_0 test is more adequate and should be used instead of the classical Shapiro–Wilk W test.

In the paper it is shown via a simulation study that the W_0 test is generally more powerful than the Kolmogorov, and W and Student t tests combined.

REFERENCES

[1] Bartoszynski, R. and Niewiadomska-Bugaj, M. (1996). *Probability and Statistical Inference*, Wiley Series in Probability and Statistics, John Wiley and Sons, New York.

[2] Bickel, P.J. and Doksum, K.A. (1980). *Mathematical Statistics*, Holden-Day, Inc., San Francisco.

[3] Chen, E.H. (1971). The power of the Shapiro–Wilk W test for normality in samples from contaminated normal distributions, *Journal of the American Statistical Association*, **66**, 760–762.

[4] Chow, G.C. (1983). *Econometrics*, Mc-Graw-Hill Book Company, New York.

[5] Greene, W.H. (2000). *Econometric Analysis*, Prentice Hall Inc., New Jersey.

[6] Knight, K. (2000). *Mathematical Statistics*, Chapman and Hall/CRC.

[7] Petrie, A. and Sabin, C. (2009). *Medical Statistics at a Glance*, Hoboken, Wiley-Blackwell, New Jersey.

[8] Quinn, G.P. and Keough, M.J. (2002). *Experimental Design and Data Analysis for Biologists*, Cambridge University Press.

[9] R Development Core Team (2008). *R: A language and environment for statistical computing*, R Foundation for Statistical Computing, Vienna, Austria, ISBN 3-900051-07-0, `http://www.R-project.org`.

[10] Royston, P. (1992). Approximating the Shapiro–Wilk W-test for non-normality, *Statistics and Computing*, **2**, 117–119.

[11] Ryan, T.P. (2007). *Modern Engineering Statistics*, Wiley-Interscience, Hoboken, New Jersey.

[12] Shapiro, S.S. and Wilk, M.B. (1965). An analysis of variance test for normality (complete samples), *Biometrika*, **52**, 591–611.

[13] Shapiro, S.S. and Wilk, M.B. (1968). Approximations for the null distribution of the W statistic, *Technometrics*, **10**, 861–866.

[14] Shapiro, S.S.; Wilk, M.B. and Chen, H.J. (1968). A comparative study of various tests for normality, *Journal of the American Statistical Association*, **63**, 1343–1372.