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Uncertainty Based Optimized Sampling Strategy for Population Mean Estimation under Correlated Measurement Errors

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Abstract:

- In sampling surveys and observational studies, the presence of correlated measurement errors (CME) under uncertainty can extremely distort the estimation of the population mean. The conventional estimators, which often assume independent and error-free measurements, may lose efficiency and lead to the biased conclusions in such complex situations. This article suggests an uncertainty based optimal class of estimator for the population mean under simple random sampling (SRS) that accounts for CMEs. Analytical properties of the proposed strategy are derived, including bias and mean square error (MSE) expressions, which highlight the advantages of the proposed method over the adapted estimators. A simulation study is executed to evaluate the performance under different correlation patterns and uncertainty levels, showing the significant gains in efficiency and stability. The practical significance of the strategy is also illustrated through a real data application. The findings suggest that the uncertainty based optimized sampling strategy offers a reliable and efficient alternative for practitioners dealing with CMEs in survey sampling.

Keywords:

- *uncertainty; correlated measurement errors; neutrosophic statistics; optimized sampling strategy.*

AMS Subject Classification:

- 62D05.

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1. INTRODUCTION

In sampling surveys, estimation of the population mean is a common issue and plays a significant role in decision-making across diverse realm such as public health, social sciences, economics, and official statistics (see, Sher et al., 2024 and Sher et al., 2025). The reliability and accuracy of such estimation, however, is often challenged by the measurement error (ME). The ME refers to the difference between the actual (true) value and the observed (recorded) value of a variable. It arises when the data collected from measuring instruments or respondents do not perfectly reflect the true characteristics of the population. In fact, these MEs may be caused by respondent biases, recording errors, or flaws in survey methodologies, which can cause observed results to differ from their true alternatives. The difficulty of estimation procedure rises considerably when these errors accumulate over time or between units, and may result in biased and inefficient estimators, if left unchecked. Cochran (1968) was among the first to draw attention to the adverse effects of such errors in survey data. Additionally, using different sampling strategies, a number of authors attempted to calibrate the impact of ME on various population parameters like mean, variance, etc., including Chandhok (1982), Manisha and Singh (2001), Singh and Karpe (2008), Singh and Karpe (2009), Singh and Karpe (2010), Zahid and Shabbir (2019), Zahid et al. (2022), and Bhushan et al. (2023b).

In many experiments or surveys, more than one characteristic is observed for the same individual (e.g., income and expenditure, crop yield and land area, weight and height). Since these responses are provided by the same source (device or respondent), the MEs are not always independent. This leads to the CME because the errors in different variables share a common source. Some real-world examples of CMEs are as follows:

- In household surveys, respondents may underreport both income and expenditure due to privacy concerns or fear of taxation. The underreporting errors in income and expenditure are likely to be positively correlated, since the same motive (tax evasion or privacy) affects both responses.
- Patients may underreport both alcohol and smoking consumption due to social desirability. The errors in alcohol and smoking data are negatively correlated with the true values but positively correlated with each other.
- In socio-economic surveys, households may overreport both number of assets and their monetary value due to prestige bias. Hence, the error in asset count is correlated with the error in asset valuation.
- Farmers may overestimate land area and crop yield when reporting to surveyors, especially if they believe it may influence subsidies or benefits. The overestimation of land area is correlated with the overestimation of crop production.

In survey sampling, Shalabh and Tsai (2017) examined the ratio and product estimators of population mean in the presence of CMEs using SRS. Later on, the impact of CMEs on population mean was calibrated by different renowned authors including, Bhushan et al. (2023a), Bhushan et al. (2023c), Kumar et al. (2023), Bhushan et al. (2024). Furthermore, Bhushan et al. (2025) developed some logarithmic imputation methods under CMEs, while Kumar et al. (2025a) assessed the impact of CMEs in the presence of missing data using ranked set sampling.

Uncertain data refers to information whose values are not known with complete accuracy due to several sources of ambiguity, incompleteness, or imprecision. In real-world applications such as sensor networks, medical diagnosis, financial forecasting, and social sciences, data are often affected by reporting errors, or variability in measurement devices, leading to uncertainty in recorded observations. Unlike precise data, uncertain data cannot be expressed by a single exact value; instead, it may be denoted by utilizing fuzzy sets, neutrosophic sets, or interval estimates to capture the degree of imprecision. In survey sampling, estimation of the population mean using fundamental estimation procedures can be severely hampered by the existence of uncertain data. Neutrosophic estimation methods furnish a robust statistical framework for handling uncertainty and indeterminacy in data that cannot be adequately addressed by the classical or fuzzy approaches. Unlike traditional estimation methods that assume precise data, the neutrosophic estimation methods incorporate three components like truth, indeterminacy, and falsity to model complex and imprecise information more realistically. Literature of survey sampling contains a sufficient amount of works for the estimation of population mean under uncertainty. Tahir et al. (2021) established the neutrosophic ratio type population mean estimation method using SRS. Later on, to estimate the population mean under uncertainty, Yadav and Smarandache (2023) proposed a generalized neutrosophic estimator. However, Yadav and Prasad (2024) developed the neutrosophic factor-type exponential estimators utilizing additional data. Alqudah et al. (2024) introduced the neutrosophic robust ratio-type estimator for the mean estimation. Singh and Kumari (2024) evaluated the neutrosophic ranked set sampling (NRSS) scheme for estimating the population mean. Kumari et al. (2024) suggested a new modification of ranked set sampling for estimating population mean. Kumar et al. (2025b) developed some efficient classes of estimators for estimating the indeterminate population mean under NRSS. Kumar and Kumar (2025c) established the uncertainty-based efficient neutrosophic imputation methods for the population mean. Al-Marzouki and Ahmad (2025a) developed a generalized class of neutrosophic estimators for estimation of the population mean, while Al-Marzouki and Ahmad (2025b) performed a case study on Islamabad stock exchange data under an enhanced class of neutrosophic estimators of population mean. Alomair and Ahmad, 2025 examined a newly developed comprehensive neutrosophic regression-cum-exponential type estimator for population mean. Kumar and Kumar (2025a) developed a neutrosophic imputation method for estimating the population mean of climate data under SRS. Purwar et al. (2025) proposed a neutrosophic regression-type estimator for the finite population mean and demonstrated its applicability using real-world datasets. Priya and Kumar (2025) introduced the robust neutrosophic exponential estimators of the population mean under uncertainty. Kumar and Kumar (2025b) conducted a simulation-based evaluation of neutrosophic exponential imputation methods for estimating the population mean using NRSS. Kumar and Priya (2026) further contributed by proposing a novel class of neutrosophic estimators for mean estimation under indeterminacy. Kumar and Kumar (2026) introduced a new neutrosophic imputation approach for the population mean estimation in the presence of indeterminacy. Basha and Usman (2026) developed an optimal neutrosophic difference-to-log-type estimator for the population mean, supported by numerical and simulation studies. Taneja et al. (2026) explored a neutrosophic exponential ratio-type estimator for estimating the finite population mean.

These neutrosophic estimation methods are particularly useful in survey sampling for estimating population mean under uncertainty. Moreover, addressing CMEs is essential for ensuring reliable statistical inferences, constructing robust estimators, and enhancing decision-making processes in the presence of uncertain data. However, to the best of our knowledge, no neutrosophic estimation methods currently exist for estimating the population mean when CMEs are present under uncertainty. Hence, the present study is conducted to achieve the following objectives:

- To incorporate neutrosophic approach and design an uncertainty-based optimized sampling strategy for estimating the population mean in the presence of CMEs.
- To obtain the theoretical properties such as bias and MSE of the proposed strategy.
- To examine the impact of CMEs on the efficiency of the adapted and proposed estimators.
- To conduct simulation study and real data illustration for comparing the performance of the proposed optimized strategy with the adapted methods in terms of efficiency.

The following sections comprise the structure of the article. The brief evaluation of the current estimators under CMEs is taken into consideration in Section 2. Section 3 lists the proposed optimized strategy and their characteristics under CMEs. By contrasting the MSE of the proposed strategy with the adapted ones in Section 4, the optimization criteria are obtained. In Section 5, a broad simulation analysis examines the theoretical results and the influence of the CMEs on the properties of different estimators. In Section 6, a real data illustration of the proposed method is presented. Finally, Section 7 concludes this study.

2. LITERATURE REVIEW

Let $\Xi = (\Xi_1, \Xi_2, \ldots, \Xi_N)$ denote a finite population. For the i^{th}, $i = 1, 2, \ldots, n_N$, unit in the population, let (X_{i_N}, Y_{i_N}) represent the true values of the auxiliary variable X_N and the study variable Y_N. The corresponding observed values are (x_{i_N}, y_{i_N}).

The observed values differ from the true values due to MEs, denoted as (u_{i_N}, v_{i_N}). Thus,

$$y_{i_N} = Y_{i_N} + u_{i_N}, \quad x_{i_N} = X_{i_N} + v_{i_N},$$

where u_{i_N} and v_{i_N} are unobservable random errors associated with Y_{i_N} and X_{i_N}, respectively. These errors have mean zero, variances $(\sigma^2_{u_N}, \sigma^2_{v_N})$, and correlation coefficient ρ_{uv_N}.

The population variances of X_N and Y_N are denoted by $\sigma^2_{x_N}$ and $\sigma^2_{y_N}$, respectively, while their coefficients of variation are C_{x_N} and C_{y_N}. The population means of study and auxiliary variables are denoted by μ_{x_N} and μ_{y_N} corresponding to the sample means as $\bar{x}_N$ and $\bar{y}_N$ respectively. Also, the correlation coefficient between X_N and Y_N is ρ_{xy_N}.

To study the properties of the neutrosophic estimators under CME, the following standardized transformations are used:

$$w_{y_N} = \frac{\sum(Y_{i_N} - \mu_{y_N})}{\sqrt{n_N}}, \quad w_{x_N} = \frac{\sum(X_{i_N} - \mu_{x_N})}{\sqrt{n_N}}, \quad w_{u_N} = \frac{\sum u_{i_N}}{\sqrt{n_N}}, \quad w_{v_N} = \frac{\sum v_{i_N}}{\sqrt{n_N}}.$$

Accordingly, the sample means can be expressed as

$$\bar{y}_N = \mu_{y_N} + n_N^{-1/2}(w_{y_N} + w_{u_N}), \quad \bar{x}_N = \mu_{x_N} + n_N^{-1/2}(w_{x_N} + w_{v_N}),$$

showing that $\bar{y}_N$ is a neutrosophic unbiased estimator of the population mean with variance

$$V(\bar{y}_N) = \gamma(\sigma_{y_N}^2 + \sigma_{u_N}^2), \quad \text{where } \gamma = \frac{1}{n_N}.$$

According to Cochran (1977), the ratio estimator performs better when study and auxiliary variables are positively correlated. However, the regression estimator performs better when the study and auxiliary variables are positively correlated but the regression line does not pass through the origine. Following Cochran (1977) and Tahir et al. (2021), the neutrosophic ratio and regression estimators are prescribed under CME for the estimation of population mean as

$$T_r = \bar{y}_N \frac{\mu_{x_N}}{\bar{x}_N},$$
$$T_{lr} = \bar{y}_N + \beta_1(\mu_{x_N} - \bar{x}_N),$$

where $\beta_{1(opt)} = -\{(\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N})/(\sigma_{x_N}^2 + \sigma_{v_N}^2)\}$ is an optimizing scalar.

The approximate MSE under CME for the neutrosophic ratio and regression estimators is given by

$$MSE(T_r) \approx \gamma \left[\begin{array}{l} \sigma_{y_N}^2 \left\{ 1 - 2\left(\frac{\sigma_{x_N}\mu_{y_N}}{\sigma_{y_N}\mu_{x_N}}\right)\rho_{xy_N} + \left(\frac{\sigma_{x_N}\mu_{y_N}}{\sigma_{y_N}\mu_{x_N}}\right)^2 \right\} \\ +\sigma_{u_N}^2 \left\{ 1 - 2\left(\frac{\sigma_{v_N}\mu_{y_N}}{\sigma_{u_N}\mu_{x_N}}\right)\rho_{uv_N} + \left(\frac{\sigma_{v_N}\mu_{y_N}}{\sigma_{u_N}\mu_{x_N}}\right)^2 \right\} \end{array} \right],$$

$$min.MSE(T_{lr}) \approx \mu_{y_N}^2 \gamma \left\{ \left(\frac{\sigma_{y_N}^2 + \sigma_{u_N}^2}{\mu_{y_N}^2}\right) - \frac{(\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N})^2}{\mu_{y_N}^2(\sigma_{x_N}^2 + \sigma_{v_N}^2)} \right\}.$$

To increase the efficiency of the estimation procedure, following Srivastava (1967), we prescribe the neutrosophic power ratio estimator for population mean under CME as follows:

$$T_{pr} = \bar{y}_N \left(\frac{\mu_{x_N}}{\bar{x}_N}\right)^{\beta_2},$$

where $\beta_{2(opt)} = \{\mu_{x_N}(\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N})/\mu_{y_N}(\sigma_{x_N}^2 + \sigma_{v_N}^2)\}$ is an optimizing scalar. The approximate minimum MSE at $\beta_{2(opt)}$ under CME for the neutrosophic power ratio estimator is given by

$$min.MSE(T_{pr}) \approx \mu_{y_N}^2 \gamma \left\{ \left(\frac{\sigma_{y_N}^2 + \sigma_{u_N}^2}{\mu_{y_N}^2}\right) - \frac{(\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N})^2}{\mu_{y_N}^2(\sigma_{x_N}^2 + \sigma_{v_N}^2)} \right\}.$$

Remark 2.1. The neutrosophic power ratio estimator T_{pr} attains the minimum MSE of the neutrosophic regression estimator T_{lr}.

Searls (1964) introduced the concept of enhancing efficiency of the estimator through the use of a multiplicative tuning constant. Specifically, the conventional estimator is scaled by a constant factor, which is determined in such a way that the MSE of the modified estimator is minimized. This approach allows for a reduction in variance, and in some cases a controlled introduction of bias, leading to a more efficient estimator under appropriate conditions. Following Searls (1964) and Bhushan et al. (2024), we prescribe the Searls-type neutrosophic regression and ratio estimators for the population mean under CME:

$$T_{slr} = \alpha_1 \bar{y}_N + \beta_1(\mu_{x_N} - \bar{x}_N),$$
$$T_{spr} = \alpha_2 \bar{y}_N \left(\frac{\mu_{x_N}}{\bar{x}_N}\right)^{\beta_2},$$

where α_1, α_2, β_1, and β_2 are optimizing scalars. These neutrosophic estimators have the following approximate MSE:

$$MSE(T_{slr}) \approx \left\{ \begin{array}{l} (\alpha_1 - 1)^2\mu_{y_N}^2 + \alpha_1^2\gamma(\sigma_{y_N}^2 + \sigma_{u_N}^2) + \beta_1^2\gamma(\sigma_{x_N}^2 + \sigma_{v_N}^2) \\ +2\alpha_1\beta_1\gamma(\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N}) \end{array} \right\}, \tag{2.1}$$

$$MSE(T_{spr}) \approx \mu_{y_N}^2(1 + \alpha_2^2 a - 2\alpha_2 b), \tag{2.2}$$

where

$$a = 1 + \gamma\left(\frac{\sigma_{y_N}^2 + \sigma_{u_N}^2}{\mu_{y_N}^2}\right) + \beta_2(2\beta_2+1)\gamma\left(\frac{\sigma_{x_N}^2 + \sigma_{v_N}^2}{\mu_{x_N}^2}\right) - 4\beta_2\gamma\left(\frac{\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N}}{\mu_{x_N}\mu_{y_N}}\right),$$

$$b = 1 + \frac{\beta_2(2\beta_2+1)}{2}\gamma\left(\frac{\sigma_{y_N}^2 + \sigma_{u_N}^2}{\mu_{y_N}^2}\right) - \beta_2\gamma\left(\frac{\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N}}{\mu_{x_N}\mu_{y_N}}\right).$$

The optimum values of scalars α_1, α_2, β_1, and β_2 are given by

$$\alpha_{1(opt)} = \frac{\mu_{y_N}^2}{\mu_{y_N}^2 + \gamma(\sigma_{y_N}^2 + \sigma_{u_N}^2) - \frac{\gamma(\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N})^2}{\sigma_{x_N}^2 + \sigma_{v_N}^2}},$$

$$\beta_{1(opt)} = -\alpha_{1(opt)}\frac{(\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N})}{\sigma_{x_N}^2 + \sigma_{v_N}^2},$$

$$\alpha_{2(opt)} = \frac{b}{a},$$

$$\beta_{2(opt)} = \frac{\mu_{x_N}}{\mu_{y_N}}\left\{\frac{(\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N})}{(\sigma_{x_N}^2 + \sigma_{v_N}^2)}\right\}.$$

The minimum MSE can be obtained by putting optimum values of α_1, α_2, β_1, and β_2 in (2.1) and (2.2) as

$$min.MSE(T_{slr}) \approx \mu_{y_N}^2(1 - \alpha_{1(opt)}),$$
$$min.MSE(T_{spr}) \approx \mu_{y_N}^2\left(1 - \frac{b^2}{a}\right).$$

3. PROPOSED SAMPLING STRATEGY

The need for proposing an optimal strategy based on SRS in the presence of CME in uncertain data arises from the fact that the traditional sampling strategy does not accurately handle the issue of CME under uncertain data, which is rarely true in practice. By explicitly accounting for CME under uncertainty, this SRS based optimized strategy reduces MSE, improves accuracy, and ensures the robust statistical inferences, which is given as

$$T_p = \left[\lambda_1 \bar{y}_N + \lambda_2 \bar{y}_N \left\{ \frac{\mu_{x_N}}{\theta \bar{x}_N + (1-\theta)\mu_{x_N}} \right\}^g \right] \left\{ 1 + \log\left(\frac{\bar{x}_N}{\mu_{x_N}} \right) \right\}^\phi,$$

where λ_1, λ_2, and ϕ are optimizing scalars. Also, g and θ are constants.

Theorem 3.1. *The bias, MSE, and minimum MSE of the proposed neutrosophic estimator T_p under the CME is given by*

$$Bias(T_p) \approx \mu_{y_N}(\lambda_1 D + \lambda_2 E - 1), \tag{3.1}$$

$$MSE(T_p) \approx \mu_{y_N}^2 (1 + \lambda_1^2 A + \lambda_2^2 B + 2\lambda_1\lambda_2 C - 2\lambda_1 D - 2\lambda_2 E), \tag{3.2}$$

$$min.MSE(T_p) \approx \mu_{y_N}^2 (1 - \Gamma). \tag{3.3}$$

where

$$\Gamma = \frac{(AE^2 + BD^2 - 2CDE)}{(AB - C^2)},$$

$$A = 1 + \gamma\left(\frac{\sigma_{y_N}^2 + \sigma_{u_N}^2}{\mu_{y_N}^2}\right) + 2\phi(\phi-1)\gamma\left(\frac{\sigma_{x_N}^2 + \sigma_{v_N}^2}{\mu_{x_N}^2}\right) + 4\phi\gamma\left(\frac{\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N}}{\mu_{x_N}\mu_{y_N}}\right),$$

$$B = \left\{ \begin{array}{l} 1 + \gamma\left(\frac{\sigma_{y_N}^2 + \sigma_{u_N}^2}{\mu_{y_N}^2}\right) + \left(\begin{array}{l} 2\phi(\phi-1) + g(g+1)\theta^2 + g^2\theta^2 \\ -4\phi g\theta \end{array} \right)\gamma\left(\frac{\sigma_{x_N}^2 + \sigma_{v_N}^2}{\mu_{x_N}^2}\right) \\ +4(\phi - g\theta)\gamma\left(\frac{\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N}}{\mu_{x_N}\mu_{y_N}}\right) \end{array} \right\},$$

$$C = \left\{ \begin{array}{l} 1 + \gamma\left(\frac{\sigma_{y_N}^2 + \sigma_{u_N}^2}{\mu_{y_N}^2}\right) + \left(\frac{g(g+1)}{2}\theta^2 - 2\phi g\theta + 2\phi(\phi-1) \right)\gamma\left(\frac{\sigma_{x_N}^2 + \sigma_{v_N}^2}{\mu_{x_N}^2}\right) \\ +2(2\phi - g\theta)\gamma\left(\frac{\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N}}{\mu_{x_N}\mu_{y_N}}\right) \end{array} \right\},$$

$$D = 1 + \phi\gamma\left(\frac{\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N}}{\mu_{x_N}\mu_{y_N}}\right) + \frac{\phi(\phi-2)}{2}\gamma\left(\frac{\sigma_{x_N}^2 + \sigma_{v_N}^2}{\mu_{x_N}^2}\right),$$

$$E = \left\{ \begin{array}{l} 1 + \left(\frac{g(g+1)}{2}\theta^2 - \phi g\theta + \frac{\phi(\phi-2)}{2} \right)\gamma\left(\frac{\sigma_{x_N}^2 + \sigma_{v_N}^2}{\mu_{x_N}^2}\right) \\ +(\phi - g\theta)\gamma\left(\frac{\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N}}{\mu_{x_N}\mu_{y_N}}\right) \end{array} \right\}.$$

Proof: Consider the proposed neutrosophic estimator under CME as

$$T_p = \left[\lambda_1 \bar{y}_N + \lambda_2 \bar{y}_N \left\{ \frac{\mu_{x_N}}{\theta \bar{x}_N + (1-\theta)\mu_{x_N}} \right\}^g \right] \left\{ 1 + \log\left(\frac{\bar{x}_N}{\mu_{x_N}} \right) \right\}^\phi.$$

We can express the above estimator using the notations provided in the previous section as

$$T_p \approx \left[\begin{array}{l} \left\{ \begin{array}{l} \lambda_1\left(\mu_{y_N} + n_N^{-1/2}(w_{y_N} + w_{u_N})\right) + \lambda_2\left(\mu_{y_N} + n_N^{-1/2}(w_{y_N} + w_{u_N})\right) \\ \left(\frac{\mu_{x_N}}{\theta\left(\mu_{x_N} + n_N^{-1/2}(w_{x_N} + w_{v_N})\right) + (1-\theta)\mu_{x_N}} \right)^g \end{array} \right\} \\ \left\{ 1 + \log\left(\frac{\mu_{x_N} + n_N^{-1/2}(w_{x_N} + w_{v_N})}{\mu_{x_N}} \right) \right\}^\phi \end{array} \right],$$

$$T_p \approx \left[\begin{array}{l} \left\{\begin{array}{l} \left(\lambda_1\mu_{y_N} + \lambda_1 n_N^{-1/2}(w_{y_N} + w_{u_N})\right) + \left(\lambda_2\mu_{y_N} + \lambda_2 n_N^{-1/2}(w_{y_N} + w_{u_N})\right) \\ \left(1 + \theta n_N^{-1/2}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)\right)^{-g} \end{array}\right\} \\ \left\{1 + \log\left(1 + n_N^{-1/2}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)\right)\right\}^{\phi} \end{array}\right],$$

$$T_p \approx \left[\begin{array}{l} \left\{\begin{array}{l} \lambda_1\mu_{y_N} + \lambda_1 n_N^{-1/2}(w_{y_N} + w_{u_N}) + \lambda_2\mu_{y_N} + \lambda_2 n_N^{-1/2}(w_{y_N} + w_{u_N}) \\ -\lambda_2 g\theta n_N^{-1/2}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)\mu_{y_N} + \lambda_2\left(\dfrac{g(g+1)}{2}\right)\theta^2 n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2\mu_{y_N} \\ -\lambda_2 g\theta n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)(w_{y_N} + w_{u_N}) \end{array}\right\} \\ \times\left\{1 + \phi n_N^{-1/2}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right) + \dfrac{\phi(\phi-2)}{2} n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2\right\} \end{array}\right],$$

$$T_p \approx \left[\begin{array}{l} \lambda_1\mu_{y_N} + \lambda_1 n_N^{-1/2}(w_{y_N} + w_{u_N}) + \lambda_2\mu_{y_N} + \lambda_2 n_N^{-1/2}(w_{y_N} + w_{u_N}) \\ -\lambda_2 g\theta n_N^{-1/2}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)\mu_{y_N} + \lambda_2\left(\dfrac{g(g+1)}{2}\right)\theta^2 n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2\mu_{y_N} \\ -\lambda_2 g\theta n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)(w_{y_N} + w_{u_N}) + \lambda_1 n_N^{-1/2}\phi\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)\mu_{y_N} \\ +\lambda_1 n_N^{-1}\phi\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)(w_{y_N} + w_{u_N}) + \lambda_2 n_N^{-1/2}\phi\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)\mu_{y_N} \\ +\lambda_2 n_N^{-1}\phi\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)(w_{y_N} + w_{u_N}) - \lambda_2 g\theta n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2\mu_{y_N} \\ +\lambda_1\left(\dfrac{\phi(\phi-2)}{2}\right)n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2\mu_{y_N} + \lambda_2\left(\dfrac{\phi(\phi-2)}{2}\right)n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2\mu_{y_N} \end{array}\right],$$

$$T_p \approx \mu_{y_N}\left[\begin{array}{l} \lambda_1 + \lambda_1 n_N^{-1/2}\left(\frac{w_{y_N}+w_{u_N}}{\mu_{y_N}}\right) + \lambda_2 + \lambda_2 n_N^{-1/2}\left(\frac{w_{y_N}+w_{u_N}}{\mu_{y_N}}\right) \\ -\lambda_2 g\theta n_N^{-1/2}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right) + \lambda_2\left(\dfrac{g(g+1)}{2}\right)\theta^2 n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2 \\ -\lambda_2 g\theta n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)\left(\frac{w_{y_N}+w_{u_N}}{\mu_{y_N}}\right) + \lambda_1 n_N^{-1/2}\phi\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right) \\ +\lambda_2 n_N^{-1/2}\phi\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right) + \lambda_1 n_N^{-1}\phi\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)\left(\frac{w_{y_N}+w_{u_N}}{\mu_{y_N}}\right) \\ +\lambda_2 n_N^{-1}\phi\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)\left(\frac{w_{y_N}+w_{u_N}}{\mu_{y_N}}\right) - \lambda_2 g\theta n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2 \\ +\lambda_1\left(\dfrac{\phi(\phi-2)}{2}\right)n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2 + \lambda_2\left(\dfrac{\phi(\phi-2)}{2}\right)n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2 \end{array}\right],$$

(3.4)

$$T_p - \mu_{y_N} \approx \mu_{y_N}\left[\begin{array}{l} \lambda_1\left\{\begin{array}{l} 1 + n_N^{-1/2}\left(\frac{w_{y_N}+w_{u_N}}{\mu_{y_N}}\right) + \phi n_N^{-1/2}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right) \\ +\phi n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)\left(\frac{w_{y_N}+w_{u_N}}{\mu_{y_N}}\right) \\ +\dfrac{\phi(\phi-2)}{2}n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2 \end{array}\right\} \\ +\lambda_2\left\{\begin{array}{l} 1 + n_N^{-1/2}\left(\frac{w_{y_N}+w_{u_N}}{\mu_{y_N}}\right) + \phi n_N^{-1/2}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right) \\ -g\theta n_N^{-1/2}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right) - \phi g\theta n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2 \\ +\dfrac{g(g+1)}{2}\theta^2 n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2 \\ +n_N^{-1}(\phi - g\theta)\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)\left(\frac{w_{y_N}+w_{u_N}}{\mu_{y_N}}\right) \\ +\dfrac{\phi(\phi-2)}{2}n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2 \end{array}\right\} - 1 \end{array}\right].$$

The bias can be obtained by employing expectation on (3.4) as

$$E(T_p-\mu_{y_N})\approx\mu_{y_N}E\left[\begin{array}{l}\lambda_1\left\{\begin{array}{l}1+\phi n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)\left(\frac{w_{y_N}+w_{u_N}}{\mu_{y_N}}\right)\\+\dfrac{\phi(\phi-2)}{2}n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2\end{array}\right\}\\+\lambda_2\left\{\begin{array}{l}1+\left(\begin{array}{l}\dfrac{g(g+1)}{2}\theta^2\\-\phi g\theta\\+\dfrac{\phi(\phi-2)}{2}\end{array}\right)n_N^{-1}\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)^2\\+n_N^{-1}(\phi-g\theta)\left(\frac{w_{x_N}+w_{v_N}}{\mu_{x_N}}\right)\left(\frac{w_{y_N}+w_{u_N}}{\mu_{y_N}}\right)\end{array}\right\}-1\end{array}\right],$$

$$Bias(T_p)\approx\mu_{y_N}\left[\begin{array}{l}\lambda_1\left\{1+\phi\gamma\left(\frac{\rho_{xy_N}\sigma_{x_N}\sigma_{y_N}+\rho_{uv_N}\sigma_{u_N}\sigma_{v_N}}{\mu_{x_N}\mu_{y_N}}\right)+\dfrac{\phi(\phi-2)}{2}\gamma\left(\dfrac{\sigma_{x_N}^2+\sigma_{v_N}^2}{\mu_{x_N}^2}\right)\right\}\\+\lambda_2\left\{\begin{array}{l}1+\left(\dfrac{g(g+1)}{2}\theta^2-\phi g\theta+\dfrac{\phi(\phi-2)}{2}\right)\gamma\left(\dfrac{\sigma_{x_N}^2+\sigma_{v_N}^2}{\mu_{x_N}^2}\right)\\+(\phi-g\theta)\gamma\left(\frac{\rho_{xy_N}\sigma_{x_N}\sigma_{y_N}+\rho_{uv_N}\sigma_{u_N}\sigma_{v_N}}{\mu_{x_N}\mu_{y_N}}\right)\end{array}\right\}-1\end{array}\right].$$

The aforementioned mathematical expression can also be expressed as

$$Bias(T_p)\approx\mu_{y_N}(\lambda_1D+\lambda_2E-1). \tag{3.5}$$

The first-order approximated MSE of the estimator T_p can be obtained by squaring and employing expectation on (3.4) as

$$MSE(T_p)\approx\mu_{y_N}^2\left[\begin{array}{l}1+\lambda_1^2\left\{\begin{array}{l}1+\gamma\left(\dfrac{\sigma_{y_N}^2+\sigma_{u_N}^2}{\mu_{y_N}^2}\right)+2\phi(\phi-1)\gamma\left(\dfrac{\sigma_{x_N}^2+\sigma_{v_N}^2}{\mu_{x_N}^2}\right)\\+4\phi\gamma\left(\frac{\rho_{xy_N}\sigma_{x_N}\sigma_{y_N}+\rho_{uv_N}\sigma_{u_N}\sigma_{v_N}}{\mu_{x_N}\mu_{y_N}}\right)\end{array}\right\}\\+\lambda_2^2\left\{\begin{array}{l}1+\gamma\left(\dfrac{\sigma_{y_N}^2+\sigma_{u_N}^2}{\mu_{y_N}^2}\right)\\+\left(\begin{array}{l}2\phi(\phi-1)+g(g+1)\theta^2\\+g_N^2\theta^2-4\phi g\theta\end{array}\right)\gamma\left(\dfrac{\sigma_{x_N}^2+\sigma_{v_N}^2}{\mu_{x_N}^2}\right)\\+4(\phi-g\theta)\gamma\left(\frac{\rho_{xy_N}\sigma_{x_N}\sigma_{y_N}+\rho_{uv_N}\sigma_{u_N}\sigma_{v_N}}{\mu_{x_N}\mu_{y_N}}\right)\end{array}\right\}\\+2\lambda_1\lambda_2\left\{\begin{array}{l}1+\gamma\left(\dfrac{\sigma_{y_N}^2+\sigma_{u_N}^2}{\mu_{y_N}^2}\right)\\+\left(\begin{array}{l}\dfrac{g(g+1)}{2}\theta^2-2\phi g\theta\\+2\phi(\phi-1)\end{array}\right)\gamma\left(\dfrac{\sigma_{x_N}^2+\sigma_{v_N}^2}{\mu_{x_N}^2}\right)\\+2(2\phi-g\theta)\gamma\left(\frac{\rho_{xy_N}\sigma_{x_N}\sigma_{y_N}+\rho_{uv_N}\sigma_{u_N}\sigma_{v_N}}{\mu_{x_N}\mu_{y_N}}\right)\end{array}\right\}\\-2\lambda_1\left\{\begin{array}{l}1+\dfrac{\phi(\phi-2)}{2}\gamma\left(\dfrac{\sigma_{x_N}^2+\sigma_{v_N}^2}{\mu_{x_N}^2}\right)\\+\phi\gamma\left(\frac{\rho_{xy_N}\sigma_{x_N}\sigma_{y_N}+\rho_{uv_N}\sigma_{u_N}\sigma_{v_N}}{\mu_{x_N}\mu_{y_N}}\right)\end{array}\right\}\\-2\lambda_2\left\{\begin{array}{l}1+\left(\begin{array}{l}\dfrac{g(g+1)}{2}\theta^2-\phi g\theta\\+\dfrac{\phi(\phi-2)}{2}\end{array}\right)\gamma\left(\dfrac{\sigma_{x_N}^2+\sigma_{v_N}^2}{\mu_{x_N}^2}\right)\\+(\phi-g\theta)\left(\frac{\rho_{xy_N}\sigma_{x_N}\sigma_{y_N}+\rho_{uv_N}\sigma_{u_N}\sigma_{v_N}}{\mu_{x_N}\mu_{y_N}}\right)\end{array}\right\}\end{array}\right]. \tag{3.6}$$

We can express the above equation as

$$MSE(T_p) \approx \mu_{y_N}^2 (1 + \lambda_1^2 A + \lambda_2^2 B + 2\lambda_1\lambda_2 C - 2\lambda_1 D - 2\lambda_2 E). \tag{3.7}$$

The optimum values of scalars λ_1, λ_2, and ϕ are given by

$$\lambda_{1(opt)} = \frac{BD - CE}{AB - C^2}, \quad \lambda_{2(opt)} = \frac{AE - CD}{AB - C^2}, \quad \text{and} \quad \phi_{(opt)} = -\frac{(\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N})}{R(\sigma_{x_N}^2 + \sigma_{v_N}^2)}.$$

Putting optimum values of λ_1 and λ_2 in (3.7), results in the minimum MSE as

$$min.MSE(T_p) \approx \mu_{y_N}^2 \left(1 - \frac{AE^2 + BD^2 - 2CDE}{AB - C^2}\right) = \mu_{y_N}^2 (1 - \Gamma).$$

□

4. OPTIMALITY CRITERIA

In this section, some optimality criteria are determined under which the proposed strategy performs better than the adapted ones.

1. Contrasting the minimum MSE of the proposed neutrosophic estimator T_p with the neutrosophic mean estimator $\bar{y}_N$, results in:

$$\begin{aligned} min.MSE(T_p) &< V(\bar{y}_N) \\ \mu_{y_N}^2 (1 - \Gamma) &< \gamma \left(\sigma_{y_N}^2 + \sigma_{u_N}^2\right) \\ \implies \Gamma &> 1 - \frac{\gamma(\sigma_{y_N}^2 + \sigma_{u_N}^2)}{\mu_{y_N}^2}. \end{aligned} \tag{4.1}$$

2. Contrasting the minimum MSE of the proposed neutrosophic estimator T_p with neutrosophic ratio estimator T_r, results in:

$$min.MSE(T_p) < MSE(T_r)$$

$$\mu_{y_N}^2 (1 - \Gamma) < \gamma \left[\begin{array}{l} \sigma_{y_N}^2 \left\{ 1 - 2\left(\frac{\sigma_{x_N}\mu_{y_N}}{\sigma_{y_N}\mu_{x_N}}\right)\rho_{xy_N} + \left(\frac{\sigma_{x_N}\mu_{y_N}}{\sigma_{y_N}\mu_{x_N}}\right)^2 \right\} \\ +\sigma_{u_N}^2 \left\{ 1 - 2\left(\frac{\sigma_{v_N}\mu_{y_N}}{\sigma_{u_N}\mu_{x_N}}\right)\rho_{uv_N} + \left(\frac{\sigma_{v_N}\mu_{y_N}}{\sigma_{u_N}\mu_{x_N}}\right)^2 \right\} \end{array} \right]$$

$$\implies \Gamma > 1 - \frac{\gamma}{\mu_{y_N}^2} \left[\begin{array}{l} \sigma_{y_N}^2 \left\{ 1 - 2\left(\frac{\sigma_{x_N}\mu_{y_N}}{\sigma_{y_N}\mu_{x_N}}\right)\rho_{xy_N} + \left(\frac{\sigma_{x_N}\mu_{y_N}}{\sigma_{y_N}\mu_{x_N}}\right)^2 \right\} \\ +\sigma_{u_N}^2 \left\{ 1 - 2\left(\frac{\sigma_{v_N}\mu_{y_N}}{\sigma_{u_N}\mu_{x_N}}\right)\rho_{uv_N} + \left(\frac{\sigma_{v_N}\mu_{y_N}}{\sigma_{u_N}\mu_{x_N}}\right)^2 \right\} \end{array} \right]. \tag{4.2}$$

3. Contrasting the minimum MSE of the proposed neutrosophic estimator T_p with the neutrosophic regression estimator T_{lr}, results in:

$$min.MSE(T_p) < min.MSE(T_{lr})$$

$$\mu_{y_N}^2 (1 - \Gamma) < \mu_{y_N}^2 \gamma \left\{ \left(\frac{\sigma_{y_N}^2 + \sigma_{u_N}^2}{\mu_{y_N}^2}\right) - \frac{(\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N})^2}{\mu_{y_N}^2(\sigma_{x_N}^2 + \sigma_{v_N}^2)} \right\}$$

$$\implies \Gamma > 1 - \gamma \left\{ \left(\frac{\sigma_{y_N}^2 + \sigma_{u_N}^2}{\mu_{y_N}^2}\right) - \frac{(\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} + \rho_{uv_N}\sigma_{u_N}\sigma_{v_N})^2}{\mu_{y_N}^2(\sigma_{x_N}^2 + \sigma_{v_N}^2)} \right\}. \tag{4.3}$$

4. Contrasting the minimum MSE of the proposed neutrosophic estimator T_p with the neutrosophic estimator T_{slr}, results in:

$$
\begin{aligned}
min.MSE(T_p) &< min.MSE(T_{slr}) \\
\mu_{y_N}^2(1-\Gamma) &< \mu_{y_N}^2(1-\alpha_{1(opt)}) \\
\Longrightarrow \Gamma &> \alpha_{1(opt)}.
\end{aligned}
\tag{4.4}
$$

5. Contrasting the minimum MSE of the proposed neutrosophic estimator T_p with the estimator T_{spr}, results in:

$$
\begin{aligned}
min.MSE(T_p) &< min.MSE(T_{spr}) \\
\mu_{y_N}^2(1-\Gamma) &< \mu_{y_N}^2\left(1-\frac{b^2}{a}\right) \\
\Longrightarrow \Gamma &> \frac{b^2}{a}.
\end{aligned}
\tag{4.5}
$$

The proposed strategy will outperform the adapted ones if the above optimality criteria are met.

5. SIMULATION STUDY

To evaluate the robustness and accuracy of the proposed neutrosophic estimator under CME, we conduct an extensive Monte Carlo simulation utilizing an artificially generated population. The primary objective was to evaluate the performance of the proposed neutrosophic estimator across varying levels of correlation structures and ME. The simulation design proceeds as follows:

(i). Use a four variate multivariate normal distribution to generate $N = 1000$ units artificially on X_N, Y_N, u_N, and v_N with a mean vector $\mu_z = (\mu_{x_N}, \mu_{y_N}, 0, 0)'$ and a covariant matrix as

$$
\begin{pmatrix}
\sigma_{x_N}^2 & \rho_{xy_N}\sigma_{x_N}\sigma_{y_N} & 0 & 0 \\
\rho_{xy_N}\sigma_{x_N}\sigma_{y_N} & \sigma_{y_N}^2 & 0 & 0 \\
0 & 0 & \sigma_{u_N}^2 & \rho_{uv_N}\sigma_{u_N}\sigma_{v_N} \\
0 & 0 & \rho_{uv_N}\sigma_{u_N}\sigma_{v_N} & \sigma_{v_N}^2
\end{pmatrix}.
$$

The parameters used are as follows: $\mu_{x_N} = [15,\ 20]$, $\mu_{y_N} = [25,\ 30]$, study-auxiliary variable's correlations $\rho_{xy_N} = ([0, 0.1], [0.45, 0.5], [0.85, 0.9])$; ME correlations $\rho_{uv_N} = ([0.0, 0.0], [0.25, 0.3], [0.45, 0.5], [0.65, 0.7], [0.85, 0.9])$; with different combinations of standard deviations given in Tables 1–8.

(ii). From the above population, simple random samples of size $n = 100$ were drawn without replacement.

(iii). For 10,000 replications, the neutrosophic percent relative efficiency (PRE_N) of the estimator T with respect to the neutrosophic mean estimator $\bar{y}_N$ was computed utilizing:

$$
PRE_N = \frac{\sum_{i=1}^{10,000}(\bar{y}_N^{(i)} - \mu_{y_N})^2}{\sum_{i=1}^{10,000}(T^{(i)} - \mu_{y_N})^2}.
$$

(iv). Simulations were conducted under multiple combinations of variance structures, correlation patterns, and different levels of MEs (5%, 10%, 15%, and 20%).

The simulation findings are provided in Tables 1–8. These findings are extensively interpreted in next section.

5.1. Interpretation of simulation findings

The simulation study is designed into two phases. The primary phase explores the impact of different CME structures, whereas the secondary phase evaluates the performance of the estimators under different levels of ME intensity (5%, 10%, 15%, and 20%).

1. The results of Table 1 are based on $\sigma_{x_N} = [25,\ 30]$, $\sigma_{y_N} = [25,\ 30]$, with different combinations of σ_{u_N}, σ_{v_N}, varying study-auxiliary correlation ρ_{xy_N}, and ME correlation ρ_{uv_N}. From these results, we observe that when $\sigma_{u_N} = [15,\ 20]$, $\sigma_{v_N} = [15,\ 20]$:

   - The simulation outcomes reveal a consistent superiority of the proposed neutrosophic estimator T_p over the adapted neutrosophic estimators such as neutrosophic mean estimator $\bar{y}_N$, neutrosophic ratio estimator T_r, neutrosophic regression estimator T_{lr}, Searls-type neutrosophic regression and ratio estimators T_{slr} and T_{spr} in the presence of CME. At lower levels of correlation between the study and auxiliary variables ($\rho_{xy_N} = [0, 0.1]$) and in the absence of CME ($\rho_{uv_N} = [0.0, 0.0]$), the PRE of the proposed neutrosophic estimator T_p is already marginally higher than the neutrosophic regression estimator (T_{lr}), and Searls-type neutrsophic regression and ratio estimators ($T_{slr},\ T_{spr}$), and substantially greater than the neutrosophic ratio estimator (T_r). This dominance highlights that even under weak association structures and uncorrelated MEs, the optimized construction of T_p offers robustness and reduced MSE.
   - As ρ_{uv_N} increases gradually from [0.0, 0.0] to [0.85, 0.9], the performance gap between the proposed neutrosophic estimator T_p and the adapted neutrosophic estimators widens. For example, while T_r only improves modestly with increasing CME correlation, T_p shows marked gains, with PRE_N escalating from approximately [113.6, 117.7] at $\rho_{uv_N} = [0.0, 0.0]$ to [121.2, 131.8] at $\rho_{uv_N} = [0.85, 0.9]$. This trend validates the theoretical expectation that the optimized weights in T_p successfully accommodate the joint influence of the study-auxiliary correlation ρ_{xy_N} and ME correlation ρ_{uv_N}, producing greater efficiency under complex error structures.
   - At moderate to high study-auxiliary correlation levels ($\rho_{xy_N} = [0.45, 0.5]$ and $\rho_{xy_N} = [0.85, 0.9]$), the advantage of T_p becomes even more prominent. Under these settings, adapted neutrosophic estimators also gain efficiency, yet their improvements plateau relative to T_p. Notably, the PRE of T_p surpasses 170 in many high-correlation-high-CME configurations, while reaching peaks above 190 when $\rho_{xy_N} = [0.85, 0.9]$ and $\rho_{uv_N} = [0.85, 0.9]$, outperforming all adapted neutrosophic estimators by a significant margin. This not only shows numerical stability but

also establishes the robustness of the proposed estimator T_p against simultaneous strengthening of the study-auxiliary correlation ρ_{xy_N} and ME correlation ρ_{uv_N}.

- The similar trend in the PRE_N of the adapted and proposed neutrosophic estimators can be observed from the other combinations σ_{u_N} and σ_{v_N}.

2. The performance and trend of the adapted and proposed neutrosophic estimators observed from Table 1, can also be observed from Tables 2–4 for different combinations of σ_{x_N}, σ_{y_N} with different combinations of σ_{u_N}, σ_{v_N}, varying study-auxiliary correlation ρ_{xy_N}, and ME correlation ρ_{uv_N}.

3. The simulation results reported in Table 5 offer an extensive assessment of the performance of the proposed neutrosophic estimator T_p at varying levels of MEs (5%, 10%, 15%, and 20%). From these results, we observe that:

   - At the lowest level of ME (5%), T_p demonstrates remarkable robustness. Even when the correlation between MEs (ρ_{uv_N}) is minimal, the PRE_N of T_p is substantially higher than that of the adapted neutrosophic ratio estimator (T_r), neutrosophic regression estimator (T_{lr}), and Searls-type neutrosophic regression and power ratio estimators (T_{slr}, T_{spr}). As ρ_{uv_N} increases from 0.0 to 0.9, the relative gain in efficiency for T_p becomes more pronounced, reflecting its ability to successfully exploit the correlation structure of the variables under indeterminacy in the data.
   - As the level of ME rises to 10%, 15%, and 20%, the PRE_N of all estimators decreases under uncorrelated errors, which is consistent with the expected loss of efficiency under larger error variances. However, unlike the competing estimators, T_p is less affected by this efficiency deterioration. In fact, the efficiency gap between T_p and the adapted estimators widens as ME increases, highlighting the optimized structure of the proposed strategy in reducing the MSE.
   - The impact of CME is particularly evident when comparing scenarios with $\rho_{uv_N} = 0$ and $\rho_{uv_N} = 0.85$. The PRE_N of T_p exhibits significant improvement as the correlation among errors (ρ_{uv_N}) increases, whereas the adapted estimators display relatively smaller gains. This reinforces that T_p not only adjusts effectively to the correlation between the true study and auxiliary variables (ρ_{xy_N}) but also captures the dependency structure in the error terms.
   - The numerical findings in Table 5 validate the optimality criteria derived earlier. The optimized weighting structure of T_p, along with its logarithmic adjustment and neutrosophic incorporation of uncertainty, collectively enhance the estimation accuracy. Thus, across all levels of ME and correlation levels, the proposed neutrosophic estimator consistently delivers the highest PRE_N values, establishing it as the most reliable and efficient strategy for population mean estimation in case of CME under uncertainty.

4. The performance and trend of the adapted and proposed neutrosophic estimators observed from Table 5, can also be observed from Tables 6–8 for different combinations of σ_{x_N}, σ_{y_N} with different % of ME, varying study-auxiliary correlation ρ_{xy_N}, and ME correlation ρ_{uv_N}.

Thus, the findings, presented in Tables 1–8, consistently show that the proposed neutrosophic estimator T_p achieves the highest PRE_N across all experimental conditions. In particular, T_p shows remarkable stability and efficiency gains when CME is present, outperforming traditional neutrosophic ratio, regression, Searls-type regression, and Searls-type power ratio estimators. Moreover, the efficiency of T_p improves with stronger correlations between study and auxiliary variables and remains robust even as the correlation among MEs increases.

These findings validate the optimality criteria derived earlier and confirm that explicitly accounting for CME under uncertainty substantially enhances performance of the estimator. The superiority of T_p is evident not only in low ME conditions but also under higher levels of ME, highlighting its practical relevance for survey sampling with CME under uncertain data.

6. REAL DATA ILLUSTRATION

The inclusion of real data illustration is essential to enhance the strength of the simulation study, as it shows how the proposed neutrosophic estimator under CME performs in practical settings. Real-world applications not only bridge the gap between theoretical findings and practical scenarios but also offer the validation of the simulation findings, thereby increasing the credibility of the research. Real data illustration offers concrete evidence of the estimator's accuracy, applicability, and robustness, fostering greater confidence in its utility. Taking this into considerations, we provide a real data illustration of both the adapted and the proposed neutrosophic estimators under CME utilizing a publicly available stock price dataset.

The stock price data inherently contains uncertainty due to market volatility, trading fluctuations, and external economic factors. Such variability makes the dataset well-suited for modeling under a neutrosophic framework to capture indeterminacy and uncertainty. Further, in stock price data, CMEs can arise when multiple reported values are systematically affected by same sources of bias or noise. For instance, errors may arise from delays in trade reporting, rounding practices, or data feed synchronization problems throughout different trading platforms. Since these errors often affect successive observations in the same way, they introduce correlation in the MEs over time. Additionally, algorithmic trading and market microstructure effects can create repeated distortions in recorded prices, reinforcing correlations in the error component of the data. Therefore, we choose the dataset pertains to the daily stock prices of "NVIDIA Corporation (NVDA)", accessible from `https://finance.yahoo.com/quote/NVDA/history/`. Specifically, the neutrosophic survey variable $y_N \in [y_L,\ y_U]$ represents the range of stock prices recorded each day from 15 May 2023 to 15 May 2024, while the neutrosophic auxiliary variable $x_N \in [x_L,\ x_U]$ corresponds to the range of stock prices recorded daily from 15 May 2022 to 15 May 2023. The descriptive statistics of this dataset are $N = [250,\ 250]$, $n_N = [20,\ 20]$, $\mu_{y_N} = [560.097,\ 580.313]$, $\mu_{x_N} = [182.255,\ 189.808]$, $\sigma_{y_N} = [180.737,\ 580.313]$, $\sigma_{x_N} = [49.447,\ 50.052]$, $\sigma_{u_N} = [18.073,\ 18.971]$, $\sigma_{v_N} = [4.944,\ 5.005]$, $\rho_{xy_N} = [0.904,\ 0.903]$, and $\rho_{uv_N} = [0.250,\ 0.780]$. Utilizing these descriptive statistics, we have calculated $PRE_N \in [PRE_L,\ PRE_U]$ for the neutrosophic estimators and reported the findings in Table 9. The PRE_N is calculated utilizing the following formula:

$$PRE_N = \frac{V(\bar{y}_N)}{MSE(T^*)} \times 100. \tag{6.1}$$

These findings exhibit that the PRE_N of the proposed estimator T_p is maximum against the PRE_N of the adapted estimators such as the neutrosophic ratio estimator T_r, neutrosophic regression estimator T_{lr}, neutrosophic Searls-type regression estimator T_{slr}, and neutrosophic Searls-type power ratio estimator T_{spr}.

7. CONCLUSION

This research work developed an uncertainty-based optimized sampling strategy for estimating the population mean in the presence of CME. By introducing neutrosophic statistics, the proposed estimator effectively handles uncertainty and reduces the adverse impacts of CME, which are often ignored in the traditional methods. The theoretical derivations of bias and MSE confirmed the superiority of the proposed estimator over adapted methods. Further validation through extensive simulation experiments and real data illustrations demonstrated that the proposed strategy consistently achieves higher accuracy, robustness, and stability across varying correlation levels and ME structures. The findings highlighted the practical significance of integrating uncertainty with optimized strategies, offering survey practitioners a reliable method for accurate and efficient estimation in real-world situations where data are inherently uncertain and prone to CME.

Moreover, the proposed strategy, although theoretically efficient, based on the availability or accurate estimation of several population parameters such as variances, correlations, and ME components, which may not always be readily accessible in practical surveys. From a computational perspective, the estimator involves optimization of multiple parameters and evaluation of complex expressions, which can increase computational burden, particularly for large-scale datasets or repeated simulations. However, this complexity remains manageable with modern computational tools.

In forthcoming studies, this article may be extended under different sampling designs, incorporation of non-response and missing data mechanisms, to further broaden its applicability.

Data availability statement

The data were taken from publicly available website `https://finance.yahoo.com/quote/NVDA/history/`. Necessary data descriptions are detailed within the article.

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Appendix A. Supplementary file

Supplementary data (Tables 1–9) to this article can be found online.